Phys 4125 HW #1 S2020

Solutions

#1.2 a) °R to °F need to find 0 K

$T_1 = 273\,K = 32°F$

$T_2 = 373\,K = 212°F$, so

$0\,K = 32 - \frac{180}{100} \cdot 273 = -459°F$

Therefore

$$T(°R) = T(°F) + 459$$

$$T(°R) = \frac{9}{5}\,T(°K)$$

and room $T = 300\,K = 540\,°R$

#1.6 The key is thermal conductivity. Answer in solution manual → bathroom rug vs. tiles on the floor, they are @ same T but feel different.

Other example: cold toilet seat (wood or plastic) vs. metallic night pot.

#1.8 $\alpha = \frac{\Delta L / L}{\Delta T}$

a) $T_{cold} \simeq -20°C$; $T_{hot} \simeq 35°C$, so

$\Delta T \simeq 55°C \Rightarrow \frac{\Delta L}{L} = 1.1 \cdot 10^{-5} \cdot 55 \simeq 6 \cdot 10^{-4}$

so $\Delta L \simeq 60\,cm$ (or something similar)

b) two metals with different thermal expansion coefficients. When you make a coil, one side shrinks or expands more, forcing more "coiling"

c) suppose $V = L_{x0} L_{y0} L_{z0}$, and

$$L_i = L_{i0} + \Delta L_i .$$

Key: linearize, use $\Delta L_x, \Delta L_y, \Delta L_z$, but not products or cubes, these are small

$$\Delta V \simeq \Delta L_x L_{y0} L_{z0} + \Delta L_y L_{x0} L_{z0} + \Delta L_z L_{x0} L_{y0}$$

$$\boxed{\frac{\Delta V}{V} = \frac{\Delta L_x}{L_{x0}} + \frac{\Delta L_y}{L_{y0}} + \frac{\Delta L_z}{L_{z0}} = 3\alpha \Delta T}$$

 # extra A quick search shows that rail length is standard @ 39 feet $\simeq 11.9\,m$

Louisiana: $T_{low} \simeq -5°C$, $T_{high} \simeq 45°C \Rightarrow \Delta T = 50K$

so we want

$$\Delta L \simeq \alpha \Delta T L \simeq 1.1 \cdot 10^{-5} \cdot 50 \cdot 11.9 \simeq$$

$$\simeq 650 \cdot 10^{-5} \simeq 6.5\,mm$$

is the minimal gap.

 # 1.9 $N_A = 6 \cdot 10^{23}$ $\quad P = 10^5\,Pa$ $\quad T = 300\,K$

$$\boxed{V = \frac{N_A k T}{P} = \frac{RT}{P} = \frac{8.3 \cdot 300}{10^5} \simeq 25\,\ell}$$

 # 1.11 Open door = same pressure

$N = \frac{PV}{kT} \Rightarrow$ <u>warmer room</u> has <u>less air</u>

#1.12

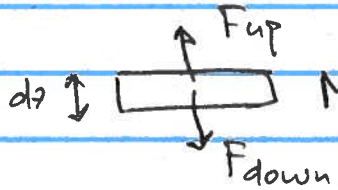
$$\frac{V}{N} = \frac{kT}{P} = \frac{1.38 \cdot 10^{-23} \cdot 300}{10^5} \simeq 4.2 \cdot 10^{-26} m^3$$

$$r_s \sim \left(\frac{V}{N}\right)^{1/3} \simeq \left(42 \cdot 10^{-27}\right)^{1/3} = \sqrt[3]{42} \cdot 10^{-9} \simeq$$

$$\simeq 3.3 \cdot 10^{-9} m \simeq 33 \mathring{A}$$

size of molecules $\sim 3-4 \mathring{A}$, so about a tenth of the distance between them

#1.16

a) 

$M = \rho A dz$, ρ is density
A = area

$F_{up} - F_{down} = -dP \cdot A$, where dP is the pressure difference

Equilibrium: $dP = -\rho g dz$, so

$$\boxed{\frac{dP}{dz} = -\rho(z) g}$$

b) For ideal gas $\rho = \frac{M}{V} = \frac{N \cdot m}{V}$ where

m is the mass of 1 molecule.

We know $\frac{N}{V} = \frac{P}{kT}$, so

$$\boxed{\rho = \frac{mP}{kT}}$$

Then

$$\frac{dP}{dz} = -\frac{mPg}{kT}$$

c) Now $\dfrac{dP}{P} = -\dfrac{mg}{kT}\,dz$

Integrate both parts $\ln\dfrac{P}{P_0} = -\dfrac{mgz}{kT}$, so

$$\boxed{P = P_0 \exp\left(-mgz/kT\right)}$$

d) $m \simeq \dfrac{29\,g}{6\cdot 10^{+23}} \simeq 5\cdot 10^{-26}\,kg$.

$$\frac{mg}{kT} = \frac{mN_A g}{RT} \simeq \frac{29\cdot 10^{-3}\cdot 10}{8.3\cdot 300} \simeq 1.16\cdot 10^{-4}\,\frac{1}{m}$$

so

$$P = 1\,atm\,\exp\left[-z/8600\,m\right] \simeq$$

$$\simeq 1\,atm\,\exp\left[-z/28000\,ft\right]$$

Then Ogden $\simeq e^{-0.17} \simeq 0.84\,atm$

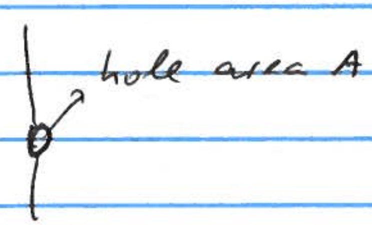
Leadville $P \simeq 0.7\,atm$

Mt. Whitney $P \simeq 0.6\,atm$

Everest $P \simeq 0.35\,atm$.

#1.18 $\quad V \simeq \sqrt{\dfrac{3RT}{M}} \simeq 500\,m/s$

#1.22



a) We saw — Eq. (1.9) : $P = -\dfrac{m\Delta V_x}{A\Delta t}$

$\Delta V_x = 2V_x$ as before

Now have N molecules hitting, so

$$P = \frac{2mV_x}{A\Delta t} N \text{, so}$$

$$\boxed{N = \frac{PA\Delta t}{2mV_x}}$$

b)

$$\overline{V_x} \simeq \sqrt{\overline{V_x^2}} = \sqrt{\frac{kT}{m}}$$

c) Now N is change in the total # of molecules inside

$$\frac{dN}{dt} = -\frac{PA}{2m}\sqrt{\frac{m}{kT}} =$$

$$= -\frac{NkT}{V}\frac{A}{2m}\sqrt{\frac{m}{kT}}$$

$$= -\frac{A}{2V} N \sqrt{\frac{kT}{m}}$$

Define $\frac{1}{\tau} = \frac{A}{2V}\sqrt{\frac{kT}{m}}$, then $\frac{dN}{dt} = -\frac{N}{\tau}$, or

$$\frac{dN}{N} = -\frac{dt}{\tau} \Rightarrow \ln\frac{N}{N_0} = -\frac{t}{\tau} \text{ or}$$

$$\boxed{N = N_0 e^{-t/\tau}}$$

(d) Need to decide on gas to know M

$A \simeq 1\,mm^2$ $V \simeq 1\ell = 10^{-3} m^3$, so $\frac{A}{2V} \simeq \frac{10^{-6}}{2\cdot 10^{-3}} = 5\cdot 10^{-4} \frac{1}{m}$

$$V = \sqrt{\frac{RT}{M}} \simeq \begin{cases} 300\,m/s, \text{ air } M = 30g \\ 1{,}100\,m/s, \text{ helium} \end{cases}$$

For air $\frac{1}{\tau} \simeq \underline{300\, m/s} \cdot 5 \cdot 10^{-4} \frac{1}{m} \simeq 0{,}15\, s^{-1}$

so

$$\tau \simeq 6.7 s$$

(e) Bicycle tire : ~~[illegible]~~

$r \simeq 30\, cm \Rightarrow 2\pi r = 1.9 m$

tube radius $\sim 1.5\, cm$, so

$$V \simeq 1.9 m \cdot \pi \cdot 2.25\, cm^2 \sim 1.5 \cdot 10^{-3} m^3$$

If $\tau = 3600\, s$, then

$$\boxed{A = \frac{2V}{\tau \cdot \bar{v}} \simeq \frac{2 \cdot 1.5 \cdot 10^{-3} m^3}{3600 s \cdot 300\, m/s} \simeq \frac{10^{-5}}{3600} \simeq 3 \cdot 10^{-9} m^2}$$

(f) Suppose (generously) $V \simeq 100\, m^3$, window area $\simeq 0.5\, m^2$, so

$$\tau \simeq \frac{2 \cdot 100}{5 \cdot 10^{-1} \cdot 3 \cdot 10^2} \simeq 1.3\, s,$$

so in ~ 3 sec that the window is open we would lose

$$N = N_0 e^{-3/1.3} \simeq N_0 e^{-2.3} \simeq \frac{N_0}{10},$$

so they would lose 90% of their air.