Phys4125 Formula sheet Exam 1

- Ideal gas law $PV = NkT = nRT$
- Gas pressure from kinetic theory $PV = Nm\overline{v_x^2}$.
- Equipartition

$$\frac{mv_x^2}{2} = \frac{1}{2}kT, \qquad U_{thermal} = \frac{f}{2}NkT, \qquad f = \text{number of degrees of freedom}$$

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- $v_{rms} = \bar{v^2} = \sqrt{\frac{3kT}{m}}$
- Heat and work

$$\Delta U = Q + W, \text{ for quasistatic processes } W = -\int_{V_i}^{V_f} PdV$$

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- Adiabatic process $VT^{f/2} = \text{const.}$
- Heat capacities $C = Q/\Delta T$, so that

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N}, \text{ and } C_P = \left(\frac{\partial U}{\partial T}\right)_{P,N} + P\left(\frac{\partial V}{\partial T}\right)_{P,N}$$

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- Ideal gas $C_P = C_V + Nk$
- Latent heat $L = Q/m$
- Enthalpy $\Delta H = \Delta U + P\Delta V$.
- Heat conduction

$$\frac{Q}{\Delta t} = -k_t A\frac{dT}{dx}, \qquad \frac{\partial T}{\partial t} = K\frac{\partial^2 T}{\partial x^2} \text{ with } K = k_t/c\rho.$$

- Diffusion

$$J_x = -D\frac{\partial n}{\partial x}$$

- Mean free path $l = (4\pi r^2)^{-1}V/N$, time between collisions $\overline{\Delta t} = l/\bar{v}$.
- Transport coefficients of gases

$$k_t = \frac{1}{2}\frac{C_V}{V}l\bar{v}, \qquad \eta = \frac{1}{2}\rho\bar{v}l, \qquad D = \frac{1}{2}l\bar{v}.$$

- Multplicity of a coin toss: n heads out of N coins

$$\Omega = \frac{N!}{n!(N-n)!} \equiv \binom{N}{n}$$

- Multiplicity of an Einstein solid: N oscillators, q quanta of energy

$$\Omega = \frac{(q+N-1)!}{q!(N-1)!} \equiv \binom{q+N-1}{q}$$

- Stirling's approximation

$$N! \approx N^N e^{-N}\sqrt{2\pi N} \qquad \text{or} \qquad \ln N! \approx N\ln N - N.$$

- Multiplicity of a large Einstein solid

$$\ln\Omega \approx (q+N)\ln(q+N) - q\ln q - N\ln N, \qquad \Omega \approx \left(\frac{eq}{N}\right)^N, \text{if } q \gg N.$$

- Multiplicity of a paramagnet

$$\Omega = \frac{N!}{N_\uparrow! N_\downarrow!} \equiv \binom{N}{N_\uparrow}$$

- Surface area of a d-dimensional sphere of radius r

$$Area = \frac{2\pi^{d/2} r^{d-1}}{(d/2-1)!}$$

- Multiplicity of an ideal gas in three dimensions

$$\Omega_N \approx \frac{1}{N!}\frac{V^N}{h^{3N}}\frac{\pi^{3N/2}}{(3N/2)!}(2mU)^{3N/2}.$$

- Entropy $S = k\ln\Omega$
- Sackur-Tetrode equation

$$S = kN\left[\ln\left(\frac{V}{N}\left(\frac{4\pi mU}{3Nh^2}\right)\right) + \frac{5}{2}\right]$$

- Temperature

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{V,N}$$

- Measuring entropies

$$dS = \frac{Q}{T} \text{ (const volume)}, \qquad \Delta S = \int_{T_i}^{T_f} \frac{C_V}{T} dT.$$

- Spin-1/2 paramagnet in a field

$$M = N\mu\tanh\frac{\mu B}{kT}$$

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- Curie's law

$$M \approx \frac{N\mu^2 B}{kT} \text{ for } \mu B \ll kT$$

Physical constants

$$k = 1.38\times10^{-23} J/K, \qquad N_A = 6.02\times10^{23}, \qquad R = 8.31 J/K$$

$$T(^\circ C) = T(^\circ K) - 273.15, \qquad 1 \text{ atm} = 1.01\times10^5 \text{Pa}$$