Phys 4125 HW #7

Solutions

4.15

(a) $\text{COP} = \dfrac{\text{useful energy}}{\text{energy cost}} = \dfrac{Q_c}{Q_f}$

(b) $\Delta U = 0$ over cycle $\Rightarrow Q_f + Q_c = Q_r$, so COP can be anything

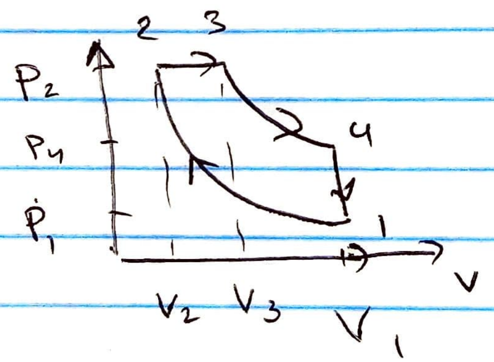
(c) $\Delta S \geq 0 \Rightarrow \dfrac{Q_r}{T_r} \geq \dfrac{Q_c}{T_c} + \dfrac{Q_f}{T_f} \Rightarrow$

$$\Rightarrow \frac{Q_f + Q_c}{T_r} \geq \frac{Q_c}{T_c} + \frac{Q_f}{T_f} \Rightarrow Q_f\left[\frac{1}{T_r} - \frac{1}{T_f}\right] \geq Q_c\left(\frac{1}{T_c} - \frac{1}{T_r}\right)$$

from this $\dfrac{Q_c}{Q_f} \leq \dfrac{\frac{1}{T_f} - \frac{1}{T_r}}{\frac{1}{T_r} - \frac{1}{T_c}} = \dfrac{T_r T_c}{T_c - T_r} \, \dfrac{T_r - T_f}{T_f T_r} =$

$$\boxed{\text{COP} \leq \frac{T_c}{T_f} \, \frac{T_r - T_f}{T_c - T_r}}$$

4.20



$1 \to 2 \quad P_1 V_1^\gamma = P_2 V_2^\gamma$

$3 \to 4 \quad P_2 V_3^\gamma = P_4 V_1^\gamma$

adiabatic $\Rightarrow$ no heat

heat output $\quad Q_c = Q_{4\to1} = \dfrac{f}{2}(P_4 - P_1)V_1$

heat input $\quad Q_h = Q_{2\to3} = \dfrac{f}{2} Nk(T_3 - T_2)$

$$+ P_2(V_3 - V_2) =$$

$$= \left(1 + \frac{f}{2}\right) P_2 (V_3 - V_2)$$

Then

$$\frac{Q_c}{Q_h} = \frac{\frac{f}{2}(P_4 - P_1)V_1}{(1+\frac{f}{2})P_2(V_3-V_2)} = \frac{\frac{f}{2}P_2V_1\left(\left(\frac{V_3}{V_1}\right)^\gamma - \left(\frac{V_2}{V_1}\right)^\gamma\right)}{(1+\frac{f}{2})P_2(V_3-V_2)} =$$

$$= \frac{1}{\gamma}\frac{V_1}{V_3-V_2}\left(\left(\frac{V_3}{V_1}\right)^\gamma - \left(\frac{V_2}{V_1}\right)^\gamma\right) =$$

$$= \frac{1}{\gamma}\frac{V_1}{V_2}\frac{1}{V_3/V_2-1}\left(\frac{V_2}{V_1}\right)^\gamma\left(\left(\frac{V_3}{V_2}\right)^\gamma - 1\right) =$$

$$= \frac{1}{\gamma}\left(\frac{V_2}{V_1}\right)^{\gamma-1}\frac{(V_3/V_2)^\gamma - 1}{V_3/V_2 - 1}$$

For Otto cycle $\frac{Q_c}{Q_h} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$, so

we need to show that for $x = \frac{V_3}{V_2} > 1$

and $\gamma > 1$

$$\frac{1}{\gamma}\frac{x^\gamma - 1}{x-1} \text{ is always } \geq 1$$

Now $\frac{\partial}{\partial x}\left\{\frac{1}{\gamma}\frac{x^\gamma-1}{x-1}\right\} = \frac{1}{\gamma}\left\{\frac{\gamma x^{\gamma-1}}{x-1} - \frac{x^\gamma-1}{(x-1)^2}\right\}$

$$= \frac{\gamma x^{\gamma-1}(x-1) - x^\gamma + 1}{(x-1)^2} = \frac{(\gamma-1)x^\gamma - (\gamma-1)}{(x-1)^2} =$$

$$= \frac{(\gamma-1)(x^\gamma-1)}{(x-1)^2} \geq 0$$

so the function is always increasing. And for $x \to 1$ $\lim_{x\to 1}\frac{1}{\gamma}\frac{x^\gamma-1}{x-1} = \frac{1}{\gamma}\gamma = 1$ (L'Hopital) this means $\frac{1}{\gamma}\frac{x^\gamma - 1}{x-1} \geq 1 \quad \forall x \geq 1, \gamma > 1,$

so $\boxed{\epsilon_{otto} \geq \epsilon_{Diesel}}$.

Take $\gamma = 7/5$, and the values in book $\boxed{\epsilon \simeq 63\%}$

$\boxed{4.26}$ The text says to take $H_2 \simeq H_1$ in Eq.(4.12), then

$$W = Q_h - Q_c = (H_3 - H_1) - (H_4 - H_1) \simeq$$

(see text above Eq. 4.13) $\simeq H_3 - H_4 \simeq 3444 - 1824 = 1620 \text{ kJ/kg}$

to get 10^9 J/sec need $\boxed{\frac{10^9}{1.62 \cdot 10^6} \simeq 620 \text{ kg}}$

but this is kg/steam → very large volume.

$\boxed{4.35}$ (a) Distance 1nm ~ 10 Å, so

$$\left[B = \frac{\mu_0}{4\pi}\frac{\mu}{r^3} = 10^{-7}\frac{9 \cdot 10^{-24}}{10^{-27}} \simeq 9 \cdot 10^{-4} T\right]$$

(b) started with 1T, drop to Earth's field 0.4 Gs $\simeq 4 \cdot 10^{-5}$ T, so would have to drop temperature by factor of 10^5. In reality probably some stray fields too.

(c) always $kT \sim \mu B$, so $T \sim 10^{-5}$ K

(d) If $\frac{\partial S}{\partial T}$ is small $\Rightarrow$ $C = T\frac{\partial S}{\partial T} = \frac{\delta Q}{\Delta T}$ is small, so small δQ gives large ΔT.

$\boxed{4.36}$ Photon $p = \frac{h}{\lambda}$, so $\Delta p_{atom} \sim p$, so

$$K \sim \frac{p^2}{2m} \sim kT \Rightarrow T \simeq \frac{h^2}{m\lambda^2 k} \simeq 5 \cdot 10^{-6}\,K$$

$\boxed{5.6}$ (a) glucose $+ 6O_2 \to 6CO_2 + 6H_2O$

$\Delta H = -2803$ kJ/mole $\Delta G = -2879$ kJ/mole

(b) assuming P, T const $\Rightarrow$
ΔG, i.e. 2879 kJ

(c) see eq. (5.12) $\Delta G = \Delta H - T\Delta S = \Delta H - Q \Rightarrow$

$\Rightarrow Q = \Delta H - \Delta G = 76$ kJ/mole

(d) $\Delta S_{products} > \Delta S_{reactants}$, hence heat can flow in

(e) not ideal, means creating more heat/entropy, but H/G are functions of state, so depend only on the initial & final states $\Rightarrow$ remain the same.

$\boxed{5.7}$

work of myosin $W \simeq 4 \cdot 10^{-12} \cdot 1.1 \cdot 10^{-8} \simeq$

$\simeq 4.4 \cdot 10^{-20} J$

per ATP.

We have 38 ATP for glucose, so

$$Q = \frac{\Delta G}{N_A \cdot 38} = \frac{2879 \cdot 10^3}{6 \cdot 10^{23} \cdot 38} \simeq 1.25 \cdot 10^{-19} J$$

so $$\boxed{\frac{W}{Q} \simeq \frac{0.44}{1.25} = 35\%}$$

$\boxed{5.18}$

Brick fell, so its potential energy decreased. As it lands, initially this energy is transferred into lattice vibrations. Then those are given in heat exchange to the air - large reservoir. So the energy of the brick decreased, and the excess energy increased the entropy of air.

$\boxed{5.23}$

(a) $\Phi = U - TS - \mu N$, so

$$d\Phi = dU - TdS - SdT - \mu dN - Nd\mu$$

$$= (TdS - PdV + \mu dN) - TdS - SdT - \mu dN - Nd\mu$$

$$= - SdT - Nd\mu - PdV, \text{ so } \Phi(T, V, \mu)$$

$$\left(\frac{\partial \Phi}{\partial T}\right)_{\mu, V} = -S; \left(\frac{\partial \Phi}{\partial \mu}\right)_{V, T} = -N; \left(\frac{\partial \Phi}{\partial V}\right)_{\mu, T} = -P$$

(b) see text and class notes. Assume V fixed, exchange particles and energy with reservoir R : $dS_t = dS + dS_R$

$$dS_R = \frac{dU_R - \mu dN_R}{T} = -\frac{dU}{T} + \frac{\mu dN}{T}$$

so $$dS_t = dS - \frac{dU}{T} + \frac{\mu dN}{T} =$$

$$= -\frac{1}{T}\left(dU - TdS - \mu dN\right) = -\frac{d\Phi}{T}$$

$dS_t \geq 0$ means $d\Phi \leq 0$.

(c) see discussion of $G = N\mu$. T, μ intensive, so V is the only extensive variable

$P = -\left(\frac{\partial \Phi}{\partial V}\right)_{T,\mu}$, but increasing V does not change T, μ $\Rightarrow$ $\boxed{\Phi = -PV}$

(d) Unoccupied : $U = N = S = 0$ no states,

Occupied: 1 state $\Rightarrow$ $S = k\ln 1 = 0$

$U = -U_0 = -13.6\,eV$

$$\mu = -kT\ln\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right], \text{ so}$$

$$\Phi = -U_0 + kT\ln\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right] \simeq -4.5\,eV$$

so occupied state is stable.

Comparable @ $T \simeq \Delta U / k\ln(\ ) \simeq 9 \cdot 10^3 K$