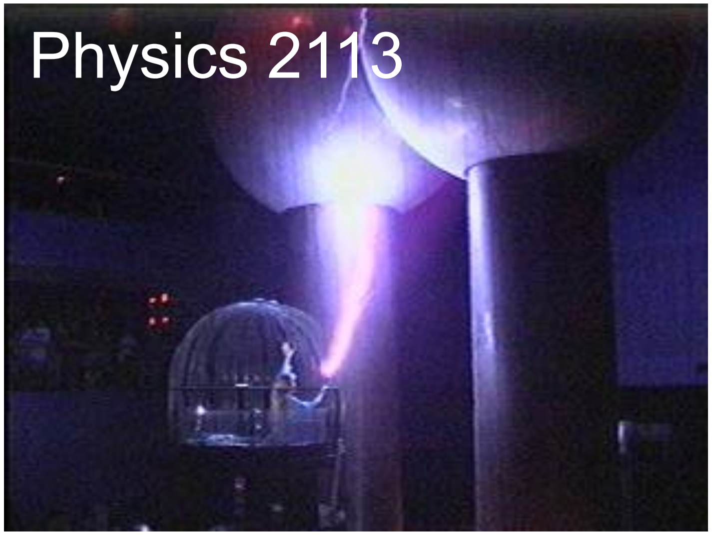


Physics 2113
Lecture 40: FRI3 DEC

Review of concepts for the final exam

Electric Fields

- Electric field E at some point in space is defined as the force divided by the electric charge.

$$\vec{F} = q\,\vec{E}$$

- Force on charge 2 at some point, by charge 1 is given by

$$|\vec{F}_{12}| = \frac{k\,|q_1|\,|q_2|}{R^2}$$

- Electric field at that point is

$$|\vec{E}_{12}| = \frac{k\,|q_1|}{R^2}$$

Sample Problem

Net electric field due to three charged particles

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

KEY IDEA

Charges q_1, q_2, and q_3 produce electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, respectively, at the origin, and the net electric field is the vector sum $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$. To find this sum, we first must find the magnitudes and orientations of the three field vectors.

Magnitudes and directions: To find the magnitude of $\vec{E}_1$, which is due to q_1, we use Eq. 22-3, substituting d for r and $2Q$ for q and obtaining

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}.$$

Similarly, we find the magnitudes of $\vec{E}_2$ and $\vec{E}_3$ to be

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

We next must find the orientations of the three electric field vectors at the origin. Because q_1 is a positive charge, the field vector it produces points directly *away* from it, and because q_2 and q_3 are both negative, the field vectors they produce point directly *toward* each of them. Thus, the three electric fields produced at the origin by the three charged particles are oriented as in Fig. 22-7*b*. (*Caution:* Note that we have placed the tails of the vectors at the point where the fields are to be evaluated; doing so decreases the chance of error. Error becomes very probable if the tails of the field vectors are placed on the particles creating the fields.)

Adding the fields: We can now add the fields vectorially just as we added force vectors in Chapter 21. However, here we can use symmetry to simplify the procedure. From Fig. 22-7*b*, we see that electric fields $\vec{E}_1$ and $\vec{E}_2$ have the same direction. Hence, their vector sum has that direction and has the magnitude

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

which happens to equal the magnitude of field $\vec{E}_3$.

We must now combine two vectors, $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$, that have the same magnitude and that are oriented symmetrically about the x axis, as shown in Fig. 22-7*c*. From the symmetry of Fig. 22-7*c*, we realize that the equal y components of our two vectors cancel (one is upward and the other is downward) and the equal x components add (both are rightward). Thus, the net electric field $\vec{E}$ at the origin is in the positive direction of the x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30°$$

$$= (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}. \qquad \text{(Answer)}$$

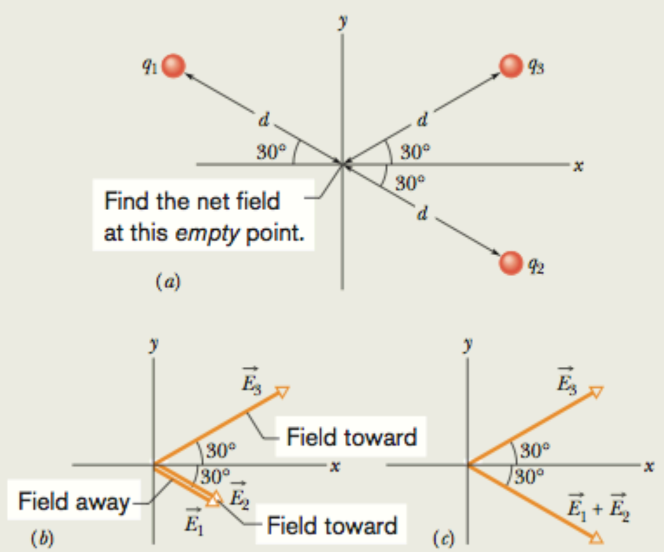
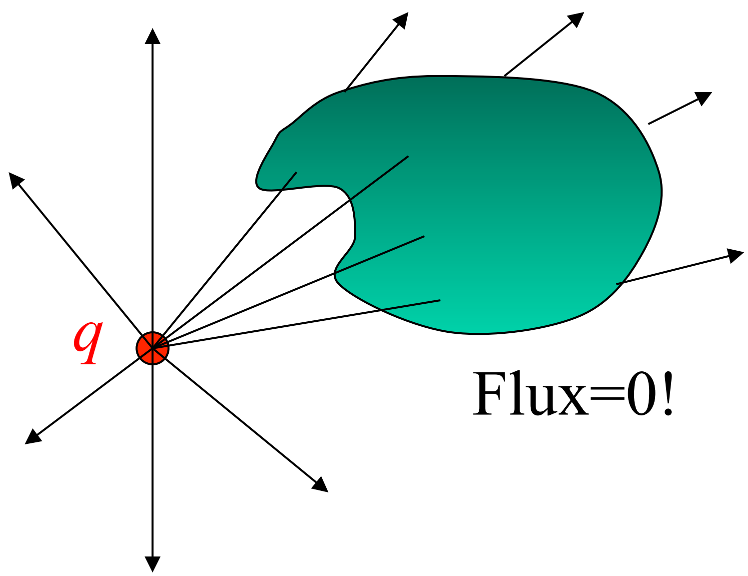
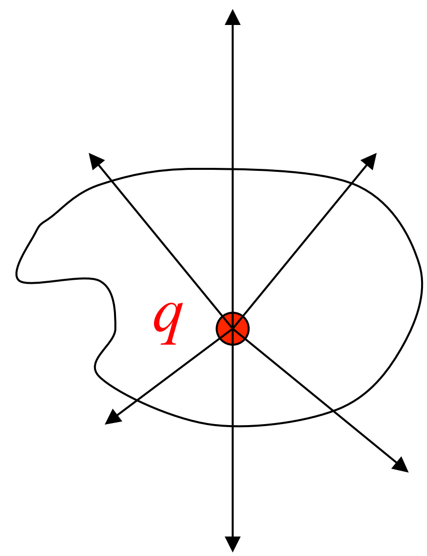


Fig. 22-7 (*a*) Three particles with charges q_1, q_2, and q_3 are at the same distance d from the origin. (*b*) The electric field vectors $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$, at the origin due to the three particles. (*c*) The electric field vector $\vec{E}_3$ and the vector sum $\vec{E}_1 + \vec{E}_2$ at the origin.

Gauss' law:

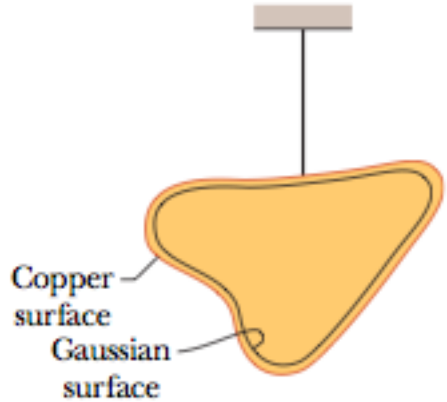
Given an arbitrary closed surface, the electric flux through it is proportional to the charge enclosed by the surface.



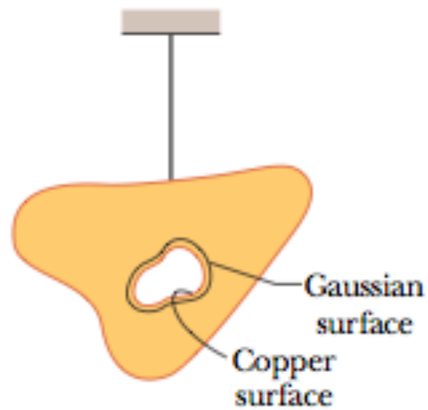


$$\Phi \equiv \oint_{\text{Surface}} \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

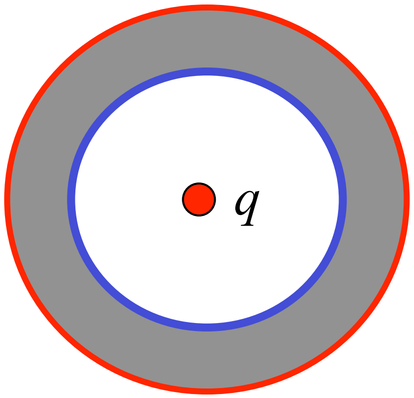
A charged conductor:



A conductor with a cavity:



Example: a charged conducting spherical sheet with a charge inside



Charge of sheet : q_s

The presence of the central charge attracts electrons to the inner surface of the metal sheet. How much charge is there on the inner surface of the sheet?

Construct a spherical Gaussian surface inside the sheet. Since it is a conductor, the field there vanishes. Therefore the flux vanishes. By Gauss' law, the enclosed charge should be zero. Therefore the amount of charge on the inner surface is -q.

Charge in the outside surface: construct a spherical Gaussian surface outside the sheet. The enclosed charge is q +qs. The external field will be equal to that of a point charge of value q+qs.

Now, the external field is entirely due to the charge on the outside of the sheet (since the field due to the inner surface cancelled with that of the point charge). Therefore the amount of charge deposited on the outside is q+qs.

Definition of electric potential:

Potential energy of a system per unit charge $V = \frac{U}{q}$

Units… Units...

$$V_f - V_i = \frac{U_f - U_i}{q} = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$\text{Units: } [\text{V}] = \frac{\text{Joule}}{\text{Coulomb}} \equiv \text{Volt}$$

Alessandro Volta (1745-1827)

$$[\text{Volt}] = \left[\frac{\text{N}}{\text{C}}\right][\text{m}] \Rightarrow \left[\frac{\text{N}}{\text{C}}\right] = \left[\frac{\text{V}}{\text{m}}\right]$$

Unit most commonly used for electric fields

$$\Delta V = \frac{\Delta U}{q} \Rightarrow \Delta U = q\Delta V$$

eV=electron-volt, the energy that an electron acquires when placed in an electric potential of *1V*

$$1\,eV = (1.6\times10^{-19}\,C)V = 1.6\times10^{-19}\,J$$

Since what matters in potential energy (and therefore in electrical potential) are differences, the potential is in general defined up to a constant. One way of fixing that constant is to declare that some point in space has zero potential. Very commonly infinity is chosen as that point.

In that case we have that $$V = -\frac{W_\infty}{q}$$

Where W_∞ is the work done by the electric field on a charged particle as it is brought from infinity to its current location.

If one moves a charge across a field exerting a force on it, there are two types of work done: the one by the external force and the one by the field. Their sum will be equal to the change in the kinetic energy of the charge. If the particle is stationary before and after the move, then $W_{app}=-W_{field}=q\Delta V$.

Potential is not a vector, orientation is irrelevant

(a) In Fig. 24-9*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?

KEY IDEAS

(1) The electric potential V at C is the algebraic sum of the electric potentials contributed by all the electrons. (Because electric potential is a scalar, the orientations of the electrons do not matter.) (2) The electric field at C is a vector quantity and thus the orientation of the electrons *is* important.

Calculations: Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, Eq. 24-27 gives us

$$V = -12\frac{1}{4\pi\varepsilon_0}\frac{e}{R}. \qquad \text{(Answer)} \quad (24\text{-}28)$$

Because of the symmetry of the arrangement in Fig. 24-9*a*, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C,

$$\vec{E} = 0. \qquad \text{(Answer)}$$

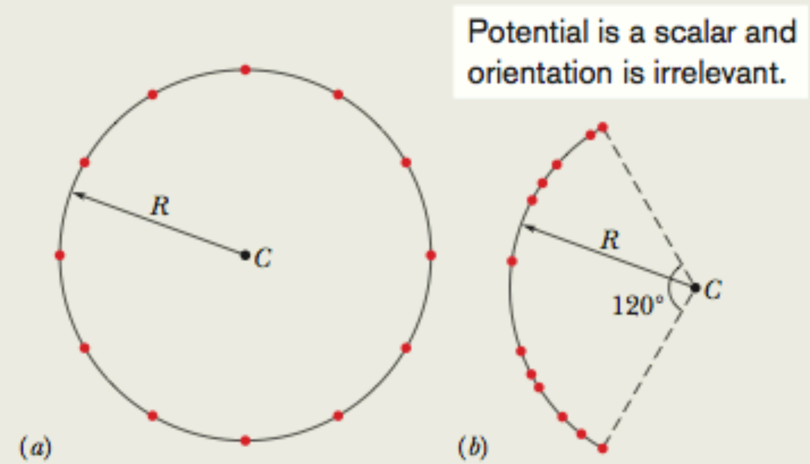


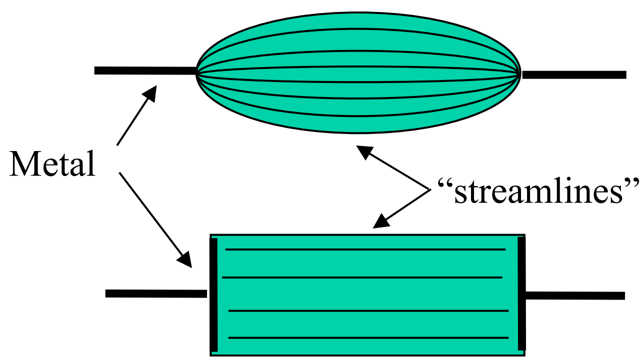
Fig. 24-9 (*a*) Twelve electrons uniformly spaced around a circle. (*b*) The electrons nonuniformly spaced along an arc of the original circle.

(b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 24-9*b*), what then is the potential at C? How does the electric field at C change (if at all)?

Reasoning: The potential is still given by Eq. 24-28, because the distance between C and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.

Resistivity:

"Resistance at a point"



These two devices could have the same resistance R, when measured on the outgoing metal leads. However, it is obvious that inside of them go on different things.

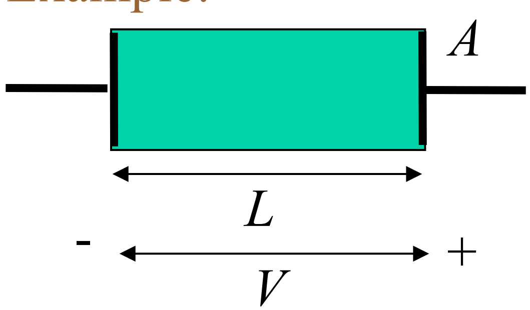
In order to quantify this, we introduce the concept of resistivity:

$$\rho = \frac{E}{J} \text{ or, as vectors, } \vec{E} = \rho \vec{J}$$

$$\text{Conductivity}: \ \sigma = \frac{1}{\rho}$$

Resistivity is associated with a material, resistance with respect to a device constructed with the material.

Example:



$$E = \frac{V}{L}, \quad J = \frac{i}{A} \quad \rho = \frac{V/L}{i/A} = R\frac{A}{L}$$

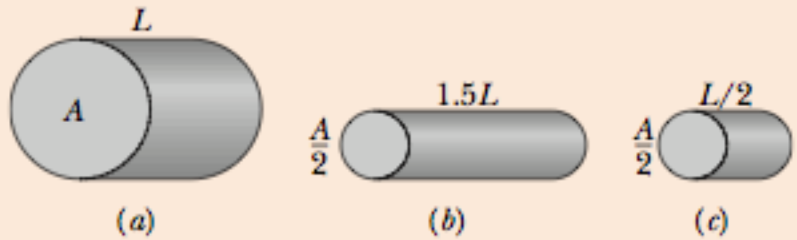
$$\boxed{R = \rho \frac{L}{A}}$$

Makes sense!
For a given material:

Longer → More resistance
Thicker → Less resistance

CHECKPOINT 3

The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference V is placed across their lengths.



$$R = \rho \frac{L}{A}$$

Ohm's laws

$$R \equiv \frac{V}{i} \quad \text{and therefore} \quad i = \frac{V}{R} \quad \text{and} \quad V = iR$$

••63 SSM WWW In the circuit of Fig. 27-65, $\mathscr{E}$ = 1.2 kV, C = 6.5 μF, $R_1 = R_2 = R_3$ = 0.73 MΩ. With C completely uncharged, switch S is suddenly closed (at $t = 0$). At $t = 0$, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2, and (c) current i_3 in resistor 3? At $t = \infty$ (that is, after many time constants), what are (d) i_1, (e) i_2, and (f) i_3? What is the potential difference V_2 across resistor 2 at (g) $t = 0$ and (h) $t = \infty$? (i) Sketch V_2 versus t between these two extreme times.

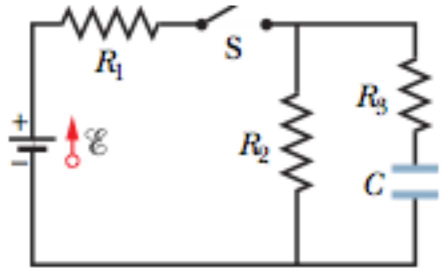
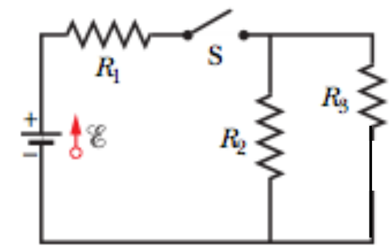
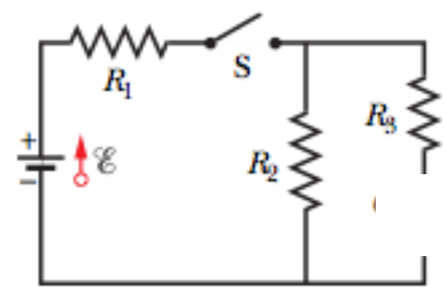


Fig. 27-65 Problem 63.

Sketch of the solution:



a,b,c) At t=0 capacitor is discharged, so it behaves as a wire: Resistors 2,3 in parallel with each other and the resulting resistor in series with R1.



d,e,f) At t=infinity, capacitor fully charged, behaves as if the circuit is open. So there is no current in R3, and R1, R2 are in series.

Magnetic versus electrostatic forces:

An important difference in electric and magnetic fields is how they act on charges.

For electrostatic forces : $\vec{F} = q\vec{E}$

For magnetic forces, $\vec{F} = q\vec{v} \times \vec{B}$

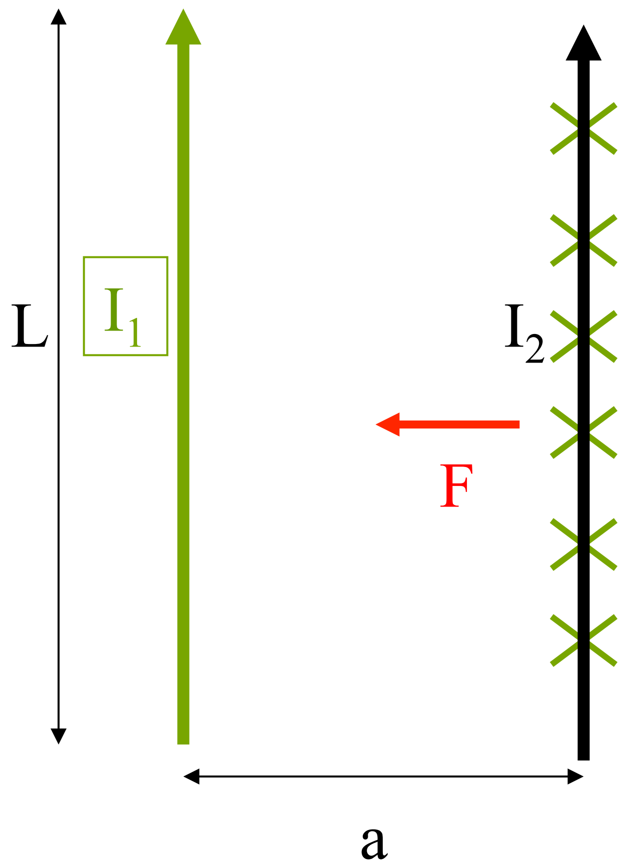
Charges that do not move, do not feel magnetic forces.

Magnetic forces are perpendicular to both the velocity of charges and to the magnetic field (electric forces are parallel to the field).

Since magnetic forces are perpendicular to the velocity, they do no work!

Speed of particles moving in a magnetic field remains constant in magnitude, the direction changes. Kinetic energy is constant (no work).

Force between parallel wires carrying current

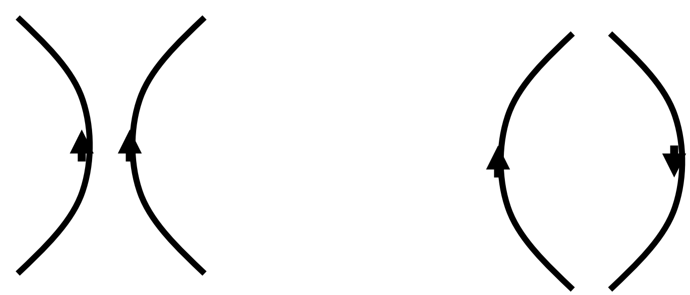


Magnetic field due to wire 1 where the wire 2 is,

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

Force on wire 2 due to this field,

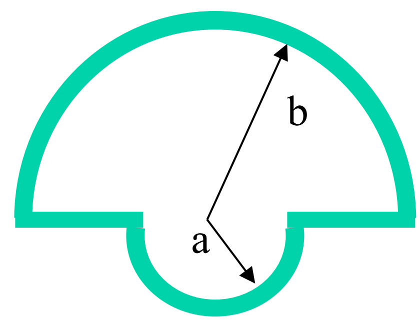
$$F_{21} = L\,I_2\,B_1 = \frac{\mu_0 L I_1 I_2}{2\pi a}$$



67 A length of wire is formed into a closed loop with radii *a* and *b*, and carries a current *i*.

a) What is the value of the magnitude and direction of B at the center?

b) Find the magnetic dipole moment of the circuit.

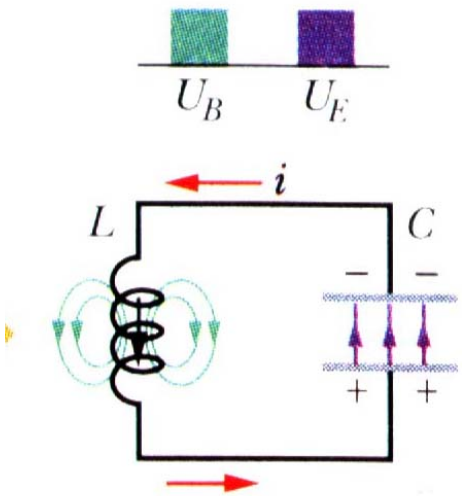


Straight pieces do not contribute to B.

Last class, arc: $B = \frac{\mu_0}{4\pi}\frac{i\phi_0}{r} = \frac{\mu_0}{4}\frac{i}{r}$ At center, $B = \frac{\mu_0 i}{4}\left(\frac{1}{a} + \frac{1}{b}\right)$

Dipole moment, $\mu = i$ Area $= i\left(\frac{\pi a^2}{2} + \frac{\pi b^2}{2}\right) = \frac{\pi i}{2}(a^2 + b^2)$

Electric oscillations: math



$$E_{tot} = E_{mag} + E_{elec}$$

$$E_{tot} = \frac{1}{2} L i^2 + \frac{1}{2} \frac{q^2}{C}$$

$$\frac{dE_{tot}}{dt} = 0 = \frac{1}{2} L\left(2i \frac{di}{dt}\right) + \frac{1}{2C}\left(2q \frac{dq}{dt}\right)$$

$$i = \frac{dq}{dt}$$

$$0 = L\left(i \frac{di}{dt}\right) + \frac{1}{C}(qi)$$

$$0 = L \frac{d^2 q}{dt^2} + \frac{q}{C}$$

Compare with:

$$M \frac{d^2 x}{dt^2} + k\,x = 0$$

So the math is exactly the same as in the case of mechanical oscillators if one makes the substitutions:

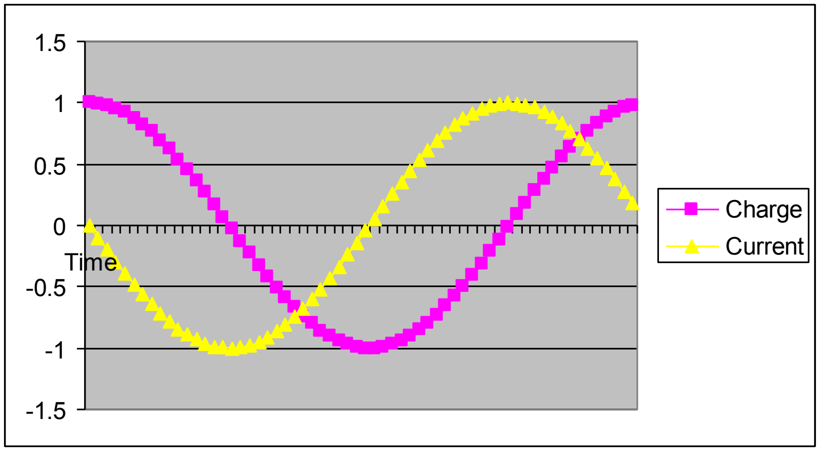
$$q \rightarrow x \qquad 1/C \rightarrow k$$

$$i \rightarrow v \qquad L \rightarrow M$$

$$q = q_0 \cos(\omega t + \phi_0)$$

$$\omega = \sqrt{\frac{1}{LC}}$$

Electric oscillations: graphs and energy

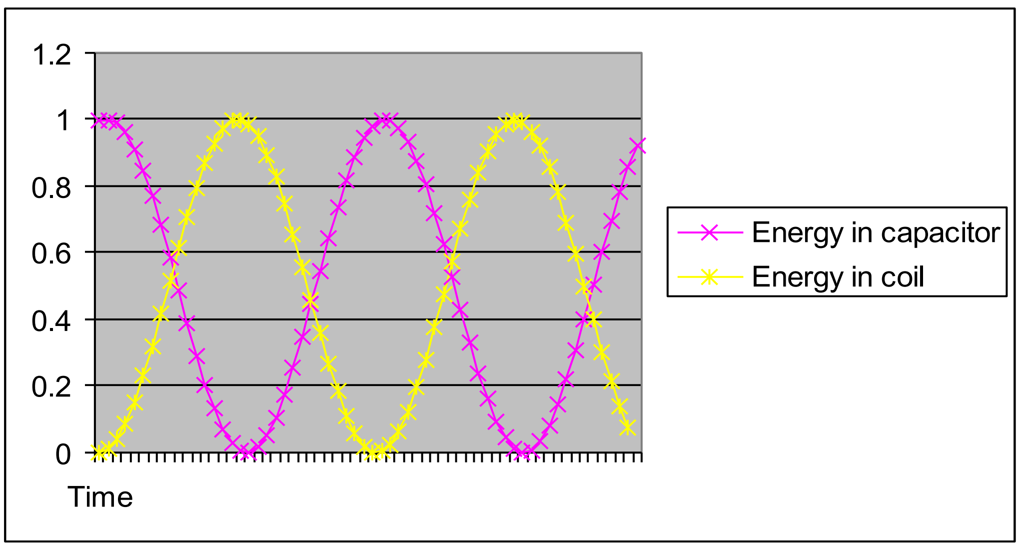


$$q = q_0 \cos(\omega t + \phi_0)$$

$$i = -\omega q_0 \sin(\omega t + \phi_0)$$

$$E_{mag} = \frac{1}{2} L i^2 = \frac{1}{2} L \omega^2 q_0^{\ 2} \sin^2(\omega t + \phi_0)$$

$$E_{ele} = \frac{1}{2} \frac{q^2}{C} = \frac{1}{2C} q_0^{\ 2} \cos^2(\omega t + \phi_0)$$



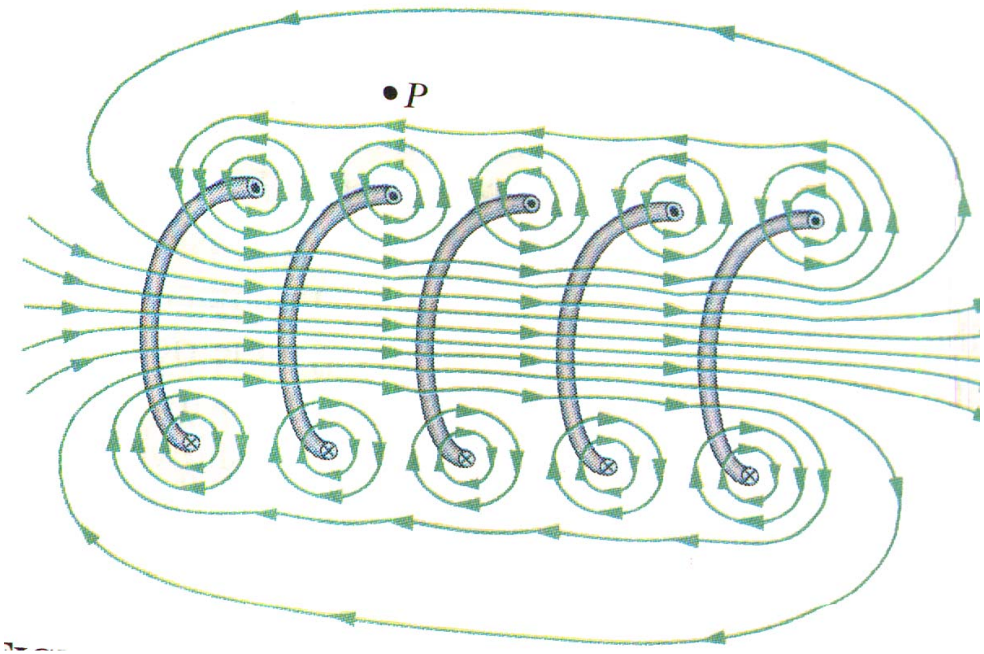
And remembering that,

$$\cos^2 x + \sin^2 x = 1, \text{ and } \omega = \sqrt{\frac{1}{LC}}$$

$$E_{tot} = E_{mag} + E_{ele} = \frac{1}{2C} q_0^{\ 2}$$

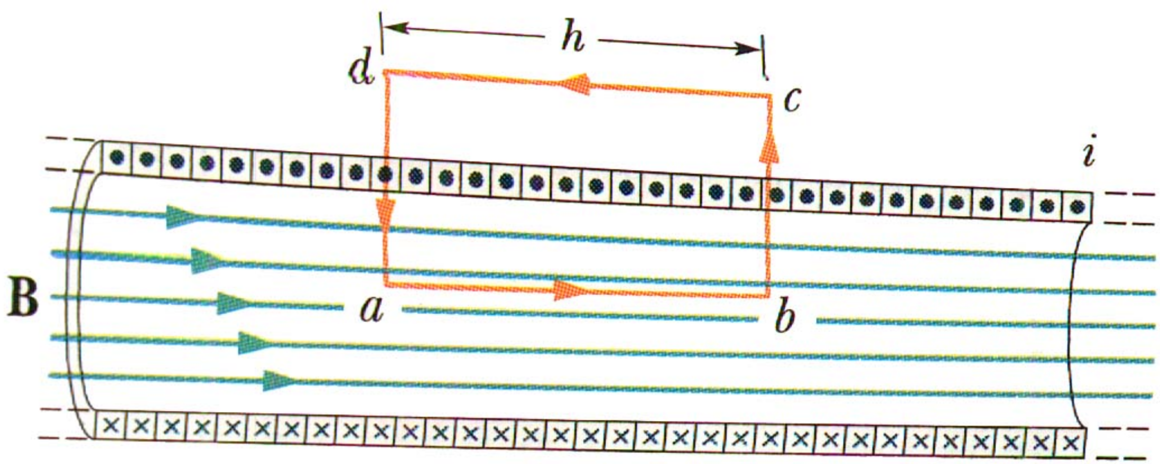
The energy is constant and equal to what we started with.

Magnetic field of a solenoid



No field outside, field concentrated inside.

Idealized: (infinitely tightly woven, infinite)



Considering Amperian loop abcd,

$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} + \int_c^d \vec{B}\cdot d\vec{s} + \int_a^b \vec{B}\cdot d\vec{s}$$

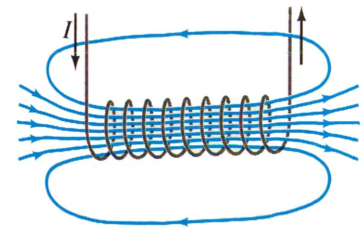
$$= Bh + 0 + 0 + 0 = Bh$$

$i_{enc} = inh,$

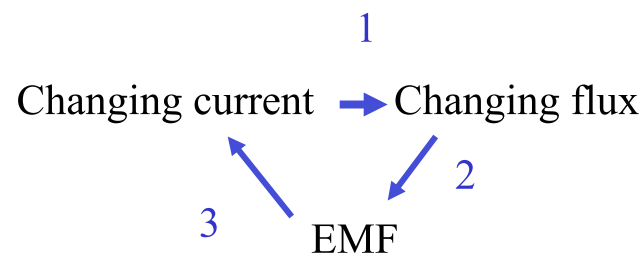
$$B = \mu_0 in$$

n=turns per unit length

Self-inductance



Suppose you have a coil with a current that changes with time. The magnetic flux in it will changewith time too. Therefore it will induce an emf in the coil!
This effect is called self-induction.



Combining, $\overset{1}{L = \dfrac{N\Phi}{i}}$ With Faraday's law, $\overset{2}{emf = -N\dfrac{d\Phi}{dt}}$

3

$$emf = -\frac{d(N\Phi)}{dt} = -\frac{d(Li)}{dt} = -L\frac{di}{dt}$$

When we "take a walk" around a circuit to solve it, every time we find a solenoid, we add a term *-L di/dt*.

If the current is constant, "the coil is invisible" (piece of wire).
If we have sudden changes of current we can get large emf's with a coil.

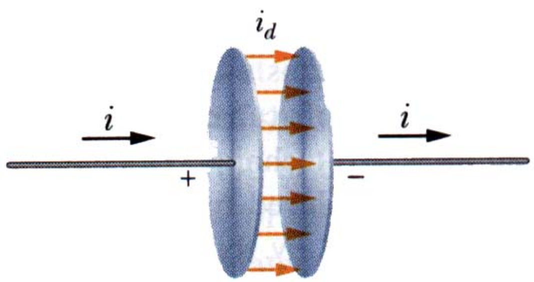
The new term in Ampere's law (Ampere-Faraday),

$$\oint_S \vec{B}\cdot d\vec{S} = \mu_0\varepsilon_0\frac{d\Phi_E}{dt} + \mu_0 i$$

$$\oint_S \vec{B}\cdot d\vec{S} = \mu_0(i + i_d)$$

Faraday's law told us that a changing magnetic flux produces an electric field. This law tells us that a changing electric flux produces a magnetic field.

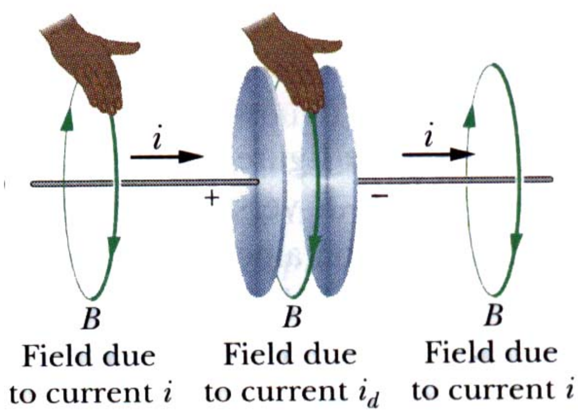
Where $i_d = \varepsilon_0\dfrac{d\Phi_E}{dt}$, "displacement current"



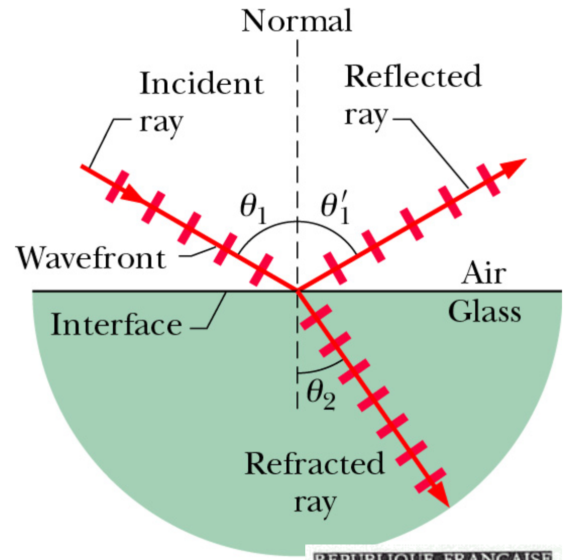
Example: a charging capacitor,

$$q = \varepsilon_0 AE \qquad \frac{dq}{dt} = \varepsilon_0 A\frac{dE}{dt} = \varepsilon_0\frac{d\Phi_E}{dt}$$

$$i = \varepsilon_0\frac{d\Phi_E}{dt} = i_d$$



Experiments show that reflection and refraction keep the outgoing rays in the same plane as the ingoing rays and the normal of the surface and are governed by two laws:



Law of reflection: the angle of incidence θ_1 equals the angle of reflection θ'_1.

Law of refraction: $n_2 \sin\theta_2 = n_1 \sin\theta_1$ Snell’s law.

Where n_1 and n_2 are called the “index of refraction” o media 1 and 2 respectively. These quantities are determined experimentally and listed in tables. For air n is very approximately 1. All other substances have larger indices of refraction.

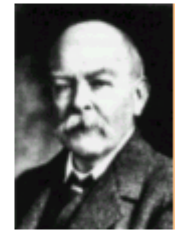
Willebrord Snell 1580-1626

René Descartes 1596-1650

If n_1 equals n_2 then light travels straight. If n_1 is smaller than n_2 then the refracted angle is smaller than the incident, otherwise larger. It can be so large that the light is actually reflected. It can never be so large that it will go beyond the normal.

Electromagnetic waves are able to transport energy from transmitter to receiver (example: from the Sun to our skin).

The amount of power transported by the wave and its direction is quantified by a vector called Poynting vector.



John Henry Poynting (1852-1914)

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$|S| = \frac{1}{\mu_0} EB = \frac{1}{c\mu_0} E^2$$

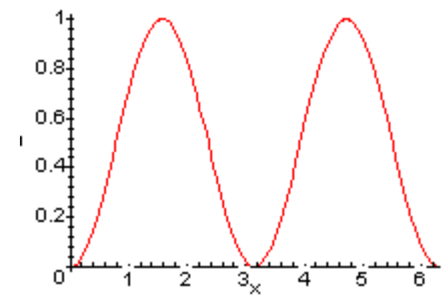
For a wave since E is perpendicular to B

The units are power per unit area, i.e. Watt/m^2

In a wave, the fields change with time in a fixed way. Therefore the Poynting vector changes too. A better measure of the amount of energy is obtained by averaginng the Poynting vector over one wave cycle. The resulting quantity is called intensity

$$I = \bar{S} = \frac{1}{c\mu_0}\overline{E^2} = \frac{1}{c\mu_0}E_m{}^2\,\overline{\sin^2(kx-\omega t)}$$

The average of $\sin^2$ over one cycle is 1/2.



$$I = \frac{1}{2c\mu_0}E_m{}^2$$

Engineers commonly use the term "root mean square" value of a quantity,

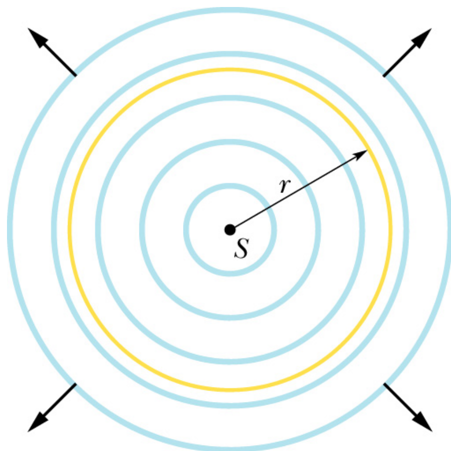
$$E_{rms} = \frac{E_m}{\sqrt{2}} = \frac{\sqrt{2}}{2}E_m \cong 1.44E_m$$

$$I = \frac{1}{c\mu_0}E_{rms}{}^2$$

Both fields have the same energy density

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (cB)^2 = \frac{1}{2}\varepsilon_0\frac{B^2}{\varepsilon_0\mu_0} = u_B$$

The intensity of a wave is power per unit area. If one has a source that emits isotropically (equally in all directions) the power emitted by the source pierces a larger and larger sphere as the wave travels outwards. Therefore,



$$I = \frac{P_s}{4\pi r^2}$$

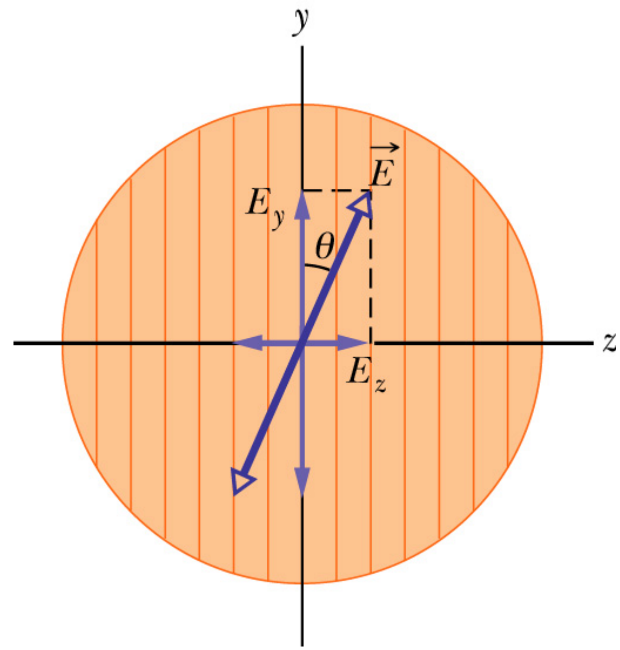
So the power per unit area decreases as the inverse of distance squared.

Waves not only carry energy but also momentum. The effect is very small (we don't ordinarily feel pressure from light). If light is completely absorbed during an interval Δt, the momentum transferred is given by,

$$\Delta p = \frac{\Delta u}{c}$$

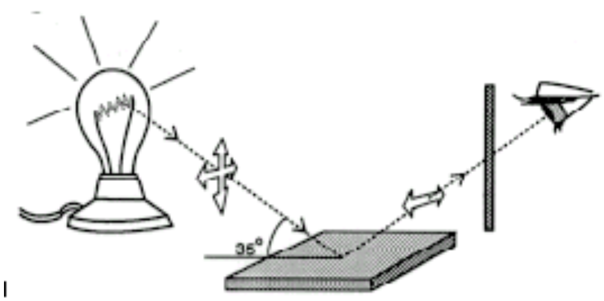
And if light is reflected, one gets double this amount.

When polarized light hits a polarizing sheet, only the component of the field aligned with the sheet will get through.



$$E_y = E\cos(\theta)$$

And therefore: $$I = I_0 \cos^2 \theta$$



Polarized sunglasses operate on this formula. They cut the horizontally polarized light from glare (reflections on roads, cars, etc).


That's all Folks!