Problem set chapter 5, Due Wednesday, February 8

This problem deals with the vibrations of the two-dimensional gas-atom surface. A monolayer of gas atoms is deposited on an atomically perfect surface. Consider first a model in which the effect of the surface is simply to constrain the atoms to move in the $z = 0$ plane. The atoms form a square lattice (with $a = 3\mathring{A}$), and for small $\mathbf{k} = (k_x, k_y)$, the equations of motion give

$$-M\omega^2 e_i(\mathbf{k}) = -A(k_x^2 + k_y^2)e_i(\mathbf{k}) \quad i = x, y$$

where $M = 6.7 \times 10^{-23}$ gm, and $A = 6.7 \times 10^{-12}$ gmcm^2/sec^2

a. Find the normalized density of states (frequencies) per unit frequency near $\omega = 0$.

b. Give an expression for the low temperature specific heat as a function of temperature. What is the specific heat at 100K?

c. Is the Debye-Waller factor finite or zero and why?

Now account for the potential of the surface, ie. allow for the corrugation of the surface by adding a potential energy

$$\phi = \frac{K}{2}\sum_{n,i} s_{ni}^2$$

where s is, as usual, the displacement from equilibrium, and $K = 6.7 \times 10^4$ gm/sec^2. The wave solutions are of the form

$$s_{ni} = e_{ni}e^{(i\mathbf{k}\cdot\mathbf{r}_n - \omega t)}$$

d. What are the new frequencies for $\mathbf{k} = 0$?

e. What is the form of the temperature dependence of the specific heat neat $T = 0$?

f. Is the Debye-Waller factor finite or zero and why?