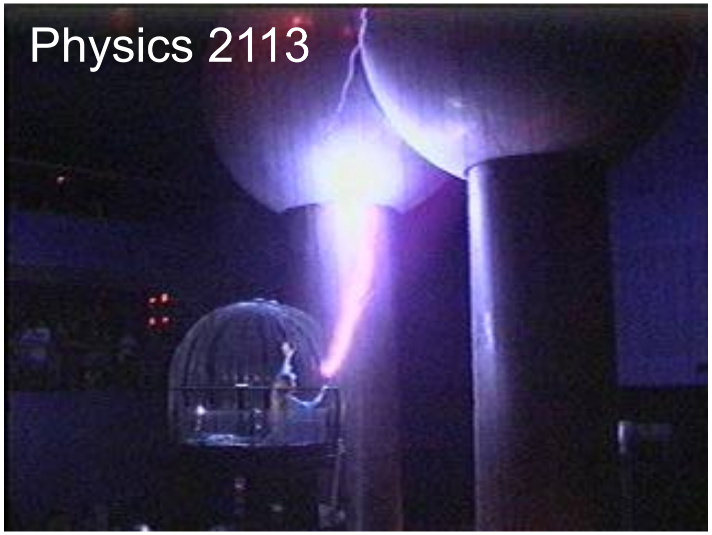


Physics 2113
Lecture 36: FRI 21 NOV

CH32: Magnetism

Sample problems

1 Figure 32-19*a* shows a capacitor, with circular plates, that is being charged. Point *a* (near one of the connecting wires) and point *b* (inside the capacitor gap) are equidistant from the central axis, as are point *c* (not so near the wire) and point *d* (between the plates but outside the gap). In Fig. 32-19*b*, one curve gives the variation with distance *r* of the magnitude of the magnetic field inside and outside the wire. The other curve gives the variation with distance *r* of the magnitude of the magnetic field inside and outside the gap. The two curves partially overlap. Which of the three points on the curves correspond to which of the four points of Fig. 32-19*a*?

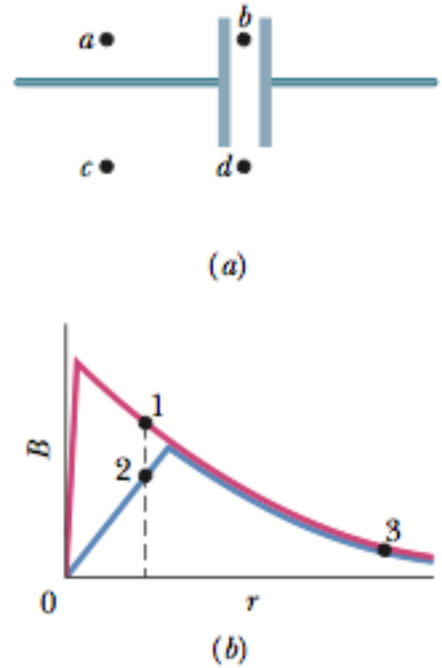
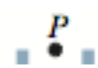


Fig. 32-19 Question 1.



Ampere's law :

$$\oint_S \vec{B} \cdot d\vec{S} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i$$

Problem 2.

••3 SSM ILW A Gaussian surface in the shape of a right circular cylinder with end caps has a radius of 12.0 cm and a length of 80.0 cm. Through one end there is an inward magnetic flux of 25.0 μWb. At the other end there is a uniform magnetic field of 1.60 mT, normal to the surface and directed outward. What are the (a) magnitude and (b) direction (inward or outward) of the net magnetic flux through the curved surface?

3. **THINK** Gauss' law for magnetism states that the net magnetic flux through any closed surface is zero.

EXPRESS Mathematically, Gauss' law for magnetism is expressed as $\oint \vec{B}\cdot d\vec{A}=0$. Now, our Gaussian surface has the shape of a right circular cylinder with two end caps and a curved surface. Thus,

$$\oint \vec{B}\cdot d\vec{A}=\Phi_1+\Phi_2+\Phi_C,$$

where Φ_1 is the magnetic flux through the first end cap, Φ_2 is the magnetic flux through the second end cap, and Φ_C is the magnetic flux through the curved surface. Over the first end the magnetic field is inward, so the flux is $\Phi_1 = -25.0\ \mu$Wb. Over the second end the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder.

••8 GO *Nonuniform electric flux.* Figure 32-29 shows a circular region of radius $R = 3.00$ cm in which an electric flux is directed out of the plane of the page. The flux encircled by a concentric circle of radius r is given by $\Phi_{E,enc} = (0.600 \text{ V}\cdot\text{m/s})(r/R)t$, where $r \le R$ and t is in seconds. What is the magnitude of the induced magnetic field at radial distances (a) 2.00 cm and (b) 5.00 cm?

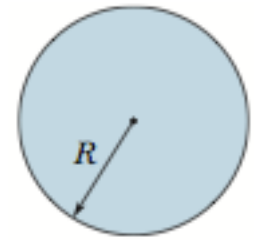


Fig. 32-29

Ampere's law: $\oint_S \vec{B}\cdot d\vec{S} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i$

8. (a) Application of Eq. 32-3 along the circle referred to in the second sentence of the problem statement (and taking the derivative of the flux expression given in that sentence) leads to

$$B(2\pi r) = \varepsilon_0\mu_0\left(0.60 \text{ V}\cdot\text{m/s}\right)\frac{r}{R}.$$

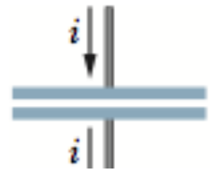
Using $r = 0.0200$ m (which, in any case, cancels out) and $R = 0.0300$ m, we obtain

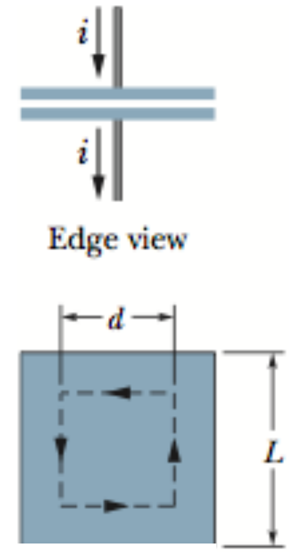
$$B = \frac{\varepsilon_0\mu_0(0.60 \text{ V}\cdot\text{m/s})}{2\pi R} = \frac{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(0.60 \text{ V}\cdot\text{m/s})}{2\pi(0.0300 \text{ m})}$$
$$= 3.54\times10^{-17} \text{ T}.$$

(b) For a value of r larger than R, we must note that the flux enclosed has already reached its full amount (when $r = R$ in the given flux expression). Referring to the equation we wrote in our solution of part (a), this means that the final fraction (r/R) should be replaced with unity. On the left hand side of that equation, we set $r = 0.0500$ m and solve. We now find

$$B = \frac{\varepsilon_0\mu_0(0.60 \text{ V}\cdot\text{m/s})}{2\pi r} = \frac{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(0.60 \text{ V}\cdot\text{m/s})}{2\pi(0.0500 \text{ m})}$$
$$= 2.13\times10^{-17} \text{ T}.$$

••22 Nonuniform displacement current. Figure 32-29 shows a circular region of radius $R = 3.00$ cm in which a displacement current i_d is directed out of the page. The magnitude of the displacement current is given by $i_d = (3.00\text{ A})(r/R)$, where r is the radial distance ($r \le R$). What is the magnitude of the magnetic field due to i_d at radial distances (a) 2.00 cm and (b) 5.00 cm?





$$\oint_S \vec{B}\cdot d\vec{S} = \mu_0 (i_d + i)$$

Where $i_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}$, "displacement current"

22. (a) Eq. 32-11 applies (though the last term is zero) but we must be careful with $i_{d,\text{enc}}$. It is the enclosed portion of the displacement current. Thus Eq. 32-17 (which derives from Eq. 32-11) becomes, with i_d replaced with $i_{d,\text{enc}}$,

$$B = \frac{\mu_0 i_{d\ \text{enc}}}{2\pi r} = \frac{\mu_0 (3.00\text{ A})(r/R)}{2\pi r}$$

which yields (after canceling r, and setting $R = 0.0300$ m) $B = 20.0\ \mu$T.

(b) Here $i_d = 3.00$ A, and we get $B = \dfrac{\mu_0 i_d}{2\pi r} = 12.0\ \mu\text{T}$.

••50 A magnetic rod with length 6.00 cm, radius 3.00 mm, and (uniform) magnetization 2.70×10^3 A/m can turn about its center like a compass needle. It is placed in a uniform magnetic field $\vec{B}$ of magnitude 35.0 mT, such that the directions of its dipole moment and $\vec{B}$ make an angle of 68.0°. (a) What is the magnitude of the torque on the rod due to $\vec{B}$? (b) What is the change in the orientation energy of the rod if the angle changes to 34.0°?

$$M = \frac{\text{measured magnetic moment}}{V}.$$

$$\vec{\tau} = \vec{\mu} \times \vec{B},$$

50. (a) Equation 29-36 gives

$$\tau = \mu_{\text{rod}} B \sin\theta = (2700 \text{ A/m})(0.06 \text{ m})\pi(0.003 \text{ m})^2(0.035 \text{ T})\sin(68°) = 1.49 \times 10^{-4} \text{ N}\cdot\text{m}.$$

We have used the fact that the volume of a cylinder is its length times its (circular) cross sectional area.

(b) Using Eq. 29-38, we have $U(\theta) = -\vec{\mu}\cdot\vec{B}.$

$$\begin{aligned}\Delta U &= -\mu_{\text{rod}} B(\cos\theta_f - \cos\theta_i) \\ &= -(2700 \text{ A/m})(0.06 \text{ m})\pi(0.003\text{m})^2(0.035\text{T})[\cos(34°) - \cos(68°)] \\ &= -72.9\ \mu\text{J}.\end{aligned}$$

Summary:

Gauss' Law for Magnetic Fields

- Gauss' law for magnetic fields,

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0,$$

Maxwell's Extension of Ampere's Law

- A changing electric field induces a magnetic field given by,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Maxwell's law and Ampere's law can be written as the single equation

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

Displacement Current

- We define the fictitious displacement current due to a changing electric field as

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}.$$

- Equation 32-5 then becomes

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}}$$

Maxwell's Equations

- Four equations are as follows:

$$\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

Summary:

Spin Magnetic Dipole Moment

- Spin angular momentum of electron is associated with spin magnetic dipole momentum through,

$$\vec{\mu}_s = -\frac{e}{m}\vec{S}.$$

- For a measurement along a *z* axis, the component S_z can have only the values given by

$$S_z = m_s \frac{h}{2\pi}, \qquad \text{for } m_s = \pm\tfrac{1}{2},$$

- Similarly,

$$\mu_{s,z} = \pm\frac{eh}{4\pi m} = \pm\mu_B,$$

- Where the Bohr magneton is

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \text{J/T}.$$

- The energy *U*

$$U = -\vec{\mu}_s \cdot \vec{B}_{ext} = -\mu_{s,z}B_{ext}.$$

Orbital Magnetic Dipole Momentum

- Angular momentum of an electron is associated with orbital magnetic dipole momentum as

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}.$$

- Orbital angular momentum is quantized,

$$L_{orb,z} = m_\ell \frac{h}{2\pi},$$
$$\text{for } m_\ell = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit}).$$

- The associated magnetic dipole moment is given by

$$\mu_{orb,z} = -m_\ell \frac{eh}{4\pi m} = -m_\ell \mu_B.$$

- The energy *U*

$$U = -\vec{\mu}_{orb} \cdot \vec{B}_{ext} = -\mu_{orb,z}B_{ext}.$$

Summary:

Diamagnetism

- Diamagnetic materials exhibit magnetism only when placed in an external magnetic field; there they form magnetic dipoles directed opposite the external field. In a nonuniform field, they are repelled from the region of greater magnetic field.

Paramagnetism

- Paramagnetic materials have atoms with a permanent magnetic dipole moment but the moments are randomly oriented unless the material is in an external magnetic field. The extent of alignment within a volume V is measured as the magnetization M, given by

$$M = \frac{\text{measured magnetic moment}}{V}.$$

- Complete alignment (saturation) of all *N* dipoles in the volume gives a maximum value $M_{max} = N\mu/V$. At low values of the ratio $\boldsymbol{B}_{ext}/T$,

$$M = C\frac{B_{\text{ext}}}{T}$$

Ferromagnetism

- The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions (domains). Alignment is eliminated at temperatures above a material's Curie temperature. In a nonuniform external field, a ferromagnetic material is attracted to the region of greater magnetic field.