

Basic Physical Processes and Principles of Free Electron Lasers (FELs)

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Abstract

A tunable coherent and high peak power sources have been very usefull in many applications. In this paper the physical process of new type of coherent, flexible and very bright radiation source called Free-Electron Lasers (FEL_s) is described. The motion of electron beam in the presence of the optical and undulator fields leads to energy exchange. This energy exchange leads to formation of bunches of electrons which will emit very bright radiation. The resonance wavelength can, in principle, be tuned to any radiation regions. This is possible by changing the undulator parameter and/or electron beam energy. The line-shape of the emitted spectrum has the characteristic form of the short time emission process. Different regions of the emitted radiation have found very wide applications.

1 Introduction

The Free electron lasers (FEL_s) was invented by by J.M.J. Madey in 1971[5]. Free Electron Lasers are very flexibile sources of coherent radiation. Due to their wide range tunabilty and high brightness they have a growing applications. The FEL_s system consists of an electron accelerator, an undulator in which the electrons emit the synchrotron radiation, and an optical resonator.

2 Principles of operation of FEL

In an FEL a beam of relativistic electrons produced by an accelerator passess through a transverse periodic field produced by a magnet called undulator and exchanges energy with an electromagnetic field (Fig. 1). As a result of energy exchange, the electrons that gain

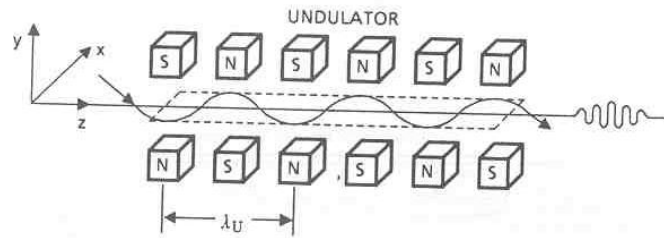


Figure 1: *Basic components of FEL [3].*

energy begin to move ahead of the average electron, while the electrons that lose energy begin to fall behind the average . The beam then becomes bunched on the scale of the radiation wavelength and this collective motion of the bunches radiatespowerful coherent synchrotron radiation. Giga watts peak power have been demonstarted. The wavelength of the emitted radiation at the resonance depends on the electron energy and the magnitude and periodicity of the undulator field according to the relation [1, 3]

$$\lambda_o = \frac{\lambda_u}{2\gamma^2}(1 + \kappa^2) \quad (1)$$

where λ_u is the undulator period, γ is the relativistic factor and κ is the so called undulator parameter which is proportional tothe magnetic field inside the undulator.

Typical values for electron beam enery ranges from few MeV ($\gamma \sim 1$) toa few GeV ($\gamma \sim 1000$); and the peak current varies from several ampersto many hundred of ampers. The peak undulator field strength B_o is usually several KG ($B_o \sim 2 - 7KG$). The undulator period λ_u is in the range of a few centimeters ($\lambda_u \sim 2 - 10cm$), whereas the undulator length $L_u = N\lambda_u$ extends for a few meters ($L_u \sim 2 - 20m$), so that the number of periods N can vary from 20 to several hundreds[1].

3 Basic Mechanism of FEL Interactions

The motion of the electron in the presence of the optical and the undulator field is governed by the Lorentz force equations [1, 3]:

$$\frac{d}{dt}(\gamma\vec{\beta}) = -\frac{e}{mc}(\vec{E} + \vec{\beta} \times (\vec{B}_u + \vec{\beta})) \quad (2)$$

$$\frac{d}{dt}\gamma = -\frac{e}{mc}\vec{\beta} \cdot \vec{E} \quad (3)$$

Where \vec{E} and \vec{B} are the electric and magnetic componenets of the e.m fields. Writting eqns.(2) and (3) interms of transverse and longitudinal components we obtain:

$$\frac{d}{dt}(\gamma\vec{\beta}_\perp) = \frac{e}{mc}(\vec{\beta}_z \times \vec{B}_u) - \frac{e}{mc}(1 - \beta_z)\vec{E} \quad (4)$$

$$\frac{d}{dt}[\gamma(1 - \beta_z)] = -\frac{e}{mc}(\vec{z} \times \vec{B}_u) \cdot \vec{\beta}_\perp \quad (5)$$

Where, $\vec{\beta}_z = \frac{1}{c} \frac{dZ}{dt} \hat{z}$. The first term on the right of eqn. (4) accounts for the interaction of the electrons with the undulator magnetic field ; whereas the the second term summerizes the effect of the transverse e.m fields,which combines almost to cancel out each other in the relativistic limit ($\beta_z \rightarrow 1$). Eqns. (4) and (5) desplay the complementary role of the undulator and optical fields in the FEL interaction.

Considering the ideal plane -polarized undulator magnetic field,

$$\vec{B}_u = \hat{y}B_o \sin k_u z, \quad k_u = \frac{2\pi}{\lambda_u}$$

and linearly ploarized emitted fields with a complex amplitude $E(\vec{r}, t) = |E(\vec{r}, t)| e^{i\phi(\vec{r}, t)}$, propagating along the z-direction,

$$\vec{E}(\vec{r}, t) = \text{Re}E(\vec{r}, t)e^{i(kz - \omega t)}\hat{x}, \quad \vec{B}(\vec{r}, t) = \hat{z} \times \vec{E} \quad (6)$$

eqns. (4)and (5) becomes, respectively:

$$\frac{d}{dt}(\gamma\beta_\perp) = -\hat{x}(\kappa \frac{d}{dt} \cos k_u z - K_R(\vec{r}, t)[\frac{d}{dt} \sin \Psi(\vec{r}, t) - \frac{d}{dt} \cos \Psi(\vec{r}, t)]) \quad (7)$$

$$\frac{d}{dt}\gamma = \frac{\omega}{\gamma}K_R[\kappa \cos k_u z \cos \Psi(\vec{r}, t) - K_R \sin \Psi(\vec{r}, t) \cos \Psi(\vec{r}, t)] \quad (8)$$

Assuming that the electron to experience a nearly constant optical field (over λ_u) eqns (7) and (8) can easily be integrated to get the longitudinal velocity ($\beta_z = [1 - \beta_\perp^2 - \frac{1}{\gamma^2}]^{1/2}$)

$$\beta_z = [1 - \frac{1}{\gamma^2}(1 + \kappa^2 \cos^2 k_u z + K_R^2 \sin^2 \Psi(\vec{r}, t) - 2\kappa K_R \cos k_u z \sin \Psi(\vec{r}, t))]^{1/2} \quad (9)$$

where,

$$\kappa = \frac{eB_o}{mc\omega_u}; \quad \text{and} \quad K_R(\vec{r}, t) = \frac{e |E(\vec{r}, t)|}{mc\omega}$$

are the undulator parameter and the dimenssionless optical vector potential, respectively; and

$$\Psi(\vec{r}, t) = (kz - \omega t) + \phi(\vec{r}, t)$$

is the phase angle combining of the fast and slow varying components of the e.m fields. Since usually $\kappa > K_R$ by the order of magnitude, the average value $\bar{\beta}_z(t)$ is given by:

$$\bar{\beta}_z(t) = 1 - \frac{1 + \kappa^2/2}{2\gamma^2(t)} \quad (10)$$

On the other hand, requiring $\vec{\beta}_\perp$ to be in phase with the optical field during the motion assures that the electron transverse velocity retains its orientation relative to the optical electric field, thus simplifying that the energy exchange persists in the same direction. The resonant condition demands therefore for vanishing the difference between the optical phase and the electron transverse velocity phase at the instantaneous electron position, thus providing the expression[1]

$$\omega - kc\bar{\beta}_z = \omega_u\bar{\beta}_z \quad (11)$$

For propagation in vacuum ($\omega = kc$) and using eqn. (10) in the above relation one finds the resonant frequency ω_o and wavelength λ_0 given by

$$\omega_0 = \frac{2\gamma_0^2\omega_u}{1 + \kappa^2/2}, \quad \text{and} \quad \lambda_0 = \frac{\lambda_u}{2\gamma_0^2} \left(1 + \frac{\kappa^2}{2}\right) \quad (12)$$

In general the electron phase in the combined optical and undulator fields is defined by

$$\xi(t) = kZ - (\omega - \omega_u)t = (k + k_u)\bar{Z} - \omega t \quad (13)$$

Introducing a dimensionless time $\tau = \frac{ct}{L_u}$, eqns. (10) and (12) gives us the parameter measuring the degree of resonanace (phase velocity):

$$v(\tau) = \frac{d}{d\tau}\xi(\tau) = 2\pi N \left[1 - \frac{1 + \kappa^2/2}{2\gamma^2(t)}\right] \frac{\omega_0(\tau) - \omega}{\omega_0(\tau)} \quad (14)$$

where, $\omega_0(\tau) = 2\frac{\gamma^2(\tau)\omega_u}{1 + \kappa^2/2}$ is the instantaneous resonance frequency.

4 Spectral characteristics of the Emmitted Radaitions

For relativistic electron moving on arbitrary curved trajectory, the electric field is given by the standard expression[2]:

$$\vec{E}(t) = e \left[\frac{\hat{n} - \beta}{R^2\gamma^2(1 - \hat{n} \cdot \beta)^3} \right]_{ret} + \frac{e}{c} \left[\frac{\hat{n} \times (\hat{n} - \beta) \times \dot{\beta}}{R(1 - \hat{n} \cdot \beta)^3} \right]_{ret} \quad (15)$$

Where the subscript ret means that the quantities in the square braket are evaluated at the retarded time $t' = t - R(t')$ and $R(t') = \vec{X} - \vec{r}(t')$ is the usual notation for observer position. Then the total energy detected during one particle passage per solid angle and per frequency, using the acceleration dependent electric field of eqn.(15)is:

$$\frac{d^2I}{d\Omega d\omega} = \frac{e^2}{4\pi^2c} \left| \int_{-\infty}^{\infty} \hat{n} \times (\hat{n} \times \beta) e^{i\omega(t' - \frac{\hat{n} \cdot \vec{r}(t')}{c})} dt' \right|^2 \quad (16)$$

In evaluating the the spectral characteristic equation we will keep contributions upto the order $(\kappa/\gamma)^2$ [5]. From Fig.2 and for $\psi \sim 1/\gamma$, $\hat{n} \cong (\psi \cos \phi, \psi \sin \phi, 1 - \frac{1}{2}\psi^2)$ then

$$[\hat{n} \times (\hat{n} \times \vec{\beta})]_z \cong \frac{\kappa}{\gamma} \psi \cos \phi \cos(\omega_u t) - \psi^2, \quad [\hat{n} \times (\hat{n} \times \vec{\beta})]_y \cong \psi \sin \phi,$$

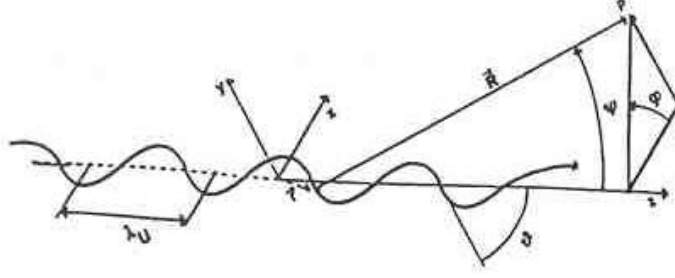


Figure 2: Coordinates and definitions of angles [5]

$$[\hat{n} \times (\hat{n} \times \vec{\beta})]_x \cong \psi \cos \phi + \frac{\kappa}{\gamma} \cos(\omega_u t) \quad (17)$$

The oscillating part in eqn.(16) can be written as:

$$e^{i\omega(t - \frac{\hat{n} \cdot \vec{r}}{c})} \cong e^{i\frac{\omega}{2\gamma^2}(1 + \frac{\kappa^2}{2} + \gamma^2\psi^2)t} \sum_{m=-\infty}^{\infty} e^{-im\omega_u t} J_m(\zeta_\omega, \xi_\omega) \quad (18)$$

with $\zeta_\omega = -\frac{\kappa\omega}{\gamma\omega_u}\psi \cos \phi$, $\xi_\omega = -\frac{\kappa^2}{8\gamma^2}\frac{\omega}{\omega_u}$.

We have denoted with $J_{m(x,y)}$ the generalized bessel function of first kind, whose generating function is provided by

$$\sum_{m=-\infty}^{\infty} t^m J_m(x,y) = e^{[\frac{x}{2}(t - \frac{1}{t}) + \frac{y}{2}(t^2 - \frac{1}{t^2})]}.$$

Using these results into our equation for spectral characteristics (eqn.16) we finally get:

$$\frac{d^2 I}{d\Omega d\omega} = \frac{4e^2 N^2 \gamma^4}{c} \sum_{m=1}^{\infty} \frac{m^2}{(1 + \kappa^2 + \gamma^2\psi^2)} [|S_m^{(1)}|^2 + |S_m^{(2)}|^2] \left(\frac{\sin(v_m/2)}{v_m/2} \right)^2 \quad (19)$$

where $v_m = 2\pi N(m - \frac{\omega}{\omega_1})$; $\omega_1 = \frac{2\gamma^2\omega_u}{(1 + \frac{\kappa^2}{2} + \gamma^2\psi^2)}$ and

$$S_m^{(1)} = \psi \cos \phi J_m(\zeta_\omega, \xi_\omega) + \frac{\kappa}{2\gamma} [J_{m-1}(\zeta_\omega, \xi_\omega) + J_{m+1}(\zeta_\omega, \xi_\omega)], S_m^{(2)} = \psi \sin \phi J_m(\zeta_\omega, \xi_\omega) \quad (20)$$

with the redefined quantities $\zeta_\omega = -m \frac{2\kappa\gamma\psi \cos \phi}{1 + \frac{\kappa^2}{2} + \gamma^2\psi^2}$, $\xi_\omega = -m \frac{\kappa^2}{4} \frac{1}{1 + \frac{\kappa^2}{2} + \gamma^2\psi^2}$

On the axis (i.e $\psi = 0$ or $\xi = \xi_{\psi=0}$) eqn.(19) reduces to:

$$\frac{d^2 I}{d\Omega d\omega} = \frac{e^2 N^2 \gamma^2}{c} \sum_{m=1}^{\infty} F_m(\xi) \left[\frac{\sin(v_m/2)}{v_m/2} \right]^2 \quad (21)$$

where,

$$F_m(\xi) = \left(\frac{4m\xi}{\kappa} \right)^2 [J_{\frac{m-1}{2}}(m\xi) - J_{\frac{m+1}{2}}(m\xi)]^2 \quad (22)$$

Equations (19) and (21) shows that the line shape of the emitted spectrum has the characteristic form of short-time emission process. The beam emittance and energy spread produce a broadening of the line spectrum and a reduction of the peak power[1, 3].

5 Tunability and Applicability of FEL in the Different Radiation Regions

In section 2 we showed that the Optical wavelength is related to the the undulator period λ_u and to the beam energies according to:

$$\lambda_0 = \frac{\lambda_u}{2\gamma_0^2} \left(1 + \frac{\kappa^2}{2}\right), \quad \kappa = \frac{eB_o\lambda_u}{2\pi mc^2}$$

Tuning the operating wavelength is possible by adjusting the electron beam energy and /or the undulator parameters. In addition to its tunability FEL_s high peak power and flexible pulse structure makes FEL a very attractive device, justified by the growing number of potential application depending on the region of radiation.

The most compelling case for FLE facilities starts with the far infrared region (1000 to $10\mu m$). Commercial laboratory lasers are not effective in these region. This is the spectral region where molecule-surface vibrations, intermolecular cluster vibrations, etc. can be excited. Some of the areas wher this region is highly involved are: surface science, chemistry, solid state physics, Biophysics and plasma physics [5]. In Vacuum Ultraviolet and Extended Ultraviolet region (200 to 10 nm) laboratory lasers become increasingly inefficient. There are important scientific need for these region, including photodissociation and single photon ionization detection. The X-ray region for the FLE (100 to $1 A^\circ$). Because there is no existing x-ray FEL, studies involving coherent X-rays are totally unexplored. Applications of this region includes imaging, such as x-ray microscopy and holography [5].

6 Conclusion

Studing the interaction of a single particle in the presence of the undulator and optical fields gave us the very practical parameters of the the Free Electron Lasers, i.e the tuning parameters. The tuning parameters are found to be the electron beam energy and/ or the undulator parameters. In principle tuning of the FEL_s to any radiation region is possible. The flexibility of FEL_s and its high peak power has found a grouing applications. Currently most of the FEL_s are in the far infrared regions (1000 to $10\mu m$). However there is a high demand for the the Vacuum Ultraviolet and extended Ultraviolet region (200 to $10nm$); and the X-ray region (100 to $1A^\circ$).

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