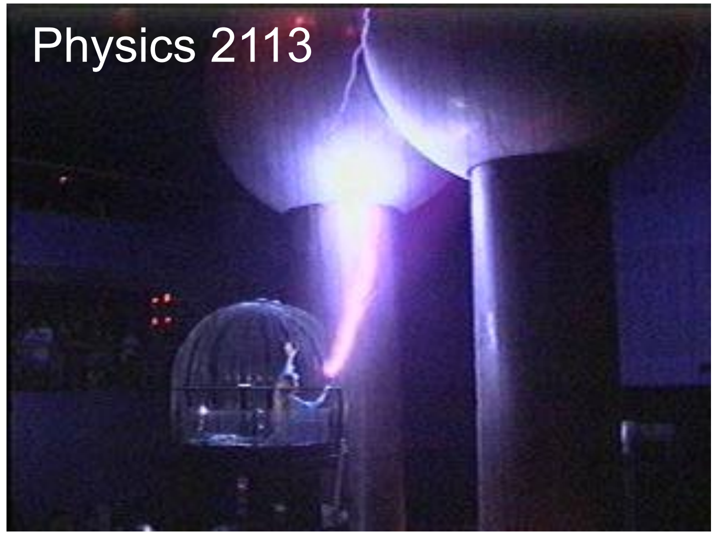
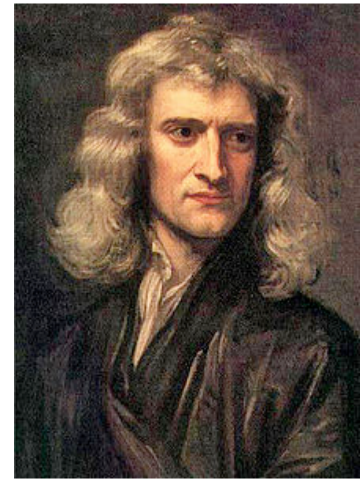
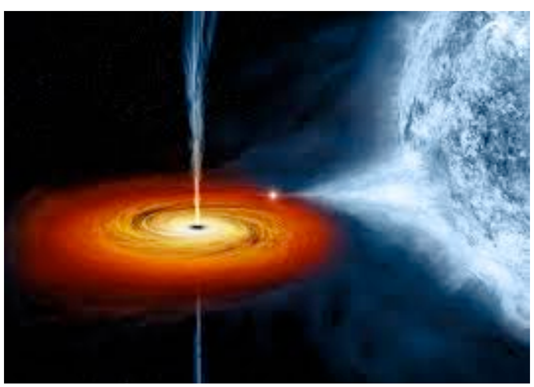




Isaac Newton
(1642–1727)

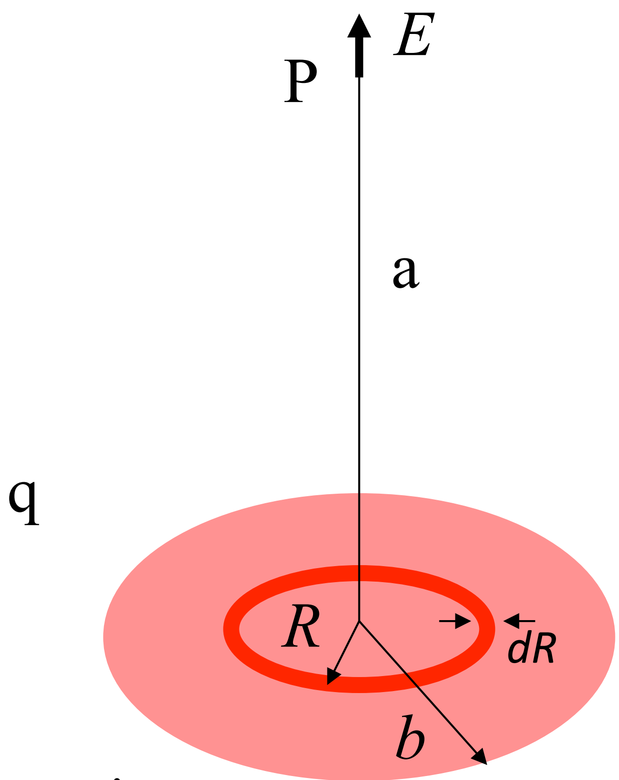
Physics 2113
Lecture 08: FRI 12 OCT

CH22: Electric Fields



Michael Faraday
(1791–1867)

Electric field due to a charged disk



Idea: superpose several rings, of infinitesimal width *dR*

Charge per unit area $\sigma = \dfrac{q}{\pi b^2}$

Charge of ring of radius *R* and width *dR*

$$dq = 2\pi R\, dR\, \sigma$$

Electric field due to ring at point P

$$dE = \frac{a}{4\pi\varepsilon_0} \frac{dq}{(R^2 + a^2)^{3/2}}$$

Integrating

$$E = \int_0^b \frac{a\sigma}{4\pi\varepsilon_0} \frac{2\pi R\, dR}{(R^2 + a^2)^{3/2}} = \frac{a\sigma}{2\varepsilon_0}\left[-\frac{1}{\sqrt{R^2 + a^2}}\right]_0^b = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{a}{\sqrt{b^2 + a^2}}\right]$$

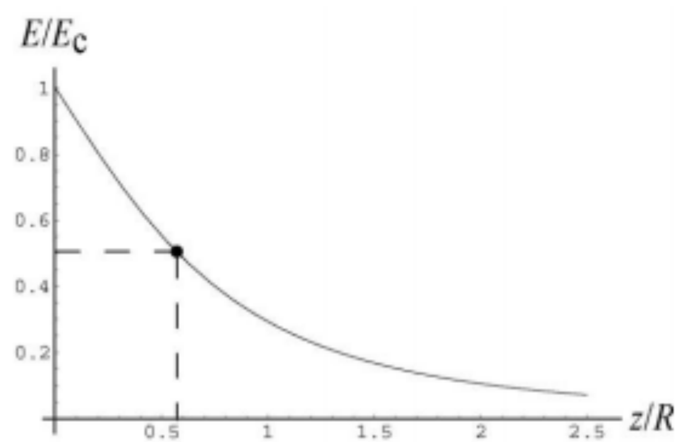
38P. At what distance along the central axis of a uniformly charged plastic disk of radius R is the magnitude of the electric field equal to one-half the magnitude of the field at the center of the surface of the disk?

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

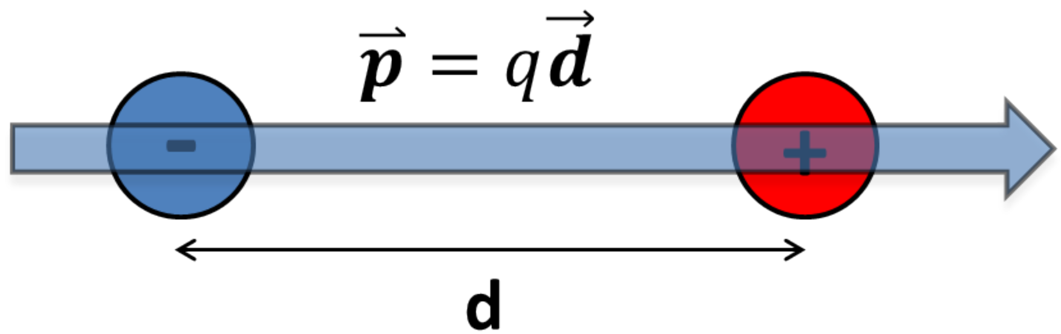
where R is the radius of the disk and σ is the surface charge density on the disk. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\varepsilon_0$. We want to solve for the value of z such that $E/E_c = 1/2$. This means

$$1 - \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2} \Rightarrow \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2}.$$

ANALYZE Squaring both sides, then multiplying them by $z^2 + R^2$, we obtain $z^2 = (z^2/4) + (R^2/4)$. Thus, $z^2 = R^2/3$, or $z = R/\sqrt{3}$. With $R = 0.600$ m, we have $z = 0.346$ m.



Electric Dipole Moment



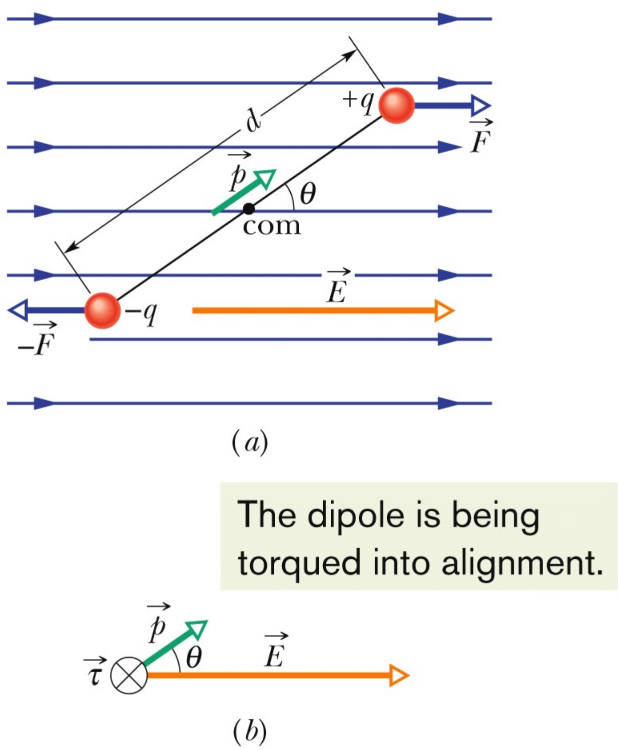
The direction of dipole moment is taken from negative to positive charge.

Using expression for the electric field of the dipole, we find

$$E = \frac{qd}{2\pi\varepsilon_0 x^3} = \frac{p}{2\pi\varepsilon_0 x^3}$$

Dipole in an electric field

A point charge feels a force proportional to the electric field $\vec{F} = q\,\vec{E}$



Assume the field is constant throughout the size of the dipole.

Forces are equal and opposite: no net force! Forces act at different points, therefore they have a nontrivial torque.

$$\tau = F\,d\sin\theta = q\,E\,d\sin\theta$$

The torque acting on the dipole can be written as $\tau = \vec{P}\times\vec{E}$

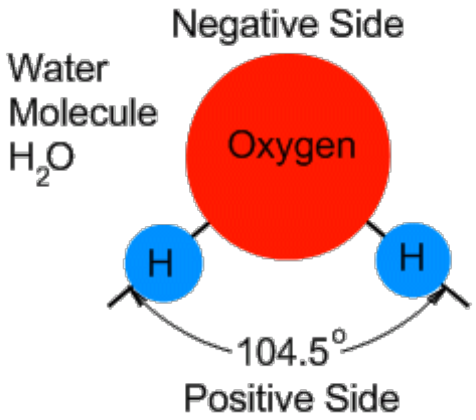
The cross product takes care of the sin θ in the magnitude and the direction is given by the right hand rule (clockwise in the figure).

The torque felt by a dipole in a field implies the ability to do work (for instance, turn the shaft of a motor). The potential energy is given by

$$U = -\vec{p} \cdot \vec{E} \quad \text{(potential energy of a dipole).}$$

It is maximum when p and E point in opposite directions, and minimum when they are parallel (note the minus sign). The dipole is "wound up" when it is opposing the field, ready to do work, and cannot do any more work when it is aligned with the field.

Microwave ovens operate by generating alternating fields and making the dipoles present in the water molecules align and realign, creating friction among the molecules.



59E. An electric dipole consists of charges $+2e$ and $-2e$ separated by 0.78 nm. It is in an electric field of strength 3.4×10^6 N/C. Calculate the magnitude of the torque on the dipole when the dipole moment is (a) parallel to, (b) perpendicular to, and (c) antiparallel to the electric field.

56. (a) Equation 22-33 leads to $\tau = pE \sin 0° = 0$.

(b) With $\theta = 90°$, the equation gives

$$\tau = pE = \left(2\left(1.6\times10^{-19}\text{ C}\right)\left(0.78\times10^{-9}\text{ m}\right)\right)\left(3.4\times10^{6}\text{ N/C}\right) = 8.5\times10^{-22}\text{ N}\cdot\text{m}.$$

(c) Now the equation gives $\tau = pE \sin 180° = 0$.

Maximum torque is at 90 degrees.

62P. An electric dipole with dipole moment

$$\mathbf{p} = (3.00\mathbf{i} + 4.00\mathbf{j})(1.24 \times 10^{-30}\ \mathrm{C \cdot m})$$

is in an electric field $\mathbf{E} = (4000\ \mathrm{N/C})\mathbf{i}$. (a) What is the potential energy of the electric dipole? (b) What is the torque acting on it? (c) If an external agent turns the dipole until its electric dipole moment is

$$\mathbf{p} = (-4.00\mathbf{i} + 3.00\mathbf{j})(1.24 \times 10^{-30}\ \mathrm{C \cdot m}),$$

how much work is done by the agent?

$$U = -\vec{p}\cdot\vec{E} = -\left[\left(3.00\hat{\mathrm{i}} + 4.00\hat{\mathrm{j}}\right)\left(1.24\times10^{-30}\,\mathrm{C\cdot m}\right)\right]\cdot\left[\left(4000\,\mathrm{N/C}\right)\hat{\mathrm{i}}\right]$$
$$= -1.49\times10^{-26}\ \mathrm{J}.$$

(b) From Eq. 22-34 (and the facts that $\hat{\mathrm{i}}\times\hat{\mathrm{i}} = 0$ and $\hat{\mathrm{j}}\times\hat{\mathrm{i}} = -\hat{\mathrm{k}}$), the torque is

$$\vec{\tau} = \vec{p}\times\vec{E} = \left[\left(3.00\hat{\mathrm{i}} + 4.00\hat{\mathrm{j}}\right)\left(1.24\times10^{-30}\,\mathrm{C\cdot m}\right)\right]\times\left[\left(4000\,\mathrm{N/C}\right)\hat{\mathrm{i}}\right] = \left(-1.98\times10^{-26}\,\mathrm{N\cdot m}\right)\hat{\mathrm{k}}.$$

(c) The work done is

$$W = \Delta U = \Delta\left(-\vec{p}\cdot\vec{E}\right) = \left(\vec{p}_i - \vec{p}_f\right)\cdot\vec{E}$$
$$= \left(3.00\hat{\mathrm{i}} + 4.00\hat{\mathrm{j}}\right) - \left(-4.00\hat{\mathrm{i}} + 3.00\hat{\mathrm{j}}\right)\left(1.24\times10^{-30}\,\mathrm{C\cdot m}\right)\cdot\left(4000\,\mathrm{N/C}\right)\hat{\mathrm{i}}$$
$$= 3.47\times10^{-26}\ \mathrm{J}.$$

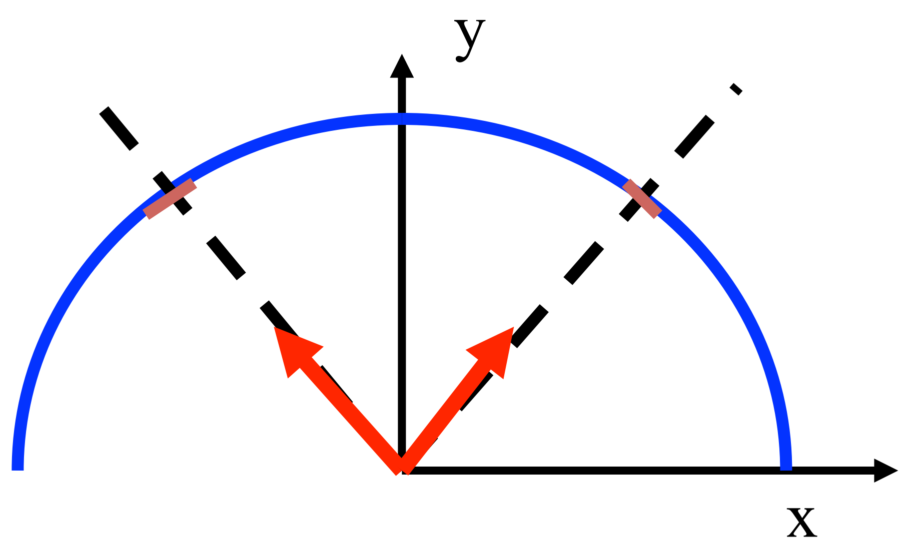
Example : Arc of Charge

- Figure shows a uniformly charged rod of charge **-Q** bent into a circular arc of radius R, centered at (0,0).
- What is the direction of the electric field at the origin?

(a) Field is 0.

(b) Along +y

(c) Along -y



- Choose symmetric elements
- x components cancel

Example : Arc of Charge

- Figure shows a uniformly charged rod of charge **-Q** bent into a circular arc of radius R, centered at (0,0).
- What is the direction of the electric field at the origin?

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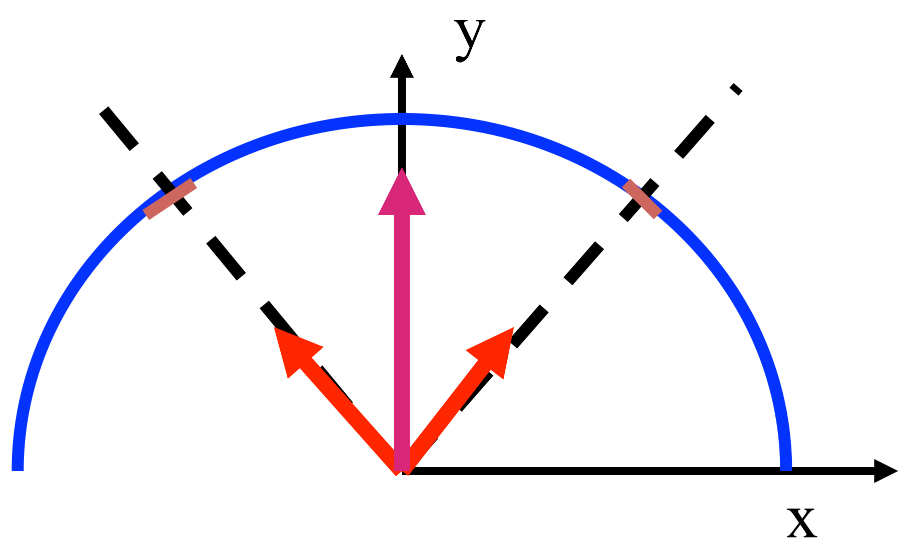
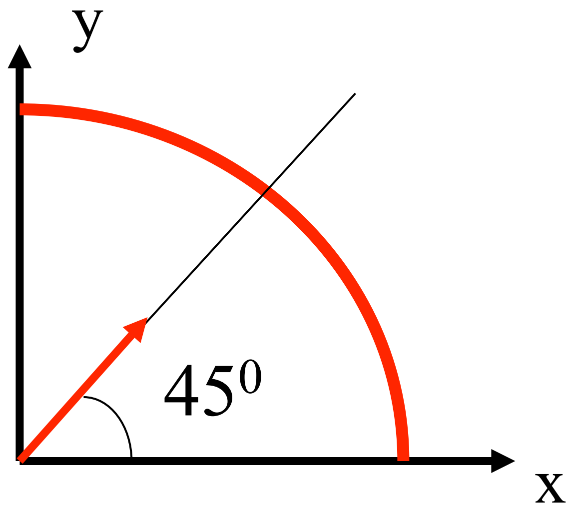


- Choose symmetric elements
- x components cancel

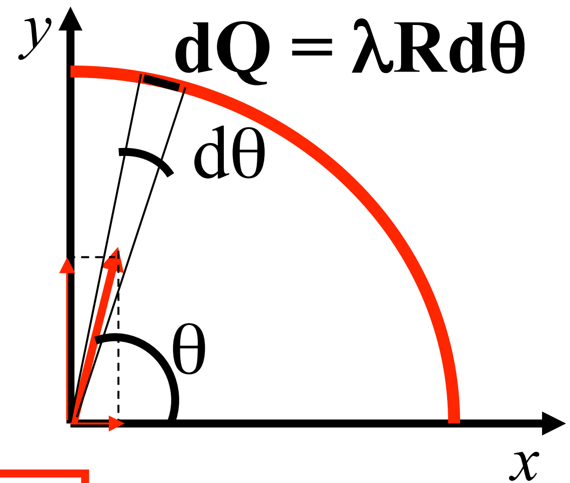
Arc of Charge: details

- Figure shows a uniformly charged rod of charge **−Q** bent into a circular arc of radius R, centered at (0,0).
- Compute the direction & magnitude of E at the origin.



$$dE_x = dE\cos\theta = \frac{kdQ}{R^2}\cos\theta$$

dQ = λRdθ



$$E_x = \int_0^{\pi/2} \frac{k(\lambda R d\theta)\cos\theta}{R^2} = \frac{k\lambda}{R}\int_0^{\pi/2}\cos\theta d\theta$$

$$E_x = \frac{k\lambda}{R} \qquad E_y = \frac{k\lambda}{R} \qquad E = \sqrt{2}\frac{k\lambda}{R}$$

λ = 2Q/(πR)

••26 ILW In Fig. 22-45, a thin glass rod forms a semicircle of radius $r = 5.00$ cm. Charge is uniformly distributed along the rod, with $+q = 4.50$ pC in the upper half and $-q = -4.50$ pC in the lower half. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the electric field $\vec{E}$ at P, the center of the semicircle?

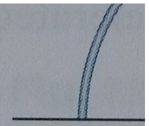


Fig. 22

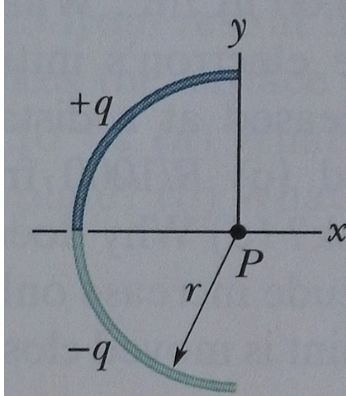


Fig. 22-45

Generalizing the previous result:

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\theta}^{\theta}$$

along the symmetry axis, with $\lambda = q/r\theta$ with θ in radians. In this problem, each charged quarter-circle produces a field of magnitude

$$|\vec{E}| = \frac{|q|}{r\pi/2}\frac{1}{4\pi\varepsilon_0 r}\sin\theta\Big|_{-\pi/4}^{\pi/4} = \frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}|q|}{\pi r^2}.$$

That produced by the positive quarter-circle points at –45°, and that of the negative quarter-circle points at +45°.

(a) The magnitude of the net field is

$$E_{\text{net},x} = 2\left(\frac{1}{4\pi\varepsilon_0}\frac{2\sqrt{2}|q|}{\pi r^2}\right)\cos 45° = \frac{1}{4\pi\varepsilon_0}\frac{4|q|}{\pi r^2}$$

$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)4(4.50\times10^{-12}\,\text{C})}{\pi(5.00\times10^{-2}\ \text{m})^2} = 20.6\ \text{N/C}.$$

(b) By symmetry, the net field points vertically downward in the $-\hat{\text{j}}$ direction, or –90° counterclockwise from the $+x$ axis.

•23 Figure 22-42 shows two parallel nonconducting rings with their central axes along a common line. Ring 1 has uniform charge q_1 and radius R; ring 2 has uniform charge q_2 and the same radius R. The rings are separated by distance $d = 3.00R$. The net electric field at point P on the common line, at distance R from ring 1, is zero. What is the ratio q_1/q_2?

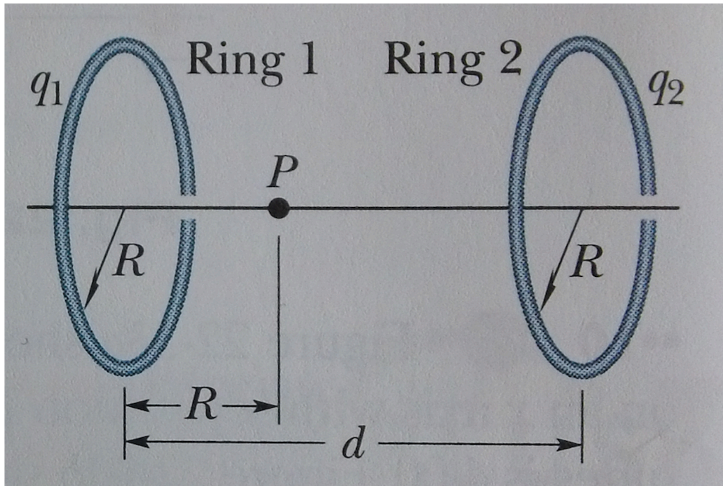


Fig. 22-42 Problem 23.

From last class:

$$E_{\text{left ring}} = E_{\text{right ring}} \Rightarrow \frac{q_1 R}{4\pi\varepsilon_0 \left(R^2 + R^2\right)^{3/2}} = \frac{q_2 (2R)}{4\pi\varepsilon_0 [(2R)^2 + R^2]^{3/2}}$$

Simplifying, we obtain

$$\frac{q_1}{q_2} = 2\left(\frac{2}{5}\right)^{3/2} \approx 0.506.$$

Summary:

Definition of Electric Field

- The electric field at any point

$$\vec{E} = \frac{\vec{F}}{q_0}. \qquad \textbf{Eq. 22-1}$$

Electric Field Lines

- provide a means for visualizing the directions and the magnitudes of electric fields

Field due to a Point Charge

- The magnitude of the electric field ***E*** set up by a point charge *q* at a distance *r* from the charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}. \qquad \textbf{Eq. 22-3}$$

Field due to an Electric Dipole

- The magnitude of the electric field set up by the dipole at a distant point on the dipole axis is

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \qquad \textbf{Eq. 22-9}$$

Field due to a Charged Disk

- The electric field magnitude at a point on the central axis through a uniformly charged disk is given by

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \qquad \textbf{Eq. 22-26}$$

Summary:

Force on a Point Charge in an Electric Field

- When a point charge q is placed in an external electric field ***E***

$$\vec{F} = q\vec{E}.$$ **Eq. 22-28**

Dipole in an Electric Field

- The electric field exerts a torque on a dipole

$$\vec{\tau} = \vec{p} \times \vec{E}.$$ **Eq. 22-34**

- The dipole has a potential energy *U* associated with its orientation in the field

$$U = -\vec{p} \cdot \vec{E}.$$ **Eq. 22-38**