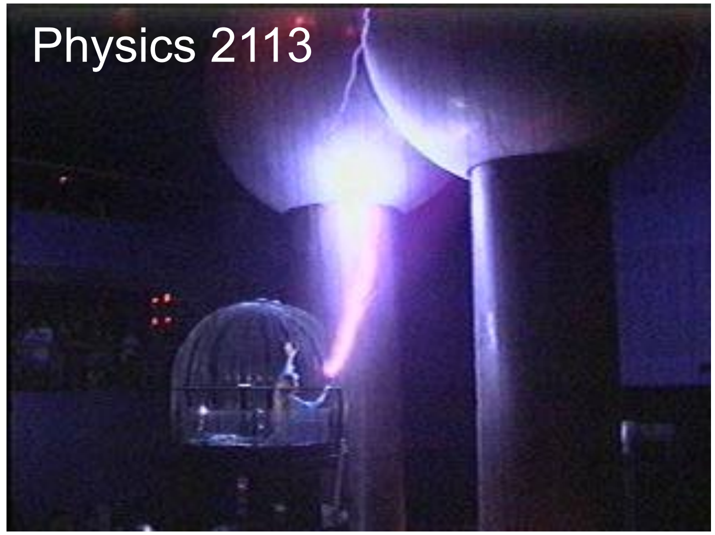
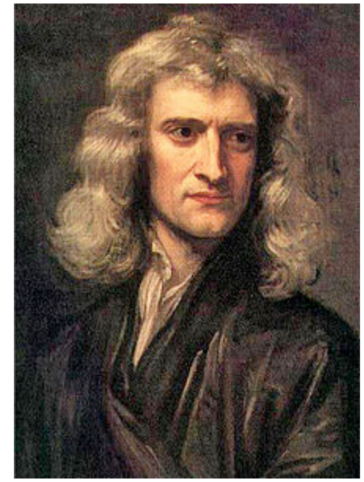




Isaac Newton
(1642–1727)

Physics 2113
Lecture 07: WED 10 OCT

CH22: Electric Fields

Michael Faraday
(1791–1867)

Computation of electric field for a continuous charge distribution

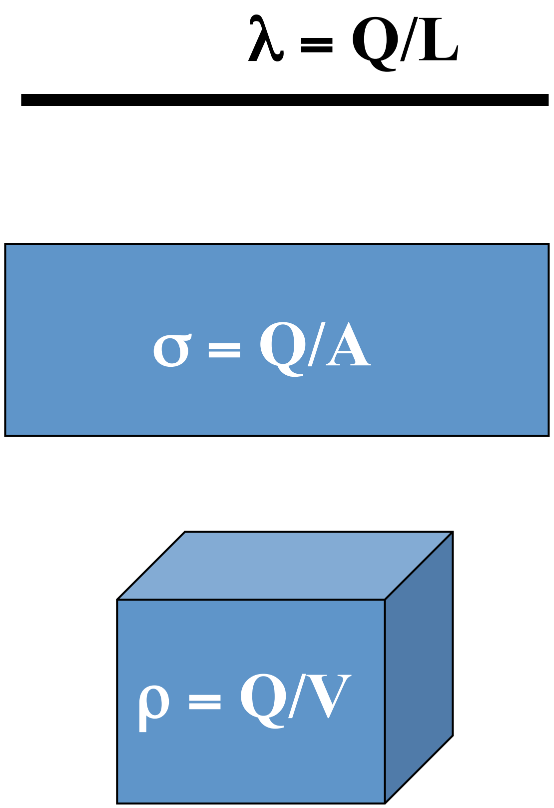
In general, a charged object has a "continuous" charge distributed on a line, a surface or a volume.

To find the electric field, we divide the continuous charge distribution into infinitesimally small parts.

Each infinitesimal part of the charge distribution is treated as a point charge. We then compute the electric field, and sum over the electric fields.

Charge Density

- Useful idea: charge density
- Line of charge:
  charge per unit length $= \lambda$
- Sheet of charge:
  charge per unit area $= \sigma$
- Volume of charge:
  charge per unit volume $= \rho$



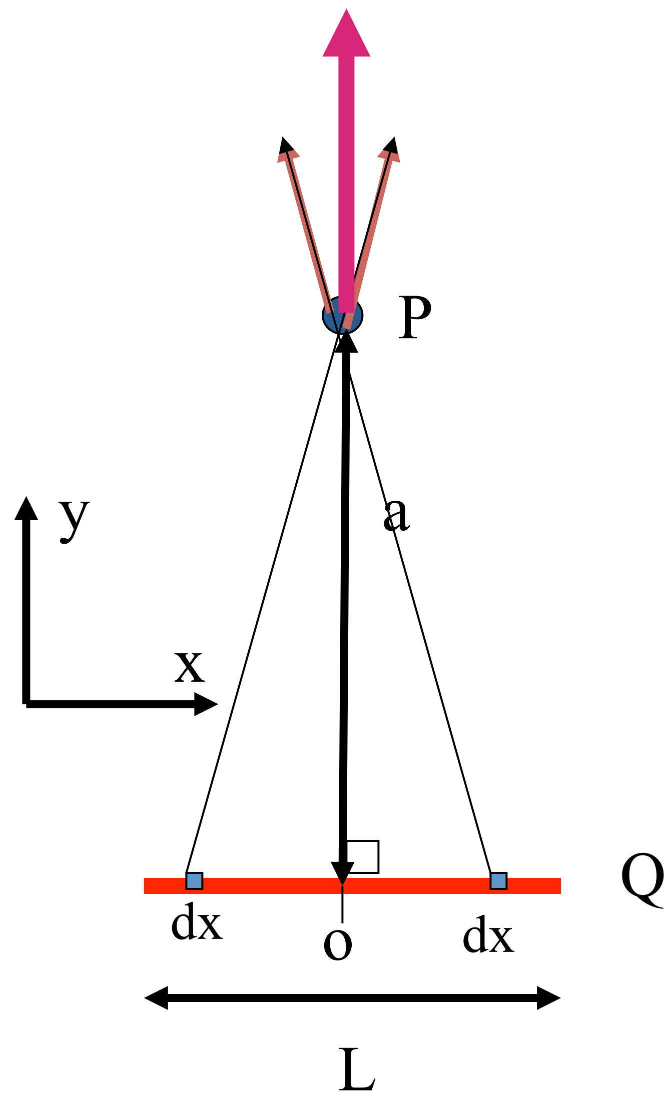
Example: Field on Bisector of Charged Rod

- Uniform line of charge +Q spread over length L
- What is the direction of the electric field at a point P on the perpendicular bisector?

(a) Field is 0.

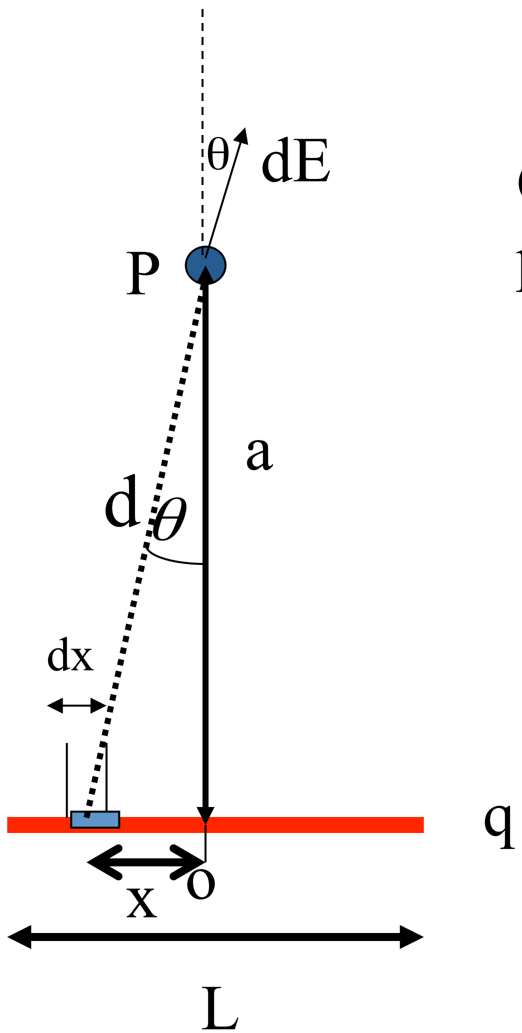
(b) Along +y

(c) Along +x



- Choose symmetrically located elements of length dx
- x components of E cancel

Line Of Charge: Field on bisector



Distance $d = \sqrt{a^2 + x^2}$

Charge per unit length $\lambda = \dfrac{q}{L}$

$$\cos\theta = \frac{a}{(a^2 + x^2)^{1/2}}$$

$$dE = \frac{k(dq)}{d^2} = \frac{k(\lambda dx)}{a^2 + x^2}$$

$$dE_y = dE\cos\theta = \frac{k(\lambda\, dx)a}{(a^2 + x^2)^{3/2}}$$

$$E_y = k\lambda\, a \int_{-L/2}^{L/2} \frac{dx}{(a^2 + x^2)^{3/2}}$$

Line Of Charge: Field on bisector

$$E_y = k\lambda\, a \int_{-L/2}^{L/2} \frac{dx}{(a^2 + x^2)^{3/2}} = k\lambda\, a \left[\frac{x}{a^2\sqrt{x^2 + a^2}} \right]_{-L/2}^{L/2} = \frac{2k\lambda L}{a\sqrt{4a^2 + L^2}}$$

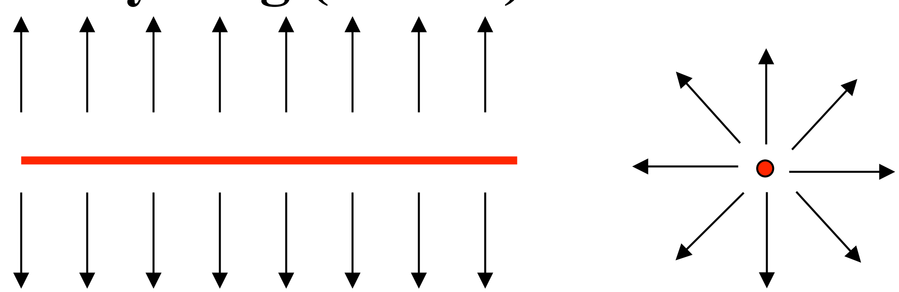
What is the field E very far away from the line (L<<a)?

$$E_y \approx \frac{2k\lambda L}{a\sqrt{4a^2}} = \frac{2k\lambda L}{2a^2} = \frac{kQ}{a^2}$$

Far away, any "localized" charge looks like a point charge

What is field E if the line is infinitely long (L >> a)?

$$E_y = \frac{2k\lambda L}{a\sqrt{L^2}} = \frac{2k\lambda}{a}$$



34P. In Fig. 23-42, a nonconducting rod of length L has charge $-q$ uniformly distributed along its length. (a) What is the linear charge density of the rod? (b) What is the electric field at point P, a distance a from the end of the rod? (c) If P were very far from the rod compared to L, the rod would look like a point charge. Show that your answer to (b) reduces to the electric field of a point charge for $a \gg L$.

EXPRESS The linear charge density λ is the charge per unit length of rod. Since the total charge $-q$ is uniformly distributed on the rod of length L, we have $\lambda = -q/L$. To calculate the electric at the point P shown in the figure, we position the x-axis along the rod with the origin at the left end of the rod, as shown in the diagram below.

Let dx be an infinitesimal length of rod at x. The charge in this segment is $dq = \lambda\, dx$. The charge dq may be considered to be a point charge. The electric field it produces at point P has only an x component and this component is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{(L+a-x)^2}.$$

The total electric field produced at P by the whole rod is the integral

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(L+a-x)^2} = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{L+a-x}\bigg|_0^L = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{L+a}\right)$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{L}{a(L+a)} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)},$$

upon substituting $-q = \lambda L$.

ANALYZE (a) With q = 4.23 × 10^{-15} C, L = 0.0815 m, and a = 0.120 m, the linear charge density of the rod is

$$\lambda = \frac{-q}{L} = \frac{-4.23\times10^{-15}\text{ C}}{0.0815\text{ m}} = -5.19\times10^{-14}\text{ C/m.}$$

(b) Similarly, we obtain

$$E_x = -\frac{1}{4\pi\varepsilon_0}\frac{q}{a(L+a)} = -\frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(4.23\times10^{-15}\text{ C})}{(0.120\text{ m})(0.0815\text{ m}+0.120\text{ m})} = -1.57\times10^{-3}\text{ N/C},$$

or $|E_x| = 1.57\times10^{-3}$ N/C.

(c) The negative sign in E_x indicates that the field points in the $-x$ direction, or $-180°$ counterclockwise from the $+x$ axis.

(d) If a is much larger than L, the quantity $L + a$ in the denominator can be approximated by a, and the expression for the electric field becomes

$$E_x = -\frac{q}{4\pi\varepsilon_0 a^2}.$$

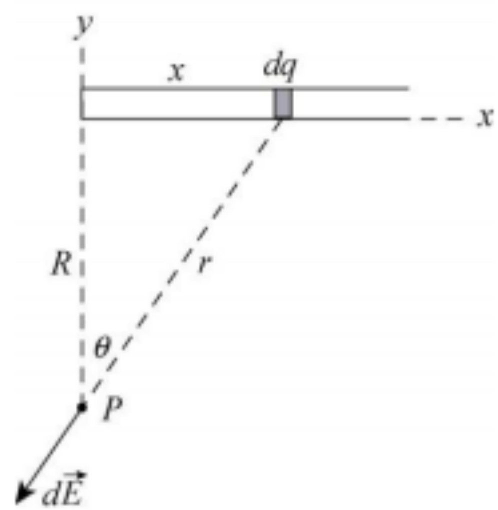
Since $a = 50\text{ m} \gg L = 0.0815\text{ m}$, the above approximation applies and we have $E_x = -1.52\times10^{-8}$ N/C, or $|E_x| = 1.52\times10^{-8}$ N/C.

(e) For a particle of charge $-q = -4.23\times10^{-15}$ C, the electric field at a distance a = 50 m away has a magnitude $|E_x| = 1.52\times10^{-8}$ N/C.

Far away…

35P*. In Fig. 23-43, a "semi-infinite" nonconducting rod (that is, infinite in one direction only) has uniform linear charge density λ. Show that the electric field at point P makes an angle of 45° with the rod and that this result is independent of the distance R. (*Hint:* Separately find the parallel and perpendicular (to the rod) components of the electric field at P, and then compare those components.)

33. Consider an infinitesimal section of the rod of length dx, a distance x from the left end, as shown in the following diagram. It contains charge $dq = \lambda dx$ and is a distance r from P. The magnitude of the field it produces at P is given by



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}.$$

The x and the y components are

$$dE_x = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\sin\theta$$

and

$$dE_y = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\cos\theta,$$

$$dE_x = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\sin\theta$$

and

$$dE_y = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r^2}\cos\theta\,,$$

respectively. We use θ as the variable of integration and substitute $r = R/\cos\,\theta$, $x = R\tan\theta$ and $dx = (R/\cos^2\theta)\,d\theta$. The limits of integration are 0 and $\pi/2$ rad. Thus,

$$E_x = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\sin\theta d\theta = \frac{\lambda}{4\pi\varepsilon_0 R}\cos\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}$$

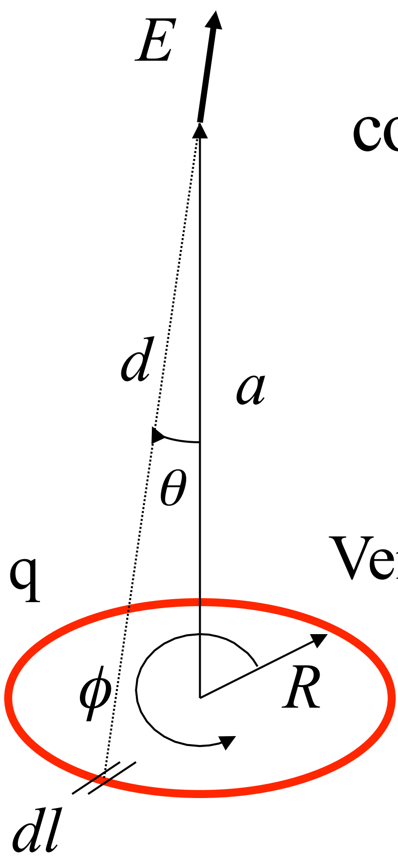
and

$$E_y = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_0^{\pi/2}\cos\theta d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\sin\theta\Big|_0^{\pi/2} = -\frac{\lambda}{4\pi\varepsilon_0 R}.$$

We notice that $E_x = E_y$ no matter what the value of R. Thus, $\vec{E}$ makes an angle of 45° with the rod for all values of R.

Electric field due to a charged ring

Due to symmetry, only the vertical component of E is needed.



$$\cos\theta = \frac{a}{d} = \frac{a}{\sqrt{a^2 + R^2}} \qquad \lambda = \frac{q}{2\pi R}, \quad dl = R\,d\phi$$

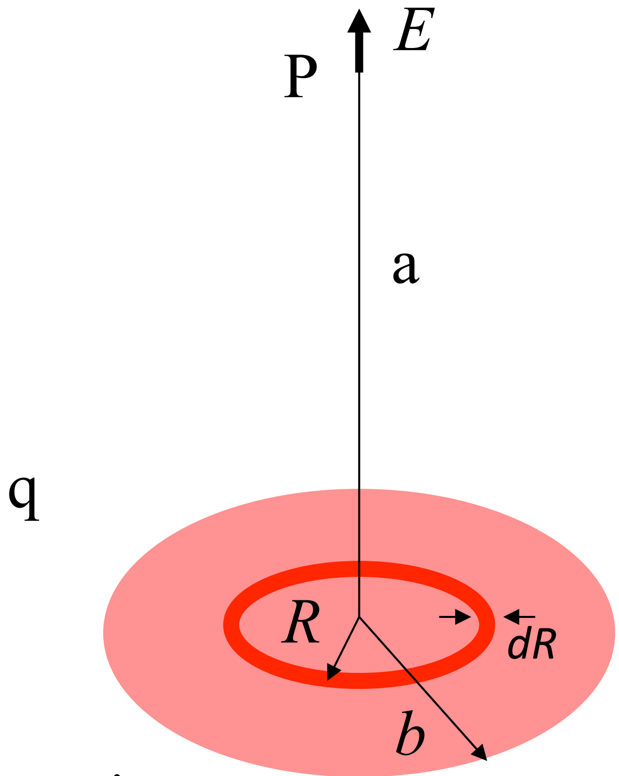
$$dE = \frac{1}{4\pi\varepsilon_0 d^2}\,dq = \frac{1}{4\pi\varepsilon_0}\,\frac{\lambda\,R\,d\phi}{(R^2 + a^2)}$$

Vertical Component: $dE_z = dE\cos\theta$

$$E = \frac{a}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{\lambda\,R\,d\phi}{(R^2 + a^2)^{3/2}} = \frac{a}{2\varepsilon_0}\,\frac{\lambda\,R}{(R^2 + a^2)^{3/2}}$$

As $R \to 0$, $E = \frac{a}{4\pi\varepsilon_0}\frac{q}{(R^2 + a^2)^{3/2}} \to \frac{q}{4\pi\varepsilon_0 a^2}$ Coulomb's law for point charge recovered.

Electric field due to a charged disk



Idea: superpose several rings, of infinitesimal width dR

Charge per unit area $\sigma = \dfrac{q}{\pi b^2}$

Charge of ring of radius R and width dR

$$dq = 2\pi R\, dR\, \sigma$$

Electric field due to ring at point P

$$dE = \frac{a}{4\pi\varepsilon_0} \frac{dq}{(R^2 + a^2)^{3/2}}$$

Integrating

$$E = \int_0^b \frac{a\sigma}{4\pi\varepsilon_0} \frac{2\pi R\, dR}{(R^2 + a^2)^{3/2}} = \frac{a\sigma}{2\varepsilon_0}\left[-\frac{1}{\sqrt{R^2 + a^2}}\right]_0^b = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{a}{\sqrt{b^2 + a^2}}\right]$$

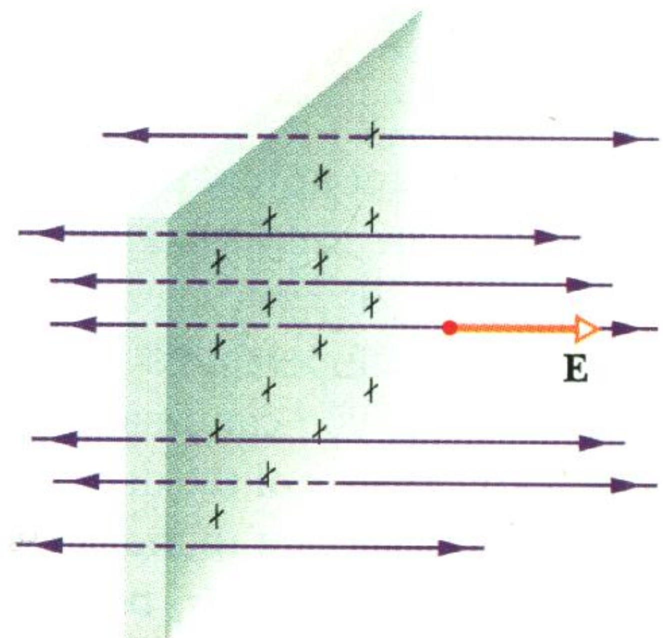
For a charged disk $$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{a}{\sqrt{b^2 + a^2}}\right]$$

As $a \to \infty$

$$\frac{a}{\sqrt{b^2 + a^2}} = \frac{1}{\sqrt{1 + \frac{b^2}{a^2}}} \approx 1 - \frac{b^2}{2a^2} \text{ when } a \to \infty \text{ so } E \to \frac{\sigma}{4\varepsilon_0}\frac{b^2}{a^2} = \frac{q}{4\pi\varepsilon_0 a^2}$$

Case of infinite disk $b \to \infty$

$$E \to \frac{\sigma}{2\varepsilon_0}$$



Electric field in this case approaches a
Constant value

Summary

- Electric field of continuous charge distribution can be computed by treating its infinitesimal parts as point charges, and then integrating.

- Far away from finite objects the field resembles that of a point charge.