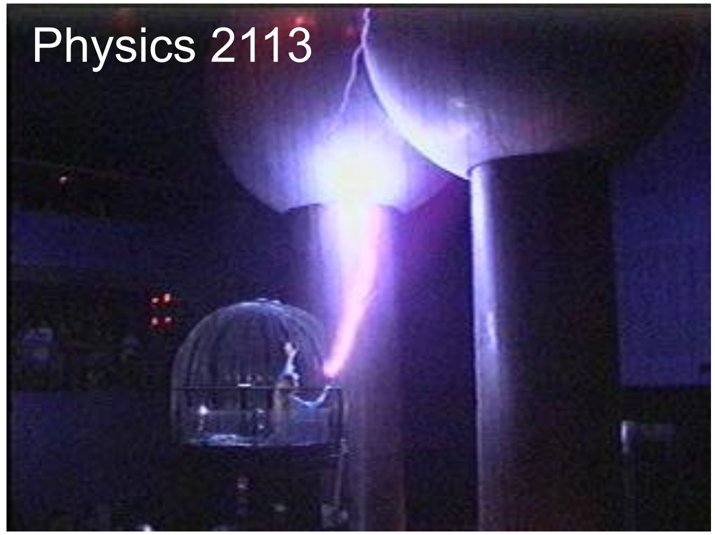

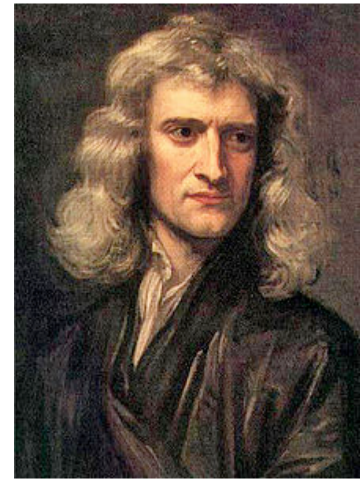

Isaac Newton
(1642–1727)

Physics 2113
Lecture 06: MON 8 OCT

CH22: Electric Fields

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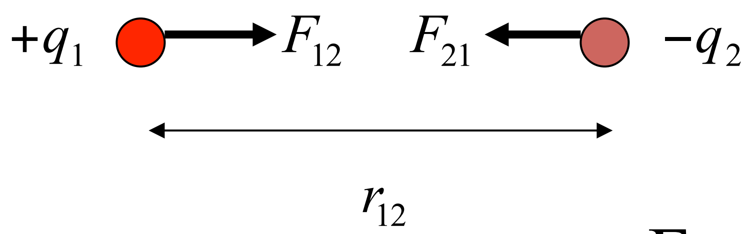
22-5 The Electric Field Due to an Electric Dipole 584

Michael Faraday
(1791–1867)

Coulomb's law



$$|F_{12}| = \frac{k\,|q_1|\,|q_2|}{r_{12}^2}$$

For charges in a VACUUM

$$\mathrm{k} = 8.99 \times 10^9 \ \frac{N\,m^2}{C^2}$$

Often, we write k as:

$$k = {}^{1}\!\!/_{4\pi\varepsilon_0} \text{ with } \varepsilon_0 = 8.85 \times 10^{-12} \ \frac{C^2}{N\,m^2}$$

Electric forces are added as vectors.

Electric Fields

- Electric field E at some point in space is defined as the force divided by the electric charge.
- Force on charge 2 at some point, by charge 1 is given by
- Electric field at that point is

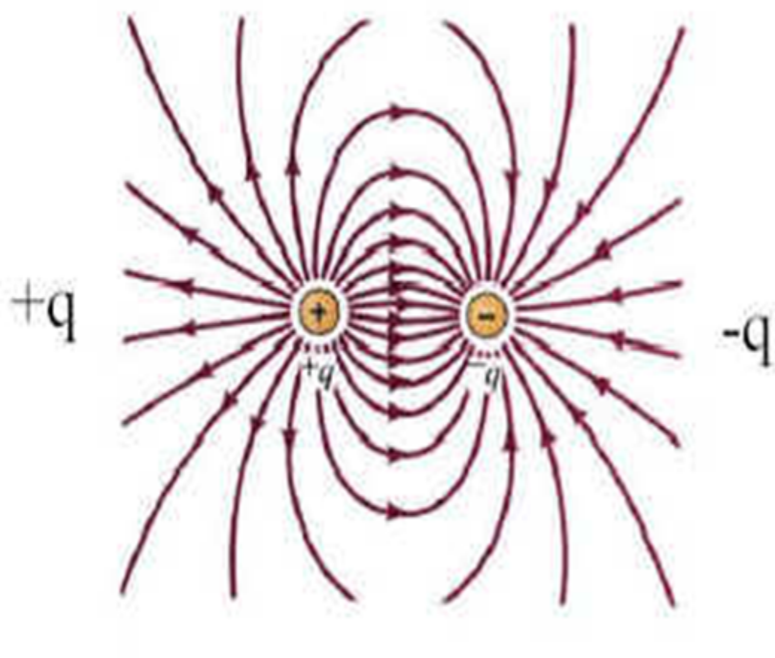
$$\vec{F} = q\,\vec{E}$$

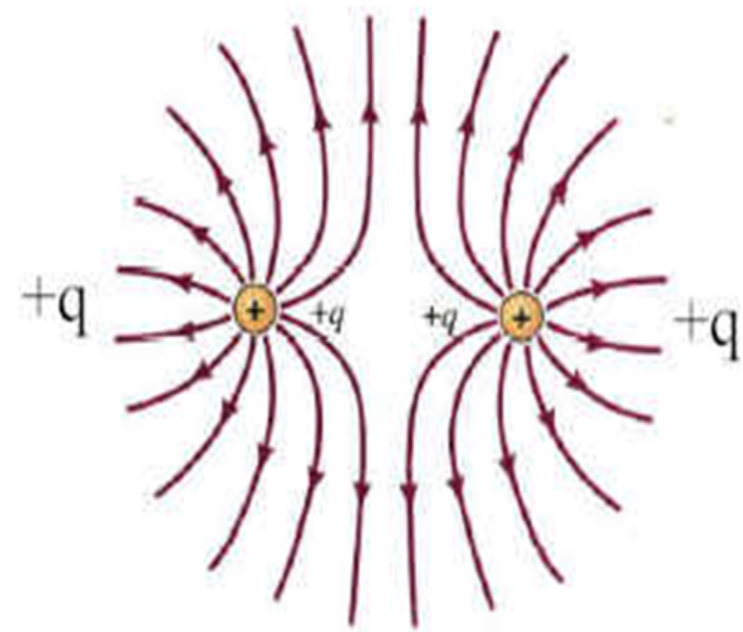
$$|\vec{F}_{12}| = \frac{k\,|q_1|\,|q_2|}{R^2}$$

$$|\vec{E}_{12}| = \frac{k\,|q_1|}{R^2}$$

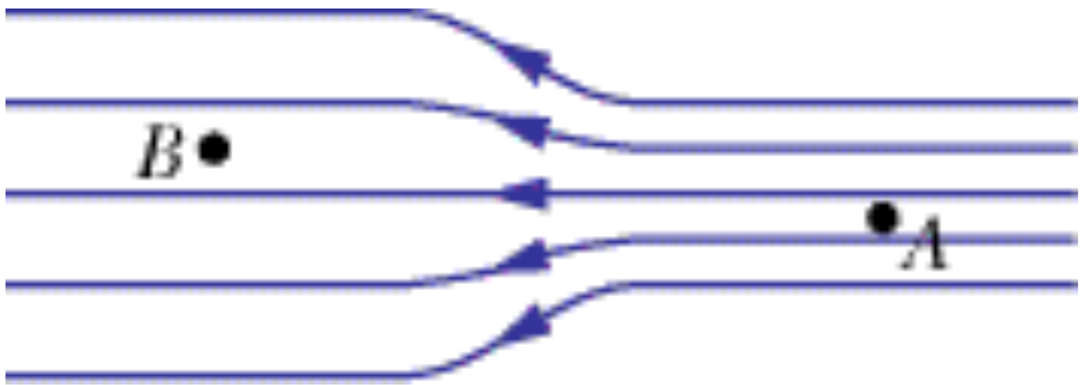
Field lines

Useful way to visualize electric field. They start at a positive charge, end at negative charge. At any point in space electric field is tangential to field line. Field is proportional to how "packed" the lines are.





Example

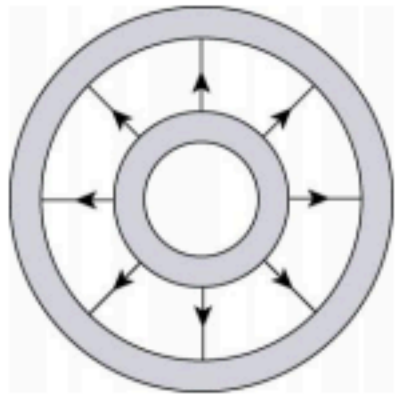


The electric field lines on the left have twice the separation as those on the right.

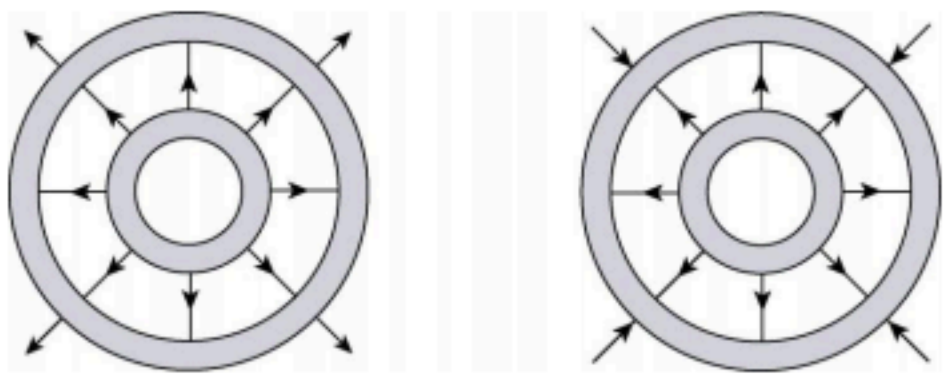
If the magnitude of the field at A is 40 N/C, what is the magnitude of field at B?

Charge q_1 on inner spherical shell, $-q_2$ on outer. Sketch the field lines.

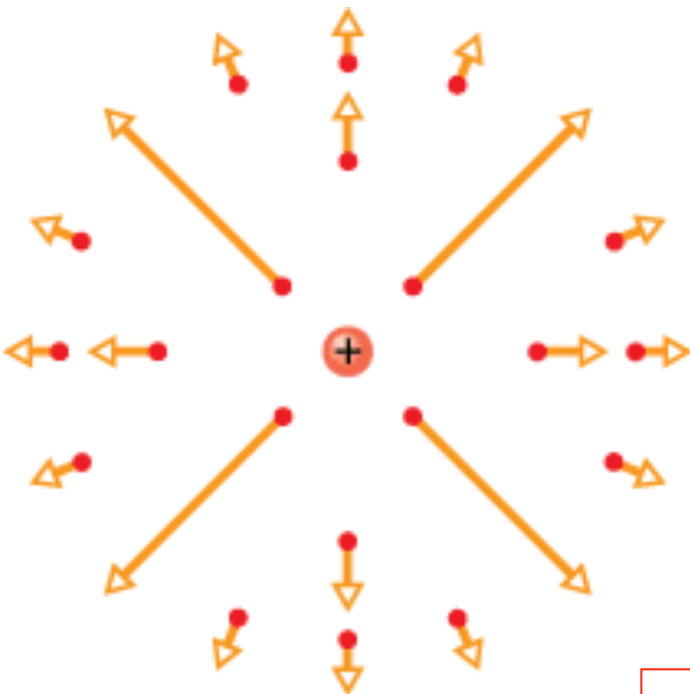
1. We note that the symbol q_2 is used in the problem statement to mean the absolute value of the negative charge that resides on the larger shell. The following sketch is for $q_1 = q_2$.



The following two sketches are for the cases $q_1 > q_2$ (left figure) and $q_1 < q_2$ (right figure).



Electric field of a point charge



The force is directed radially from the point charge. At any point, at a distance r, the force on charge q_0 is

$$\vec{F} = k\,\frac{q q_0}{r^2}\,\hat{r},$$

where $\hat{r}$ is a unit radial vector.

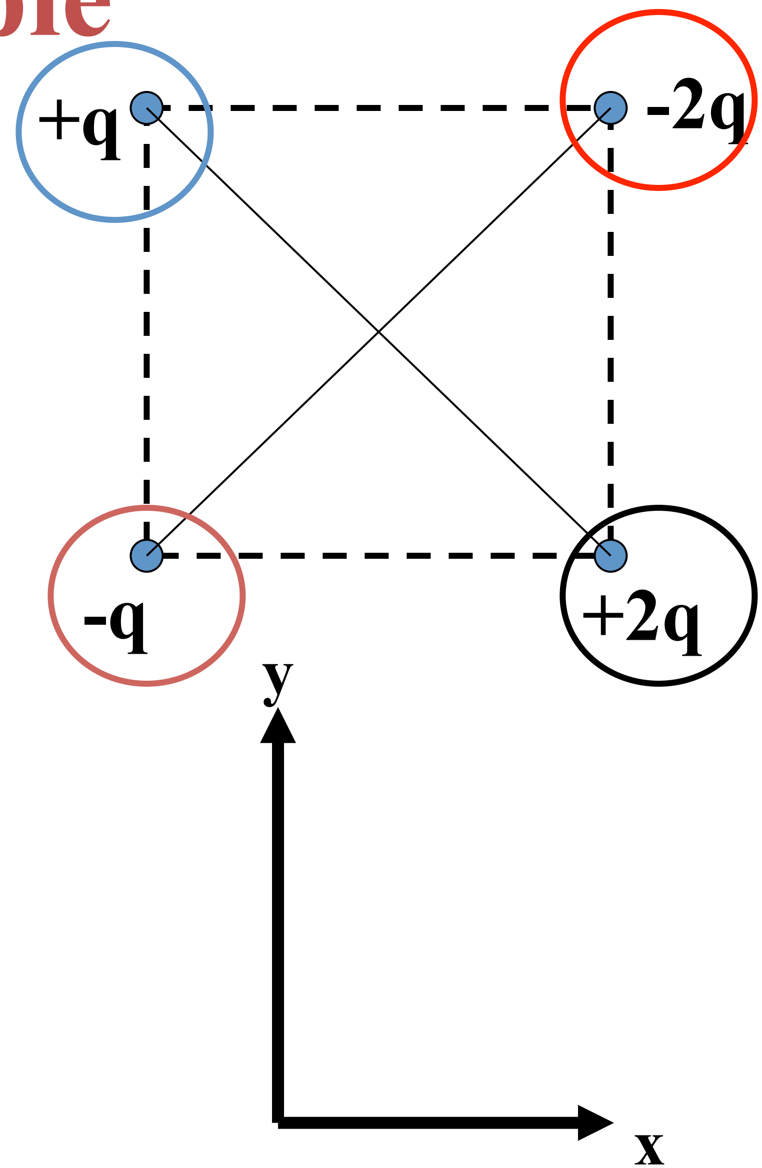
The electric field is given by

$$\vec{E} = k\,\frac{q}{r^2}\,\hat{r} = k\,\frac{q}{r^3}\,\vec{r}$$

To compute electric field of more than one charges, one should try to simplify the problem by breaking it to simpler problem.

Example

- 4 charges are placed at the corners of a square as shown.
- What is the direction of the electric field at the center of the square?



(a) Field vanishes at the center

(b) Along +y

(c) Along +x

Sample Problem

Net electric field due to three charged particles

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

KEY IDEA

Charges q_1, q_2, and q_3 produce electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, respectively, at the origin, and the net electric field is the vector sum $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. To find this sum, we first must find the magnitudes and orientations of the three field vectors.

Magnitudes and directions: To find the magnitude of $\vec{E}_1$, which is due to q_1, we use Eq. 22-3, substituting d for r and $2Q$ for q and obtaining

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

Similarly, we find the magnitudes of $\vec{E}_2$ and $\vec{E}_3$ to be

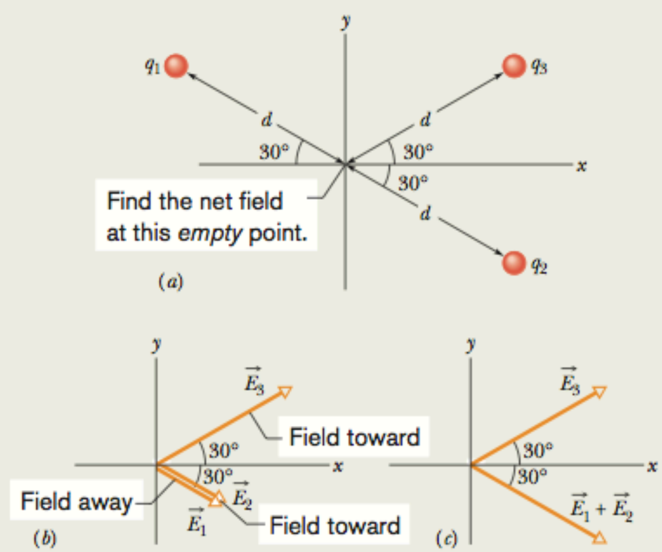


Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

We next must find the orientations of the three electric field vectors at the origin. Because q_1 is a positive charge, the field vector it produces points directly *away* from it, and because q_2 and q_3 are both negative, the field vectors they produce point directly *toward* each of them. Thus, the three electric fields produced at the origin by the three charged particles are oriented as in Fig. 22-7*b*. (*Caution:* Note that we have placed the tails of the vectors at the point where the fields are to be evaluated; doing so decreases the chance of error. Error becomes very probable if the tails of the field vectors are placed on the particles creating the fields.)

Adding the fields: We can now add the fields vectorially just as we added force vectors in Chapter 21. However, here we can use symmetry to simplify the procedure. From Fig. 22-7*b*, we see that electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction. Hence, their vector sum has that direction and has the magnitude

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

which happens to equal the magnitude of field $\vec{E}_3$.

We must now combine two vectors, $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$, that have the same magnitude and that are oriented symmetrically about the x axis, as shown in Fig. 22-7*c*. From the symmetry of Fig. 22-7*c*, we realize that the equal y components of our two vectors cancel (one is upward and the other is downward) and the equal x components add (both are rightward). Thus, the net electric field $\vec{E}$ at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30°$$

$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}. \qquad \text{(Answer)}$$

Two particles are fixed to an x axis: particle 1 of charge $q_1=2.1\times10^{-8}$ C at x=20cm and particle 2 of charge $q_2=-4q_1$ at x=70cm. At what coordinate on the axis is the net electric field zero?

11. **THINK** Our system consists of two point charges of opposite signs fixed to the x axis. Since the net electric field at a point is the vector sum of the electric fields of individual charges, there exists a location where the net field is zero.

EXPRESS At points between the charges, the individual electric fields are in the same direction and do not cancel. Since charge $q_2 = -4.00\, q_1$ located at $x_2 = 70$ cm has a greater magnitude than $q_1 = 2.1\times10^{-8}$ C located at $x_1 = 20$ cm, a point of zero field must be closer to q_1 than to q_2. It must be to the left of q_1.

Let x be the coordinate of P, the point where the field vanishes. Then, the total electric field at P is given by

$$E=\frac{1}{4\pi\varepsilon_0}\left(\frac{|q_2|}{(x-x_2)^2}-\frac{|q_1|}{(x-x_1)^2}\right).$$

ANALYZE If the field is to vanish, then

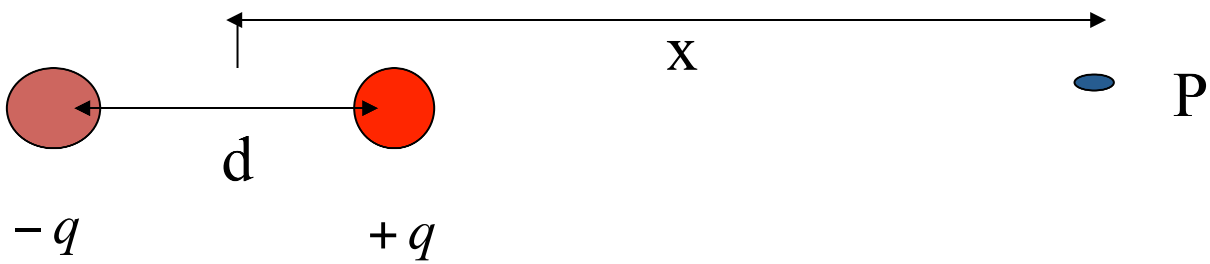
$$\frac{|q_2|}{(x-x_2)^2}=\frac{|q_1|}{(x-x_1)^2} \quad\Rightarrow\quad \frac{|q_2|}{|q_1|}=\frac{(x-x_2)^2}{(x-x_1)^2}.$$

Taking the square root of both sides, noting that $|q_2|/|q_1| = 4$, we obtain

$$\frac{x-70\text{ cm}}{x-20\text{ cm}}=\pm2.0\,.$$

Choosing -2.0 for consistency, the value of x is found to be $x = -30$ cm.

Electric field due to a dipole



What is the electric field at P due to the dipole?

To simplify things, we only compute the field along the axis.

Superposition of fields: $\vec{E} = \vec{E}_+ + \vec{E}_-$

$$\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x - \frac{d}{2}\right)^2}, \quad \vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{q}{\left(x + \frac{d}{2}\right)^2}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left(x-\frac{d}{2}\right)^2} - \frac{1}{\left(x+\frac{d}{2}\right)^2}\right] = \frac{q}{4\pi\varepsilon_0}\left[\left(x-\frac{d}{2}\right)^{-2} - \left(x+\frac{d}{2}\right)^{-2}\right]$$

When x>>d, at distances much greater than the dipole

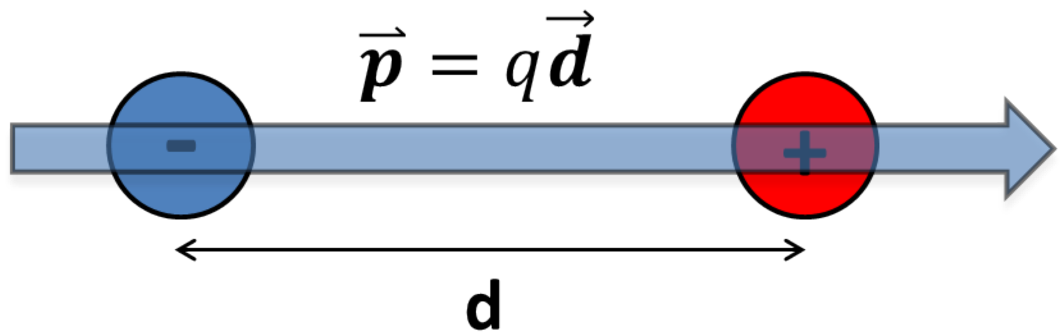
$$\left(x \pm \frac{d}{2}\right)^{-2} = x^{-2}\left(1 \pm \frac{d}{2x}\right)^{-2} = x^{-2}\left(1 \mp \frac{d}{x} + \cdots\right)$$

Then

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 x^2}\left[\left(1+\frac{d}{x}+\cdots\right)-\left(1-\frac{d}{x}+\cdots\right)\right] = \frac{q}{4\pi\varepsilon_0 x^2}\frac{2d}{x} = \frac{qd}{2\pi\varepsilon_0 x^3}$$

- Since, 2d << x, the field is weaker than that of the point charge at large x.
- Field scales with charges, vanishes if $d \longrightarrow 0$

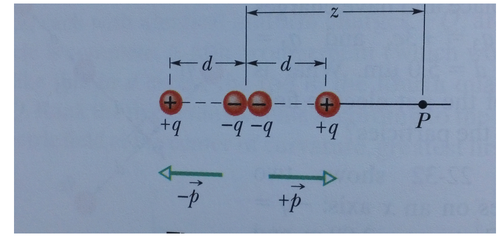
Electric Dipole Moment



The direction of dipole moment is taken from negative to positive charge.

Using expression for the electric field of the dipole, we find

$$E = \frac{qd}{2\pi\varepsilon_0 x^3} = \frac{p}{2\pi\varepsilon_0 x^3}$$



The electric quadrupole

It consists of two dipoles of equal magnitude place close to each other and in opposite directions.

EXPRESS Consider the point P on the axis, a distance z to the right of the quadrupole center and take a rightward pointing field to be positive. Then the field produced by the right dipole of the pair is given by $qd/2\pi\varepsilon_0(z - d/2)^3$ while the field produced by the left dipole is $-qd/2\pi\varepsilon_0(z + d/2)^3$.

ANALYZE Use the binomial expansions

$$(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$$

$$(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$$

we obtain

$$E = \frac{qd}{2\pi\varepsilon_0(z-d/2)^3} - \frac{qd}{2\pi\varepsilon_0(z+d/2)^3} \approx \frac{qd}{2\pi\varepsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\varepsilon_0 z^4}.$$

Since the quadrupole moment is $Q = 2qd^2$, we have $E = \dfrac{3Q}{4\pi\varepsilon_0 z^4}$.

The story can be continued: superposed two quadrupoles, obtain an octupole… hexadecupole… All with fields that decay faster and faster with distance.

Summary

- Electric is a property of space that characterizes how charges placed in it react to it.

- Field lines are a quick way to visualize fields.

- In dipoles and higher multipoles, the leading contribution to the fields cancel, leading to faster dropoff with distance in the magnitude of the fields.