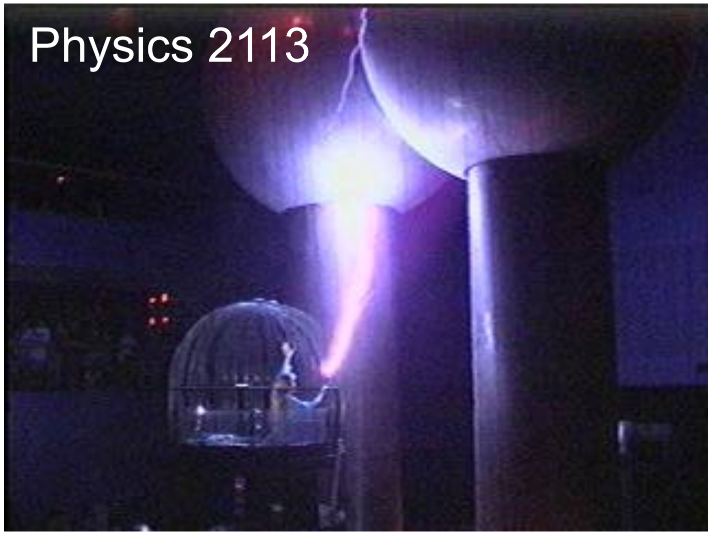


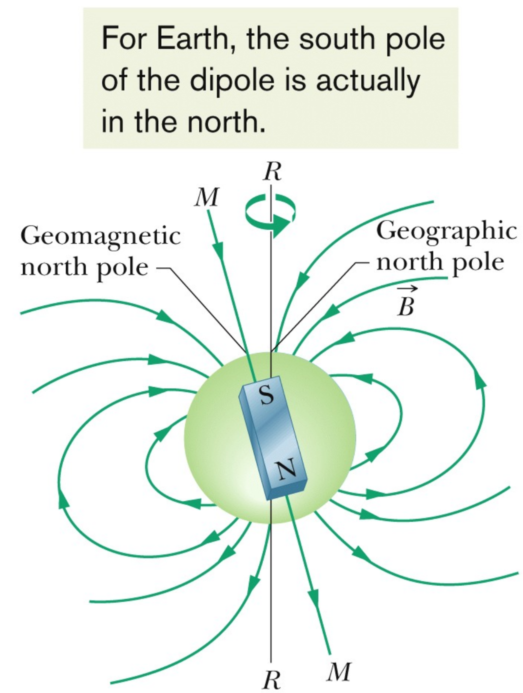
Physics 2113
Lecture 35: WED 19 NOV

CH32: Magnetism

The Magnetism of Earth

The Earth is a huge magnet; for points near Earth's surface, its magnetic field can be approximated as the field of a huge bar magnet — a magnetic dipole — that straddles the center of the planet. Figure shown here is an idealized symmetric depiction of the dipole field, without the distortion caused by passing charged particles from the Sun.

The direction of the magnetic field at any location on Earth's surface is commonly specified in terms of two angles. The **field declination** is the angle (left or right) between geographic north (which is toward 90° latitude) and the horizontal component of the field. The **field inclination** is the angle (up or down) between a horizontal plane and the field's direction.



Earth's magnetic field represented as a dipole field. The dipole axis MM makes an angle of 11.5° with Earth's rotational axis RR. The south pole of the dipole is in Earth's Northern Hemisphere.

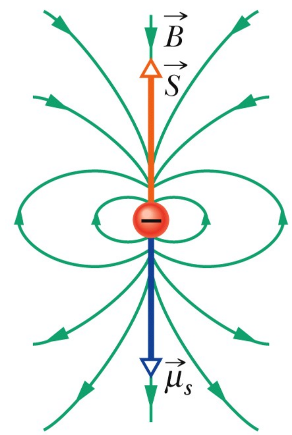
Magnetism of matter

The magnetic properties of matter are determined by the behavior of the electrons in the atoms that constitute matter.

Spin Magnetic Dipole Moment. An electron has an intrinsic angular momentum called its spin angular momentum (or just spin) $\boldsymbol{S}$; associated with this spin is an intrinsic spin magnetic dipole moment $\boldsymbol{\mu}_s$. (By intrinsic, we mean that $\boldsymbol{S}$ and $\boldsymbol{\mu}_s$ are basic characteristics of an electron, like its mass and electric charge.) Vectors $\boldsymbol{S}$ and $\boldsymbol{\mu}_s$ are related by

$$\vec{\mu}_s = -\frac{e}{m}\vec{S},$$

For an electron, the spin is opposite the magnetic dipole moment.



in which e is the elementary charge (1.60×10^{-19} C) and m is the mass of an electron (9.11×10^{-31} kg). The minus sign means that $\boldsymbol{\mu}_s$ and $\boldsymbol{S}$ are oppositely directed.

For a measurement along a z axis, the component $\boldsymbol{S}_z$ can have only the values given by

$$S_z = m_s \frac{h}{2\pi}, \text{ for } m_s = \pm ½$$

Similarly, $\mu_{s,z} = \pm \frac{eh}{4\pi m} = \pm \mu_B,$ where μ_B is the Bohr magneton:

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24} \text{ J/T}.$$

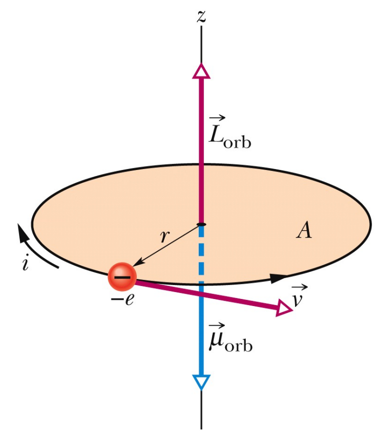
Energy. When an electron is placed in an external magnetic field $\boldsymbol{B}_{ext}$, an energy U can be associated with the orientation of the electron's spin magnetic dipole moment $\boldsymbol{\mu}_s$ just as an energy can be associated with the orientation of the magnetic dipole moment $\boldsymbol{\mu}$ of a current loop placed in $\boldsymbol{B}$. The orientation energy for the electron is

$$U = -\vec{\mu}_s \cdot \vec{B}_{\text{ext}} = -\mu_{s,z} B_{\text{ext}},$$

where the z axis is taken to be in the direction of $\boldsymbol{B}_{ext}$.

Orbital Magnetic Dipole Moment. When it is in an atom, an electron has an additional angular momentum called its orbital angular momentum $\boldsymbol{L}_{orb}$. Associated with $\boldsymbol{L}_{orb}$ is an orbital magnetic dipole moment $\boldsymbol{\mu}_{orb}$ the two are related by

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}}.$$



The minus sign means that $\boldsymbol{\mu}_{orb}$ and $\boldsymbol{L}_{orb}$ have opposite directions. Orbital angular momentum is quantized and can have only measured values given by

$$L_{\text{orb},z} = m_\ell \frac{h}{2\pi}, \qquad \text{for } m_l=0, \pm1, \pm2, \ldots, \pm(\text{limit integer})$$

The associated magnetic dipole moment is given by

$$\mu_{\text{orb},z} = -m_\ell \frac{eh}{4\pi m} = -m_\ell \mu_{\text{B}}.$$

An electron moving at constant speed *v* in a circular path of radius *r* that encloses an area *A*.

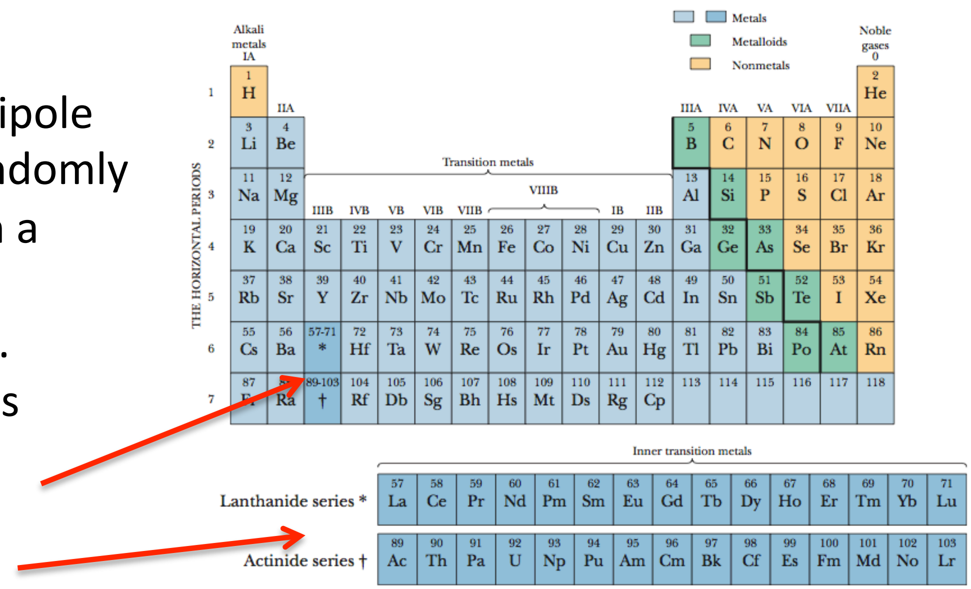
The energy U associated with the orientation of the orbital magnetic dipole moment in an external magnetic field B_{ext} is

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}}.$$

Most materials can be categorized into three different behaviors from the magnetic point of view:

-Diamagnetic materials. Most materials exhibit this behavior. Their atoms do not have permanent magnetic dipoles, but they react to a magnetic field by creating magnetic dipole moments. The induced dipole moments oppose the external field. For most materials this effect is very weak. It disappears when the field is removed.

-Paramagnetic materials. Have permanent dipole moments in their molecules but they are randomly oriented, giving no net magnetic field. When a Magnetic field is applied, they can realign themselves, producing a net dipole moment. Rare earth elements and transition elements exhibit this behavior.

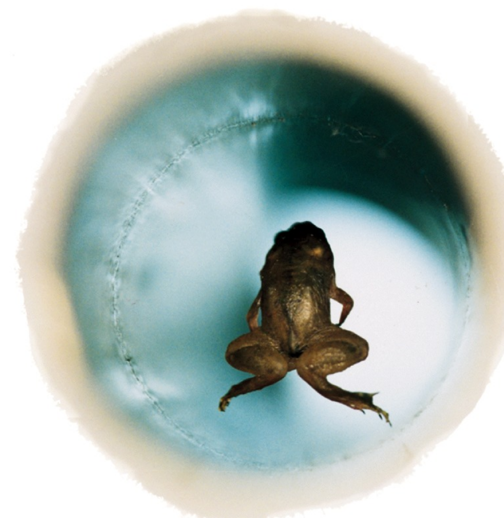


-Ferromagnetic materials. Have permanent dipole moments in their atoms that are aligned, giving a net dipole moment. This is what we normally call “magnets”. Iron, cobalt, nickel, gadolinium, dysprosium and their alloys.

Diamagnetism

A diamagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment directed opposite $\vec{B}_{ext}$. If the field is nonuniform, the diamagnetic material is repelled *from* a region of greater magnetic field *toward* a region of lesser field.

Levitating Frog: The frog in the figure is diamagnetic (as is any other animal). When the frog was placed in the diverging magnetic field near the top end of a vertical current- carrying solenoid, every atom in the frog was repelled upward, away from the region of stronger magnetic field at that end of the solenoid. The frog moved upward into weaker and weaker magnetic field until the upward magnetic force balanced the gravitational force on it, and there it hung in midair. The frog is not in discomfort because every atom is subject to the same forces and thus there is no force variation within the frog.



Courtesy A.K. Geim, University of Manchester, UK

Paramagnetic materials

A paramagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the paramagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

Paramagnetic materials have atoms with a permanent magnetic dipole moment but the moments are randomly oriented, with no net moment, unless the material is in an external magnetic field $\mathbf{B}_{ext}$, where the dipoles tend to align with that field. The extent of alignment within a volume V is measured as the magnetization M,

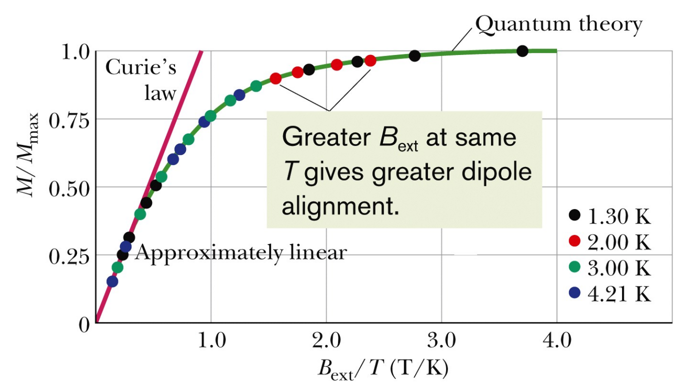
given by

$$M = \frac{\text{measured magnetic moment}}{V}.$$

Complete alignment (saturation) of all N dipoles in the volume gives a maximum value $M_{max} = N\mu/V$.

At low values of the ratio $\mathbf{B}_{ext}/T$,

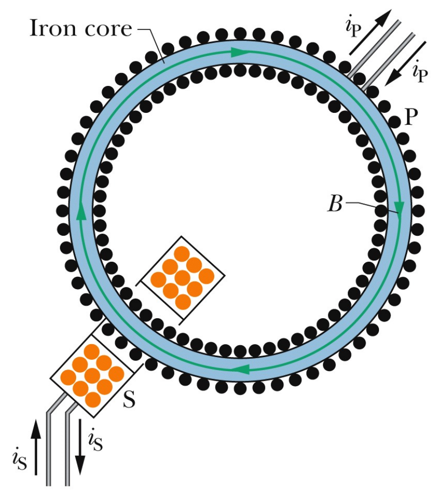
$$M = C\frac{B_{ext}}{T}$$



where T is the temperature (in kelvins) and C is a material's Curie constant.

In a nonuniform external field, a paramagnetic material is attracted to the region of greater magnetic field.

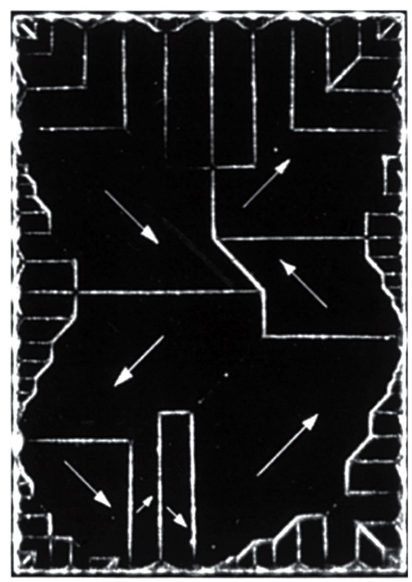
Ferromagnetic materials



A Rowland ring. A primary coil P has a core made of the ferromagnetic material to be studied (here iron). The core is magnetized by a current i_P sent through coil P. (The turns of the coil are represented by dots.) The extent to which the core is magnetized determines the total magnetic field ***B*** within coil P. Field ***B*** can be measured by means of a secondary coil S.

A ferromagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a strong magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the ferromagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

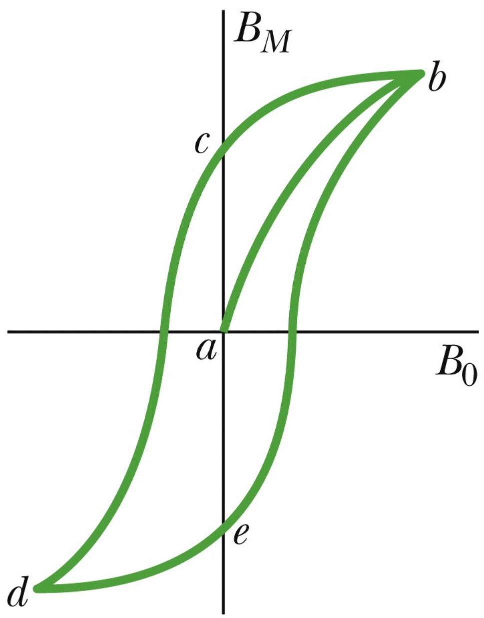
The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions know as **magnetic domains**.



Courtesy Ralph W. DeBlois

A photograph of domain patterns within a single crystal of nickel; white lines reveal the boundaries of the domains. The white arrows superimposed on the photograph show the orientations of the magnetic dipoles within the domains and thus the orientations of the net magnetic dipoles of the domains. The crystal as a whole is unmagnetized if the net magnetic field (the vector sum over all the domains) is zero.

Hysteresis loop



A ferromagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a strong magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the ferromagnetic material is attracted *toward* a region of greater magnetic field *from* a region of lesser field.

The lack of retraceability shown in the Figure is called **hysteresis**, and the curve *bcdeb* is called a **hysteresis loop**. Note that at points *c* and *e* the iron core is magnetized, even though there is no current in the toroid windings; this is the familiar phenomenon of permanent magnetism.

Hysteresis can be understood through the concept of magnetic domains. Evidently the motions of the domain boundaries and the reorientations of the domain directions are not totally reversible. When the applied magnetic field B0 is increased and then decreased back to its initial value, the domains do not return completely to their original configuration but retain some “memory” of their alignment after the initial increase. This memory of magnetic materials is essential for the magnetic storage of information.

Summary:

Gauss' Law for Magnetic Fields

- Gauss' law for magnetic fields,

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0,$$

Maxwell's Extension of Ampere's Law

- A changing electric field induces a magnetic field given by,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- Maxwell's law and Ampere's law can be written as the single equation

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

Displacement Current

- We define the fictitious displacement current due to a changing electric field as

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}.$$

- Equation 32-5 then becomes

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,enc} + \mu_0 i_{enc}$$

Maxwell's Equations

- Four equations are as follows:

$$\oint \vec{E} \cdot d\vec{A} = q_{enc}/\varepsilon_0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

Summary:

Spin Magnetic Dipole Moment

- Spin angular momentum of electron is associated with spin magnetic dipole momentum through,

$$\vec{\mu}_s = -\frac{e}{m}\vec{S}.$$

- For a measurement along a *z* axis, the component S_z can have only the values given by

$$S_z = m_s\frac{h}{2\pi}, \qquad \text{for } m_s = \pm\tfrac{1}{2},$$

- Similarly,

$$\mu_{s,z} = \pm\frac{eh}{4\pi m} = \pm\mu_B,$$

- Where the Bohr magneton is

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \text{J/T}.$$

- The energy *U*

$$U = -\vec{\mu}_s \cdot \vec{B}_{ext} = -\mu_{s,z}B_{ext}.$$

Orbital Magnetic Dipole Momentum

- Angular momentum of an electron is associated with orbital magnetic dipole momentum as

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}.$$

- Orbital angular momentum is quantized,

$$L_{orb,z} = m_\ell\frac{h}{2\pi},$$
$$\text{for } m_\ell = 0, \pm 1, \pm 2, \ldots, \pm(\text{limit}).$$

- The associated magnetic dipole moment is given by

$$\mu_{orb,z} = -m_\ell\frac{eh}{4\pi m} = -m_\ell\mu_B.$$

- The energy *U*

$$U = -\vec{\mu}_{orb} \cdot \vec{B}_{ext} = -\mu_{orb,z}B_{ext}.$$

Summary:

Diamagnetism

- Diamagnetic materials exhibit magnetism only when placed in an external magnetic field; there they form magnetic dipoles directed opposite the external field. In a nonuniform field, they are repelled from the region of greater magnetic field.

Paramagnetism

- Paramagnetic materials have atoms with a permanent magnetic dipole moment but the moments are randomly oriented unless the material is in an external magnetic field. The extent of alignment within a volume V is measured as the magnetization M, given by

$$M = \frac{\text{measured magnetic moment}}{V}.$$

- Complete alignment (saturation) of all *N* dipoles in the volume gives a maximum value $M_{max} = N\mu/V$. At low values of the ratio $\boldsymbol{B}_{ext}/T$,

$$M = C\frac{B_{\text{ext}}}{T}$$

Ferromagnetism

- The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions (domains). Alignment is eliminated at temperatures above a material's Curie temperature. In a nonuniform external field, a ferromagnetic material is attracted to the region of greater magnetic field.