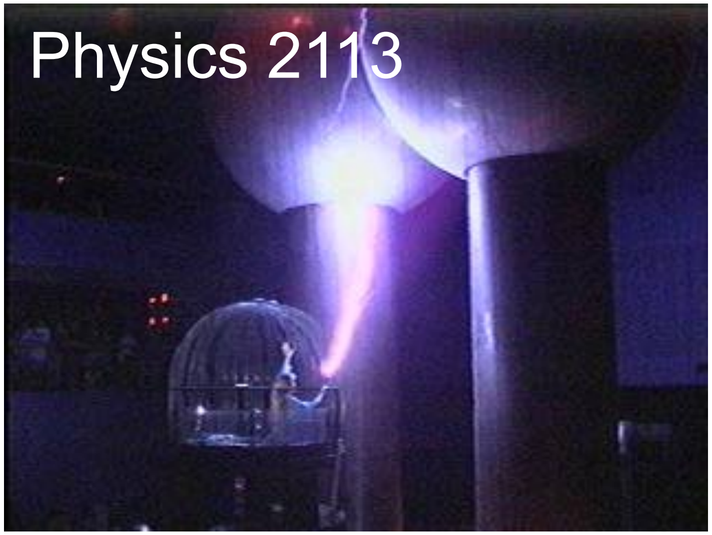

Physics 2113
Lecture 32: WED 12 NOV

CH31: Oscillations and AC

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Last class we discussed how one could construct electrical oscillators: by hooking together capacitors and coils one could get them to shuttle energy back and forth between them.

In realistic electrical circuits, energy is dissipated in the resistance of wires. As a consequence, oscillations do not continue forever but die off in time.

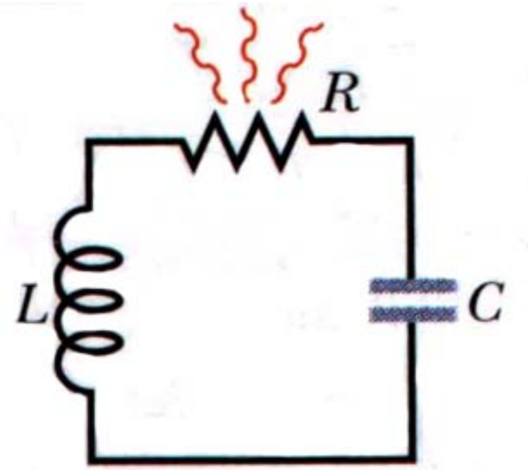
To keep electrical oscillators going, one needs to drive them by pumping energy into them. As when one pushes someone on a swing set, it is best to push in a rythmic mode.

In terms of electrical currents, this leads us to consider current sources that produce currents that go back and forth rythmically, this is what is called “alternating currents” AC.

The electricity in everyday outlet sockets is AC.

Damped electrical oscillators:

Let us start by studying how oscillations die off in a real life circuit.



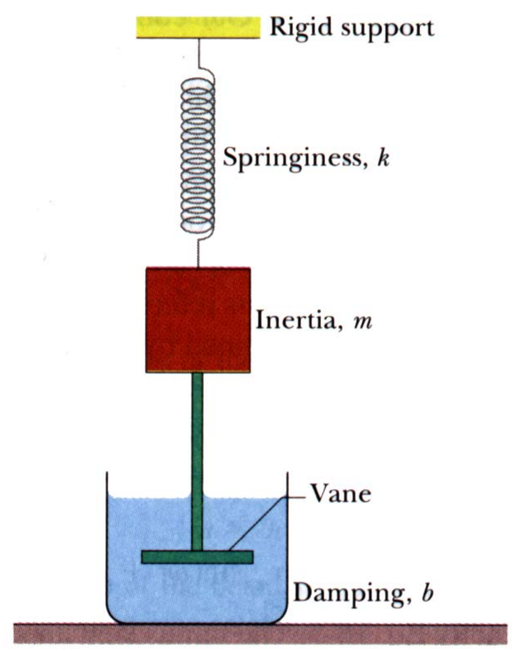
The capacitor and coil store energy, the resistor dissipates. It can be a real resistor, or just the resistance of the interconnecting wires.

$$E_{tot} = E_{ele} + E_{mag} \quad \text{but} \quad \frac{dE_{tot}}{dt} \neq 0 \qquad \boxed{\frac{dE_{tot}}{dt} = -i^2 R}$$

$$\frac{d}{dt}\left(\frac{1}{2}Li^2 + \frac{1}{2C}q^2\right) = -i^2 R \quad \Rightarrow L\frac{di}{dt} + \frac{q}{C} + iR = 0$$

$$\boxed{L\frac{d^2q}{dt^2} + \frac{q}{C} + \frac{dq}{dt}R = 0}$$

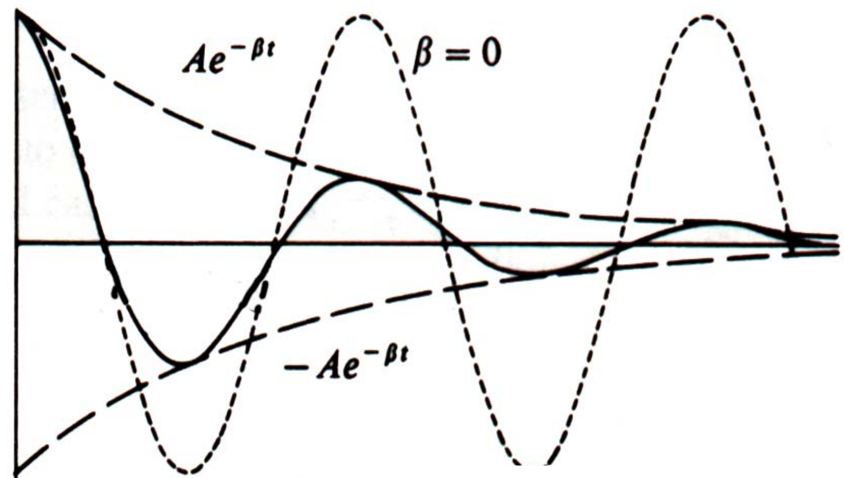
$$\text{Similar to:} \quad M\frac{d^2x}{dt^2} + k\,x + \beta\,v = 0$$



$$L\frac{d^2q}{dt^2}+\frac{q}{C}+\frac{dq}{dt}R=0$$

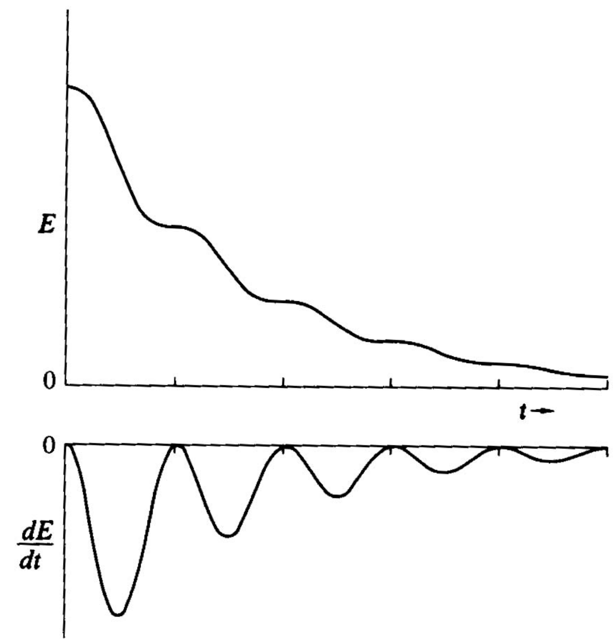
Similar to: $$M\frac{d^2x}{dt^2}+kx+\beta v=0$$

Solution: $$q=q_0e^{-\frac{R}{2L}t}\cos(\omega' t+\phi_0)$$



Where: $$\omega'=\sqrt{\omega^2-\left(\frac{R}{2L}\right)^2}$$

$$\omega=\sqrt{\frac{1}{LC}}$$ like last class.



Presence of damping shifts the frequency of oscillations.

Circuit oscillates with an amplitude that dies off exponentially in time.

Loss of energy is maximum when current moves through the circuit.

••25 ILW What resistance R should be connected in series with an inductance $L = 220$ mH and capacitance $C = 12.0\ \mu$F for the maximum charge on the capacitor to decay to 99.0% of its initial value in 50.0 cycles? (Assume $\omega' \approx \omega$.)

25. Since $\omega \approx \omega'$, we may write $T = 2\pi/\omega$ as the period and $\omega = 1/\sqrt{LC}$ as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$t = 50T = 50\left(\frac{2\pi}{\omega}\right) = 50\left(2\pi\sqrt{LC}\right) = 50\left(2\pi\sqrt{(220\times10^{-3}\,\text{H})(12.0\times10^{-6}\,\text{F})}\right)$$
$$= 0.5104\,\text{s}.$$

The maximum charge on the capacitor decays according to $q_{\max} = Qe^{-Rt/2L}$ (this is called the *exponentially decaying amplitude* in Section 31-5), where Q is the charge at time $t = 0$ (if we take $\phi = 0$ in Eq. 31-25). Dividing by Q and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}$$

which leads to

$$R = -\frac{2L}{t}\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2(220\times10^{-3}\,\text{H})}{0.5104\,\text{s}}\ln(0.99) = 8.66\times10^{-3}\ \Omega.$$

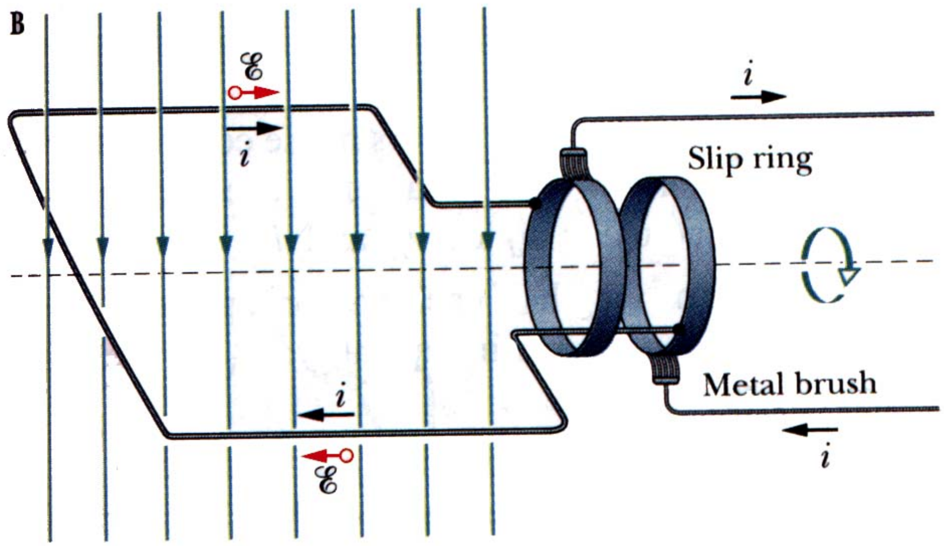
Alternating current:

To keep oscillations going we need to drive the circuit with an external emf that produces a current that goes back and forth.

Notice that there are two frequencies involved: one at which the circuit would oscillate “naturally”. The other is the frequency at which we drive the oscillation.

The interplay of these two frequencies can be very complex. However, as we just discussed the “natural” oscillation usually dies off quickly (exponentially) with time. Therefore in the long run, circuits actually oscillate with the frequency at which they are driven. (All this is true for the gentleman trying to make the lady swing back and forth in the picture too).

Alternating current:



We have studied that a loop of wire, spinning in a constant magnetic field will have an induced emf that oscillates with time,

$$E = E_m \sin(\omega_d t)$$

That is, it is an AC generator.

AC's are very easy to generate, they are also easy to amplify and decrease in voltage. This in turn makes them easy to send in distribution grids like the ones that power our homes. We will get to these issues very soon.

Because the interplay of AC and oscillating circuits can be quite complex, we will start by steps, studying how currents and voltages respond in various simple circuits to AC's.

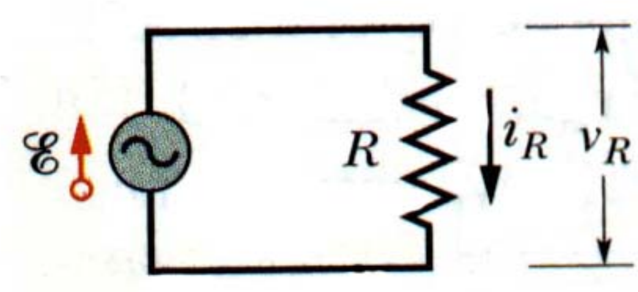
AC driven circuits:

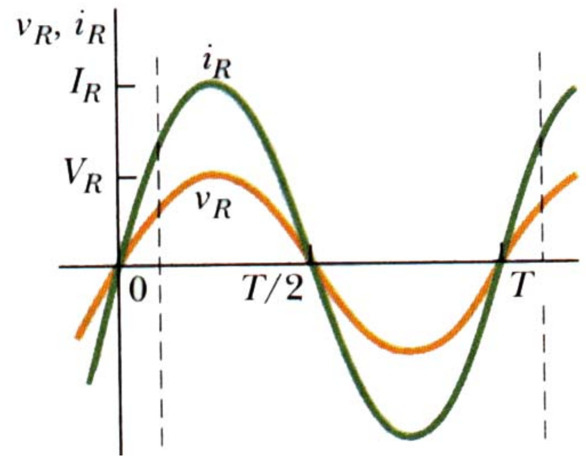
1) A resistor:

$$emf - v_R = 0$$

$$v_R = emf = E_m \sin(\omega_d t)$$

$$i_R = \frac{v_R}{R} = \frac{E_m}{R} \sin(\omega_d t)$$

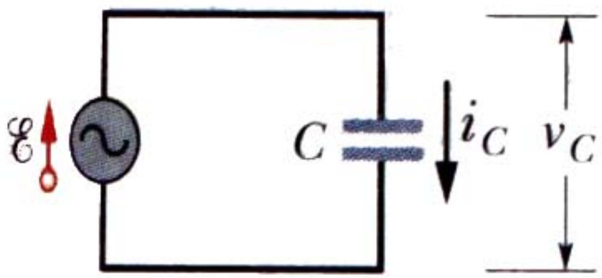




Resistors behave in AC very much as in DC, current and voltage are proportional (as functions of time in the case of AC), that is, they are "in phase".

AC driven circuits:

2) Capacitors:

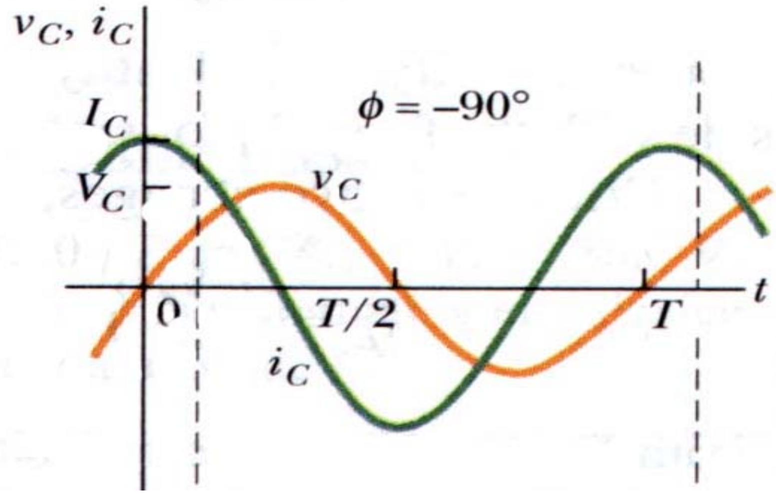


$$v_C = emf = E_m \sin(\omega_d t)$$

$$q_C = C\,emf = CE_m \sin(\omega_d t)$$

$$i_C = \frac{dq_C}{dt} = \omega_d CE_m \cos(\omega_d t)$$

$$i_C = \omega_d CE_m \sin(\omega_d t + 90^0)$$



$$i_C = \frac{E_m}{X} \sin(\omega_d t + 90^0)$$

where $X = \dfrac{1}{\omega_d C}$ "reactance"

$$\boxed{i_m = \frac{E_m}{X}}$$ looks like $i = \dfrac{E}{R}$

Capacitors "oppose a resistance" to AC (reactance) of frequency-dependent magnitude $1/\omega_d C$

(this idea is true only for maximum amplitudes, the instantaneous story is more complex).

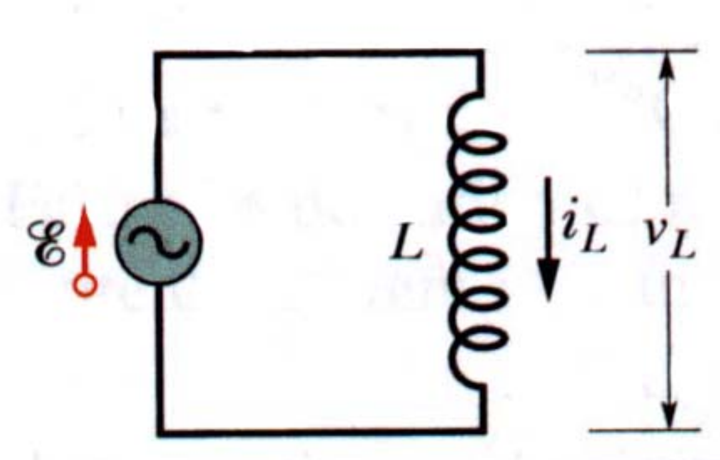
AC driven circuits:

3) Inductors:

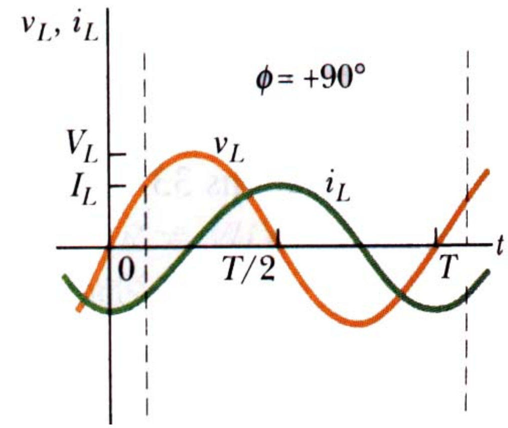
$$v_L = emf = E_m \sin(\omega_d t)$$

$$v_L = L\frac{di_L}{dt} \Rightarrow i_L = \frac{\int v_L dt}{L}$$

$$i_L = -\frac{E_m}{L\omega_d}\cos(\omega_d t) = \frac{E_m}{L\omega_d}\sin(\omega_d t - 90^0)$$



$$i_L = \frac{E_m}{X}\sin(\omega_d t - 90^0)$$



$$i_m = \frac{E_m}{X} \quad \text{where} \quad X = L\omega_d$$

Inductors "oppose a resistance" to AC (reactance) of frequency-dependent magnitude ω_d L (this idea is true only for maximum amplitudes, the instantaneous story is more complex).

How to remember all this????

• In capacitors, it takes time to build up a potential. Therefore the voltage trails the current. Current gets going immediately, potential comes later, as the device gets charged. Capacitors initially behave as short circuits, lots of current, little emf.

• In coils, as one applies an emf, the coil reacts back (Lenz' law) preventing the establishment of an immediate current. Therefore the current trails the emf. Coils initially behave like open circuits, lots of emf, little current.

• In resistors, "no room for funny behavior" since they cannot store energy, what you put in you get out, current and emf are in phase.

Reactances:

Resistors: $X = R$

Inductors: $X = \omega_d L$

Capacitors: $X = 1/(\omega_d C)$

Big inductance: lots of opposition to AC.

Big capacitance: little opposition to AC.

Summary:

- Electrical oscillators have “friction” given by the resistance, that makes oscillations die off exponentially.
- To keep them going we “drive” them with AC.
- Coils and capacitors “react” to AC and produce currents that are out of phase with voltage. In capacitors, ahead, in coils trailing. In resistors both are in phase.