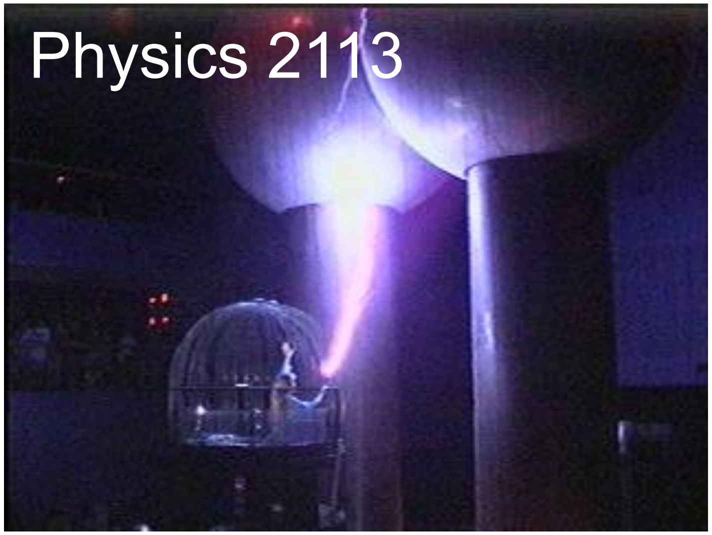
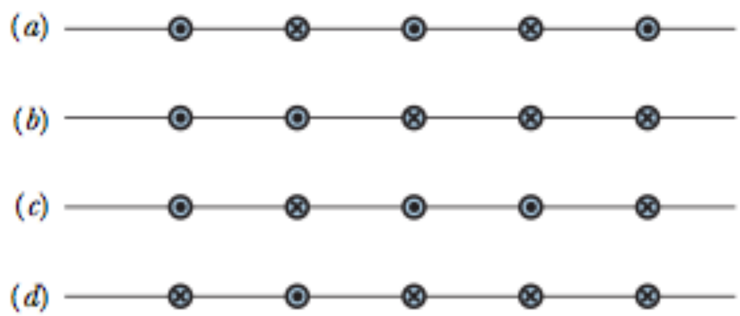


Physics 2113
Lecture 27: FRI 31 OCT

CH28: Magnetic fields due to curents

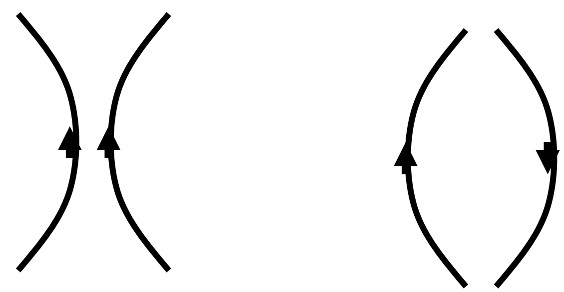
Problems

Rank according to force on central wire. All carry the same current.



$$F_{21} = L I_2 B_1 = \frac{\mu_0 L I_1 I_2}{2\pi a}$$

a) F=0.
b) All pull to the right.
c) Outermost pair undercuts innermost neighbors.
d) Three pull to the right, one to the left.



•3 SSM At a certain location in the Philippines, Earth's magnetic field of 39 μT is horizontal and directed due north. Suppose the net field is zero exactly 8.0 cm above a long, straight, horizontal wire that carries a constant current. What are the (a) magnitude and (b) direction of the current?

ANALYZE (a) The field due to the wire, at a point 8.0 cm from the wire, must be 39 μT and must be directed due south. Therefore,

$$i = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.080\,\text{m})(39\times10^{-6}\,\text{T})}{4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}} = 16\,\text{A}.$$

(b) The current must be from west to east to produce a field that is directed southward at points below it.

••29 SSM In Fig. 29-56, four long straight wires are perpendicular to the page, and their cross sections form a square of edge length a = 20 cm. The currents are out of the page in wires 1 and 4 and into the page in wires 2 and 3, and each wire carries 20 A. In unit-vector notation, what is the net magnetic field at the square's center?

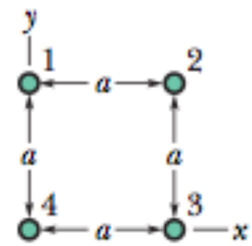
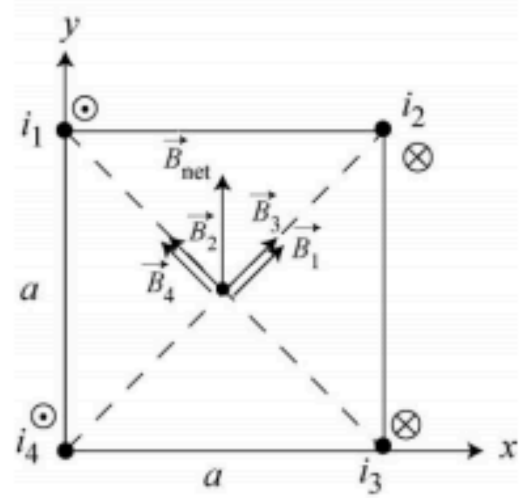


Fig. 29-56

EXPRESS Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where r is the distance from the corner of the square to the center. According to the Pythagorean theorem, the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left (wire 1) and lower right (wire 3) corners both point toward the upper right corner of the square. The fields due to the wires at the upper right (wire 2) and lower left (wire 4) corners both point toward the upper left corner.

ANALYZE The horizontal components of the fields cancel and the vertical components sum to

$$B_{\text{net}} = 4\frac{\mu_0 i}{\sqrt{2}\pi a}\cos 45° = \frac{2\mu_0 i}{\pi a} = \frac{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(20\ \text{A})}{\pi(0.20\ \text{m})}$$
$$= 8.0\times10^{-5}\ \text{T}.$$



In the calculation, cos 45° was replaced with $1/\sqrt{2}$. The total field points upward, or in the $+y$ direction. Thus, $\vec{B}_{\text{net}} = (8.0\times10^{-5}\ \text{T})\hat{\text{j}}$.

LEARN In the figure to the right, we show the contributions from the individual wires. The directions of the fields are deduced using the right-hand rule.

••18 A current is set up in a wire loop consisting of a semicircle of radius 4.00 cm, a smaller concentric semicircle, and two radial straight lengths, all in the same plane. Figure 29-46*a* shows the arrangement but is not drawn to scale. The magnitude of the magnetic field produced at the center of curvature is 47.25 μT. The smaller semicircle is then flipped over (rotated) until the loop is again entirely in the same plane (Fig. 29-46*b*). The magnetic field produced at the (same) center of curvature now has magnitude 15.75 μT, and its direction is reversed. What is the radius of the smaller semicircle?

Fig. 29-46 Problem 18.

Straight pieces do not contribute.

$$B = \frac{\mu_0}{4\pi}\frac{i\phi_0}{r}$$

18. In the one case we have $B_{small} + B_{big} = 47.25\ \mu$T, and the other case gives $B_{small} - B_{big} = 15.75\ \mu$T (cautionary note about our notation: B_{small} refers to the field at the center of the small-radius arc, which is actually a bigger field than B_{big}!). Dividing one of these equations by the other and canceling out common factors (see Eq. 29-9) we obtain

$$\frac{(1/r_{small}) + (1/r_{big})}{(1/r_{small}) - (1/r_{big})} = \frac{1 + (r_{small}/r_{big})}{1 - (r_{small}/r_{big})} = 3 .$$

The solution of this is straightforward: $r_{small} = r_{big}/2$. Using the given fact that the $r_{big} = 4.00$ cm, then we conclude that the small radius is $r_{small} = 2.00$ cm.

••48 In Fig. 29-70, a long circular pipe with outside radius $R = 2.6$ cm carries a (uniformly distributed) current $i = 8.00$ mA into the page. A wire runs parallel to the pipe at a distance of $3.00R$ from center to center. Find the (a) magnitude and (b) direction (into or out of the page) of the current in the wire such that the net magnetic field at point P has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction.

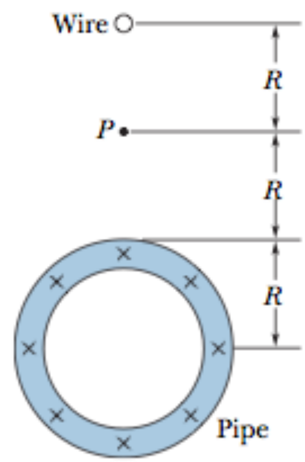


Fig. 29-70 Problem 48.

48. (a) The field at the center of the pipe (point C) is due to the wire alone, with a magnitude of

$$B_C = \frac{\mu_0 i_{\text{wire}}}{2\pi(3R)} = \frac{\mu_0 i_{\text{wire}}}{6\pi R}.$$

For the wire we have $B_{P,\text{ wire}} > B_{C,\text{ wire}}$. Thus, for $B_P = B_C = B_{C,\text{ wire}}$, i_{wire} must be into the page:

$$B_P = B_{P,\text{wire}} - B_{P,\text{pipe}} = \frac{\mu_0 i_{\text{wire}}}{2\pi R} - \frac{\mu_0 i}{2\pi(2R)}.$$

Setting $B_C = -B_P$ we obtain $i_{\text{wire}} = 3i/8 = 3(8.00\times10^{-3}\,\text{A})/8 = 3.00\times10^{-3}\,\text{A}$.

(b) The direction is into the page.

•45 SSM Each of the eight conductors in Fig. 29-68 carries 2.0 A of current into or out of the page. Two paths are indicated for the line integral $\oint \vec{B} \cdot d\vec{s}$. What is the value of the integral for (a) path 1 and (b) path 2?

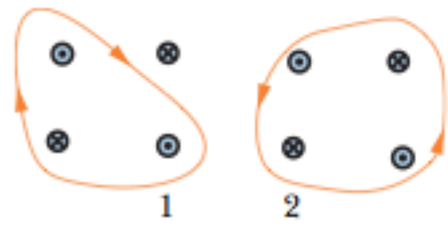


Fig. 29-68 Problem 45.

45. **THINK** The value of the line integral $\oint \vec{B} \cdot d\vec{s}$ is proportional to the net current enclosed.

EXPRESS By Ampere's law, we have $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$, where i_{enc} is the current enclosed by the closed path.

ANALYZE (a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path, or "Amperian loop" is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus,

$$\oint \vec{B} \cdot d\vec{s} = -\mu_0 i = -(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(2.0\,\text{A}) = -2.5 \times 10^{-6}\,\text{T} \cdot \text{m}.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} = 0$.

LEARN The value of $\oint \vec{B} \cdot d\vec{s}$ depends only on the current enclosed, and not the shape of the Amperian loop.

••55 SSM ILW WWW A long solenoid with 10.0 turns/cm and a radius of 7.00 cm carries a current of 20.0 mA. A current of 6.00 A exists in a straight conductor located along the central axis of the solenoid. (a) At what radial distance from the axis will the direction of the resulting magnetic field be at 45.0° to the axial direction? (b) What is the magnitude of the magnetic field there?

55. **THINK** The net field at a point inside the solenoid is the vector sum of the fields of the solenoid and that of the long straight wire along the central axis of the solenoid.

EXPRESS The magnetic field at a point P is given by $\vec{B} = \vec{B}_s + \vec{B}_w$, where $\vec{B}_s$ and $\vec{B}_w$ are the fields due to the solenoid and the wire, respectively. The direction of $\vec{B}_s$ is along the axis of the solenoid, and the direction of $\vec{B}_w$ is perpendicular to it, so the two fields are perpendicular to each other, $\vec{B}_s \perp \vec{B}_w$. For the net field $\vec{B}$ to be at 45° with the axis, we must have $B_s = B_w$.

ANALYZE (a) Thus,

$$B_s = B_w \Rightarrow \quad \mu_0 i_s n = \frac{\mu_0 i_w}{2\pi d},$$

which gives the separation d to point P on the axis:

$$d = \frac{i_w}{2\pi i_s n} = \frac{6.00\,\text{A}}{2\pi(20.0\times10^{-3}\,\text{A})(10\,\text{turns/cm})} = 4.77\,\text{cm}.$$

(b) The magnetic field strength is

$$B = \sqrt{2}B_s = \sqrt{2}\left(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A}\right)\left(20.0\times10^{-3}\,\text{A}\right)\left(10\text{ turns}/0.0100\text{ m}\right) = 3.55\times10^{-5}\,\text{T}.$$

LEARN In general, the angle $\vec{B}$ makes with the solenoid axis is give by

$$\phi = \tan^{-1}\left(\frac{B_w}{B_s}\right) = \tan^{-1}\left(\frac{\mu_0 i_w / 2\pi d}{\mu_0 i_s n}\right) = \tan^{-1}\left(\frac{i_w}{2\pi d\, n i_s}\right).$$

•57 SSM A student makes a short electromagnet by winding 300 turns of wire around a wooden cylinder of diameter $d = 5.0$ cm. The coil is connected to a battery producing a current of 4.0 A in the wire. (a) What is the magnitude of the magnetic dipole moment of this device? (b) At what axial distance $z \gg d$ will the magnetic field have the magnitude 5.0 μT (approximately one-tenth that of Earth's magnetic field)?

57. **THINK** The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area.

EXPRESS The cross-sectional area is a circle, so $A = \pi R^2$, where R is the radius. The magnetic field on the axis of a magnetic dipole, a distance z away, is given by Eq. 29-27: $B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}$.

ANALYZE (a) Substituting the values given, we find the magnitude of the dipole moment to be

$$\mu = Ni\pi R^2 = (300)(4.0\,\text{A})\pi(0.025\,\text{m})^2 = 2.4\,\text{A}\cdot\text{m}^2.$$

(b) Solving for z, we obtain

$$z = \left(\frac{\mu_0}{2\pi}\frac{\mu}{B}\right)^{1/3} = \left(\frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.36\,\text{A}\cdot\text{m}^2)}{2\pi(5.0\times10^{-6}\,\text{T})}\right)^{1/3} = 46\,\text{cm}.$$

LEARN Note the similarity between $B = \frac{\mu_0}{2\pi}\frac{\mu}{z^3}$, the magnetic field of a magnetic dipole μ and $E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$, the electric field of an electric dipole p (see Eq. 22-9).

Summary:

The Biot-Savart Law

- The magnetic field set up by a current-carrying conductor can be found from the Biot–Savart law.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,d\vec{s} \times \hat{r}}{r^2}$$

- The quantity μ_0, called the permeability constant, has the value

$$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T \cdot m/A}.$$

Magnetic Field of a Long Straight Wire

- For a long straight wire carrying a current *i*, the Biot–Savart law gives,

$$B = \frac{\mu_0 i}{2\pi R}$$

Magnetic Field of a Circular Arc

- The magnitude of the magnetic field at the center of a circular arc,

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

Force Between Parallel Currents

- The magnitude of the force on a length L of either wire is

$$F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}$$

Ampere's Law

- Ampere's law states that,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

Summary:

Fields of a Solenoid and a Toroid

- Inside a long solenoid carrying current *i*, at points not near its ends, the magnitude *B* of the magnetic field is

$$B = \mu_0 i n$$

- At a point inside a toroid, the magnitude *B* of the magnetic field is

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r}$$

Field of a Magnetic Dipole

- The magnetic field produced by a current-carrying coil, which is a magnetic dipole, at a point P located a distance *z* along the coil's perpendicular central axis is parallel to the axis and is given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$$