Phys 4125 HW#2 — S2020

Solutions

#1.23

He is monoatomic $\Rightarrow U = \frac{3}{2} NkT = \frac{3}{2} PV$

so

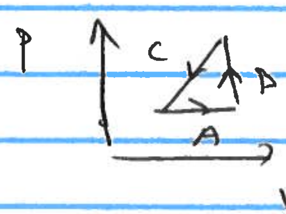
$$U = \frac{3}{2} \cdot 10^5 \cdot 10^{-3} = 150\,\mathrm{J}$$

#1.27

1) Temperature increase, no heat added: compress air pumping tire, pump gets hot.

2) Heating with no change in T: boiling water, discussed in class

#1.33



(a) work done on gas

$-\int P\,dV$

+ for C

− for A

0 for B

(b) internal energy $U = \frac{f}{2} PV$, so increases when either P or V increased

+ for A

+ for B

− for C

(c) heat $Q = \Delta U - W$, so obviously

Q: + for A

+ for B

− for C

whole cycle W ⊕

ΔU 0 → any cycle

$Q = ⊖$ → conservation law.

$\boxed{\# 1.36}$

(a) $PV^\gamma = \text{const} \Rightarrow$

$$\Rightarrow V_f = \left(\frac{P_i}{P_f}\right)^{1/\gamma} V_i = \left(\frac{P_i}{P_f}\right)^{5/7} V_i \simeq \frac{1}{4}\,\ell$$

(b)

$$W = -\int P\,dV = -\int_{V_i}^{V_f} \frac{P_i V_i^\gamma}{V^\gamma}\,dV =$$

$$= P_i V_i^\gamma \frac{1}{1-\gamma}\left\{\frac{1}{V_f^{\gamma-1}} - \frac{1}{V_i^{\gamma-1}}\right\} =$$

$$= \frac{P_i V_i}{\gamma-1} \cdot \left\{\left(\frac{V_i}{V_f}\right)^{\gamma-1} - 1\right\} \simeq 190\ \text{J}$$

(c) $V T^{5/2} = \text{const}$, so

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{2/f} = 300 \cdot (4)^{2/5} \simeq 500\ \text{K}$$

$\boxed{\# 1.39}$

(a) $T = \text{const}$ $\left(\frac{\partial P}{\partial V}\right)_T = -\frac{NkT}{V^2}$, so

$$\left[B = -V\left(\frac{\partial P}{\partial V}\right)_T = \frac{NkT}{V} = P\right]$$

<u>adiabatic</u>: $PV^\gamma = \text{const} \Rightarrow P = \frac{c}{V^\gamma}$, so

$$\left(\frac{\partial P}{\partial V}\right)_{ad} = -\gamma \frac{c}{V^{\gamma+1}} \Rightarrow$$

$$\Rightarrow \left[B = -V\left(\frac{\partial P}{\partial V}\right)_{ad} = +\gamma \frac{c}{V^\gamma} = \gamma P\right]$$

(b) Presumably if you compress air locally, heat does not have time to travel in/out, so $B = \gamma P$ is a better choice. Thermalization is slow

(c) $\rho = \frac{Nm}{V} = \frac{mP}{kT}$, so

$$\left[C_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P kT}{mP}} = \sqrt{\gamma \frac{RT}{M}} \right]$$

so $\left[\frac{C_s}{\bar{v}} \sim \sqrt{\frac{\gamma}{3}} \sim 1 \right]$ $\quad 1 < \gamma < \frac{5}{3}$

air @ room T

$$C_s = \sqrt{\frac{7}{5} \cdot \frac{8.3 \cdot 300}{3 \cdot 10^{-2}}} \simeq 330 \, m/s$$

(d) C_s depends on T, not P by itself. So @ high altitude it is colder, but the altitude does not matter.

$\boxed{\#1.42}$ We have M_{H_2O} : mass water

M_p : mass pasta

$T_b = 100°C$

T_i = room T

T_f = temperature @ end

water cools $\Rightarrow$ released Q goes to pasta

$$M_{H_2O} C_{H_2O} (T_B - T_f) = M_p C_p (T_f - T_i), \text{ so}$$

$$T_f = \frac{M_{H_2O} C_{H_2O} T_B + M_p C_p T_i}{M_{H_2O} C_{H_2O} + M_p C_p} \simeq 94°C$$

$\boxed{\#1.43}$

1 mole = 18g $\quad C_{mole} = 4.2 \cdot 18 = 75.6 \ J/_{mole\,k}$

so per molecule

$$C = \frac{C_{mole}}{N_A} = \frac{75.6}{6 \cdot 10^{23}} = 12.6 \cdot 10^{-23} \ J/k$$

Then assume $C = \frac{f}{2} k$, so

$$\left[f = \frac{2C}{k} = \frac{2.5 \cdot 10^{-22}}{1.38 \cdot 10^{-23}} \simeq 18 \right]$$

so we have non-quadratic part.

$\boxed{\#1.45}$

a) $W = xy \qquad x = yz \Rightarrow W = \frac{x^2}{z}\ ;\ W = y^2 z$

b) $\left(\frac{\partial W}{\partial x}\right)_y = y \qquad \left(\frac{\partial W}{\partial x}\right)_z = \frac{2x}{z}$

c) $\left(\frac{\partial W}{\partial y}\right)_x = x \qquad \left(\frac{\partial W}{\partial y}\right)_z = 2yz = 2x$

$\left(\frac{\partial W}{\partial z}\right)_x = -\frac{x^2}{z^2} \qquad \left(\frac{\partial W}{\partial z}\right)_y = y^2 = \frac{x^2}{z^2}$

#1.47 Want $Q = M c_w (T_b - T_f)$ removed

Need to a) warm ice to melting

$$Q_1 = M_{ice} C_{ice} (T_i - T_m)$$

$$T_m = 0^\circ C \quad : \text{melting}$$

b) melt

$$Q_2 = M_{ice} \cdot L_{ice}$$

c) warm water

$$Q_3 = M_{ice} C_w (T_f - T_m)$$

so

$$M_{ice} \left(C_w (T_f - T_m) + L_{ice} + C_{ice} (T_i - T_m) \right) =$$

$$= M_w C_w (T_b - T_f), \text{ so}$$

$$M_{ice} = \frac{M_w C_w (T_b - T_f)}{C_w (T_f - T_m) + L_{ice} + C_{ice} (T_i - T_m)} =$$

$$\simeq 45\,g$$