Phys 4125 - 2020

HW # 3 Solutions

#1.49 Text says enthalpy change is 286 kJ
Work done by atmosphere : constant pressure
$P = 10^5$ Pa $\Rightarrow$ $P(V_i - V_f) \simeq PV_i$ (volume of 1 mole of water ~ 18 ml, volume of gas 22.4 l/mole;
so $W \simeq PV_{gas} \simeq nRT = 1.5 \cdot 8.3 \cdot 300 \simeq 3.7$ kJ
(1.5 since we have 1 mole of H_2 and $\frac{1}{2}$ mole of O_2)
so $W \simeq 3.7$ kJ and $\Delta U \simeq 282$ kJ.

#1.54 a) $W = mgh \simeq 60 \cdot 9.8 \cdot 1500 \simeq 900\,kJ \simeq \cancel{205}\;215$ kcal
(assuming 4.2 J/cal).
Efficiency 25% $\Rightarrow$ need 860 kcal
so around 8.5 bowls of cereal
b) our body mostly water $\Rightarrow$
$\Rightarrow C \simeq 60 \cdot 1\,kcal/K \simeq 60$ kcal/K

need to absorb $0.75 \cdot 860 = 645$ kcal, so
$\Delta T \simeq \frac{645}{60} \simeq 11°C$: certain death.

c) Evaporation energy : ~580 kcal/kg
@ 25°C, so needs to evaporate just over 1 kg $\simeq$ 1.1 l.

#1.55

a) $U = mv^2$ ~~at~~ $V = -\frac{Gm^2}{2r}$

since in the center of mass frame the distance between objects is $2r$, they both orbit CM.

Radius of the orbit and velocity are related

$$m\frac{v^2}{r} = Gm^2/4r^2 \Rightarrow$$

$$\Rightarrow mv^2 = \frac{Gm^2}{4r}, \text{ so}$$

$$\left[U + V = \frac{Gm^2}{4r} - \frac{Gm^2}{2r} = -\frac{Gm^2}{4r} = -\boxed{U} \right]$$

~~a)~~ so $\boxed{V = -2U}$

b) We just saw the answer $U_{tot} = -U$, so increasing total energy decreases kinetic energy. The average separation r increases as the total energy grows which decreases v^2, see eqns above.

c) If we believe the kinetic energy $U = \frac{3}{2}NkT$, then $U_{total} = -\frac{3}{2}NkT$, so

$$C = \left(\frac{\partial U_{total}}{\partial T}\right)_N = -\frac{3}{2}Nk \quad \text{negative}$$

d) We only can have G, M, and R in the expression for energy, and we know from mechanics GM^2/R has the right units.

e) Let's assume

$$U = \frac{3}{2}NkT \simeq -\frac{1}{2}V \simeq \frac{GM^2}{2R}, \text{ so}$$

(note R is R_{sun}, not $R = kN_A$)

$$T \simeq \frac{GM^2}{3kRN}$$

Taking table values and assuming sun is mostly hydrogen, so $M/N \simeq 1.7\cdot10^{-27}$ kg, we find $\boxed{T \simeq 4\cdot10^{6}\ K}$

$\boxed{\#1.63}$

$$\ell \simeq \frac{kT}{P}\frac{1}{4\pi r^2} \quad \text{(see class), so}$$

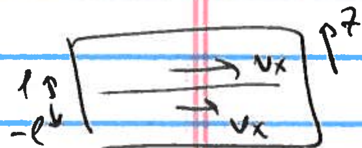
$$P = \frac{1}{4\pi r^2}\frac{kT}{\ell}$$

(note solution manual has $\frac{1}{2\pi r^2}$)

$$\text{so} \quad P \simeq \frac{1}{4\pi\cdot4\cdot10^{-20}\,m^2}\,\frac{1.38\cdot10^{-23}\cdot300}{10^{-1}} \simeq$$

$$\boxed{\simeq 0.08\,Pa \simeq 10^{-6}\,atm}$$

$\boxed{\#1.66}$ as discussed in class, viscosity is the transfer of momentum



$$\Delta p \simeq \frac{1}{2}\tilde{N}m\{v_x(-\ell) - v_x(\ell)\} \text{ s}$$

$$\simeq \frac{1}{2}\tilde{N}m\frac{\partial v_x}{\partial z}\ell, \text{ so}$$

$$F = \frac{\Delta p}{\Delta t}, \quad \text{assume } \Delta t \sim \ell/\bar{v}, \text{ so}$$

$$F \simeq \frac{1}{2}\tilde{N} m \bar{v} \frac{\partial v_x}{\partial z}$$

According to Eq. (1.69) $\eta = \frac{1}{2}\tilde{N} m \bar{v}/A$, we know that $\tilde{N}$ is the number of molecules in a box $A \cdot \ell$ volume, so $\tilde{N} = nA\ell$, so

$$\left[\eta = \frac{1}{2} n m \bar{v} \ell = \frac{1}{2} \rho \bar{v} \ell\right]$$ since

density $\rho = nm$

We derived $\ell \simeq \frac{1}{4\pi r^2}\frac{V}{N}$, and

density $\rho = \frac{Nm}{V}$, (here N is the # of molecules), so

$$\rho\ell = \frac{m}{4\pi r^2}$$: depends on molecular properties only, then

$$\eta \sim \bar{v} \sim \sqrt{T}$$

Air @ room T $\rho = \frac{29g}{22.4\ell} \sim 1.3 \, g/\ell = 1.3 \, kg/m^3$

so $\left[\eta \simeq 1 \, kg/m^3 \cdot 10^{-7} m \cdot 500 \, m/s \simeq \right.$

$$\left. \simeq 5 \cdot 10^{-5} \, Pa \cdot s. \right]$$

#2.2

(a) $2^{20} \simeq (2^{10})^2 \sim 10^6$

(b) This is one microstate: all are equally likely $\Rightarrow \frac{1}{2^{20}} \sim 10^{-6}$

(c) $\frac{20!}{8!\,12!} = \frac{20\cdot 19\cdot 18\cdot 17\cdot \cancel{16}\cdot \cancel{15}\cdot 14\cdot 13}{1\cdot \cancel{2}\cdot \cancel{3}\cdot 4\cdot \cancel{5}\cdot 6\cdot 7\cdot \cancel{8}} =$

$= \frac{20\cdot 19\cdot 18\cdot 17\cdot 14\cdot 13}{4\cdot 6\cdot 7} = 5\cdot 19\cdot 3\cdot 17\cdot 2\cdot 13 =$

$= 26 \times 51 \times 95 = 125{,}970$

so we have $\frac{125{,}970}{10^6} \simeq 12\%$

#2.3

(a) $2^{50} \simeq (2^{10})^5 \simeq 10^{15}$

(b) $\frac{50!}{(25!)^2}$ $\Omega(25) \simeq 10^{14}$ (I used Stirling)

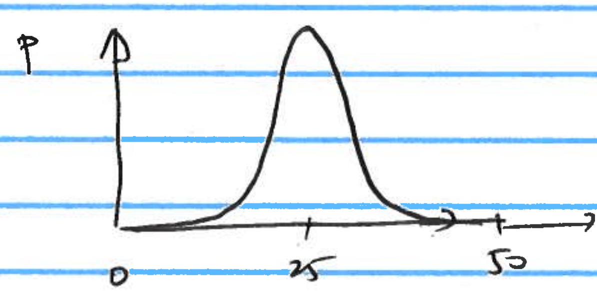
(c) $P(25) \simeq 10^{-1}$

(d) $P(30) \simeq 5\cdot 10^{-2}$

(e) $P(40) \simeq 10^{-6}$

(f) $P(50) = \frac{1}{2^{50}} \simeq 10^{-15}$

(g)



#2.4

52 cards, select 5 in any order (6)

$$\frac{52!}{5!\,47!} \simeq 3 \cdot 10^6$$

Royal flush : 4 choices (each suit), so

$$\boxed{P \simeq \frac{4}{3} \cdot 10^{-6} \simeq 1.3 \cdot 10^{-6}}$$

#2.5

(a) $N = 3$, $q = 4$

400	310	220	130	211
040	301	202	103	121
004	031	022	013	112

$$15 \text{ states} = \frac{(4+3-1)!}{4!\,2!} = \frac{6 \cdot 5}{2} = 15$$

(b) $N = 3$ $q = 5$

500	410	320	230	140	050
	041	032	023	014	
005	401	302	203	104	

(15)

311	131	113
221	212	122

(6)

$$21 \text{ state} = \frac{7!}{5!\,2!} = \frac{7 \cdot 6}{2} = 21$$

(c) $N = 3$ $q = 6$: 28, can check.

(d) $N=4 \quad q=2$

$$\left.\begin{array}{llll} 2000 & 0200 & 0020 & 0002 \\ 1100 & 1010 & 1001 & \\ 0110 & 0101 & & \\ 0011 & & & \end{array}\right\} 10$$

(e) $N=4 \quad q=3$: 20 total

(f) $N=1$: all q in state we have.

so 1 possible state

(g) $q=1$: any of N selections

N choices

2.6

$$\Omega(30,30) = \frac{59!}{30!\,29!} \simeq 6 \cdot 10^{16}$$

(I used Stirling).