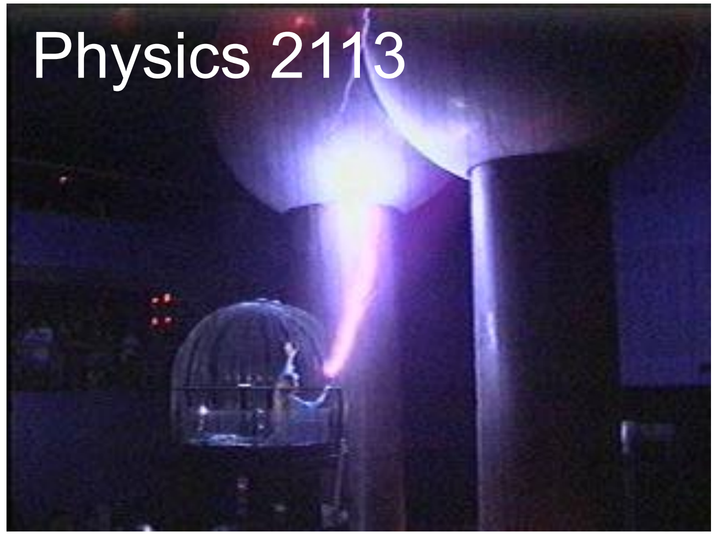
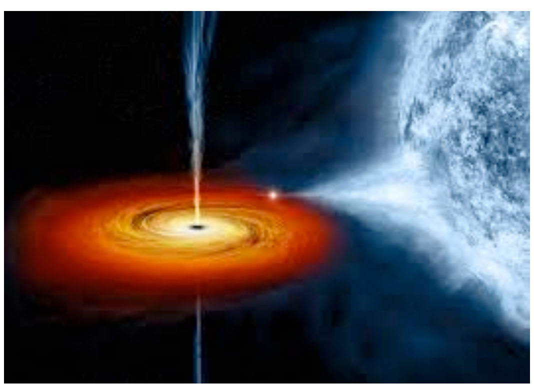


Physics 2113
Lecture 26: WED 29 OCT

CH28: Magnetic fields due to curents



To compute magnetic fields, one uses the Biot-Savart law:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \vec{r}}{r^3}$$

And it is evident that the Biot-Savart law is harder to use than Coulomb’s law.

Now, even Coulomb’s law we considered hard enough to handle that we went and considered an alternative path to compute electric fields, Gauss’ law.

Therefore it is not surprising that in the magnetic case we will also use an alternative approach to computing magnetic fields. This approach is called Ampere’s law.

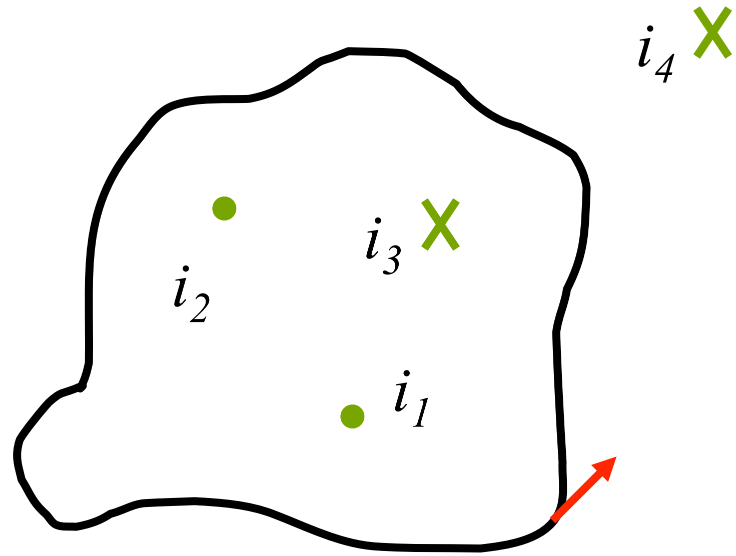
André-Marie Ampère
(1775-1836)

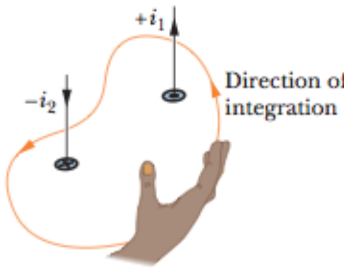
Hans Christian Ørsted
(1777-1851)

Ampere's law:

$$\oint_{\text{loop}} \vec{B}\cdot d\vec{s} = \mu_0 I$$

The circulation of B (the integral of B scalar ds) along an imaginary closed loop is proportional to the net amount of current traversing the loop.





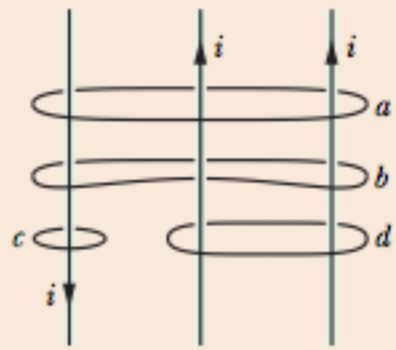
$$\oint_{\text{loop}} \vec{B}\cdot d\vec{s} = \mu_0 (i_1 + i_2 - i_3)$$

Thumb rule for sign; ignore i_4

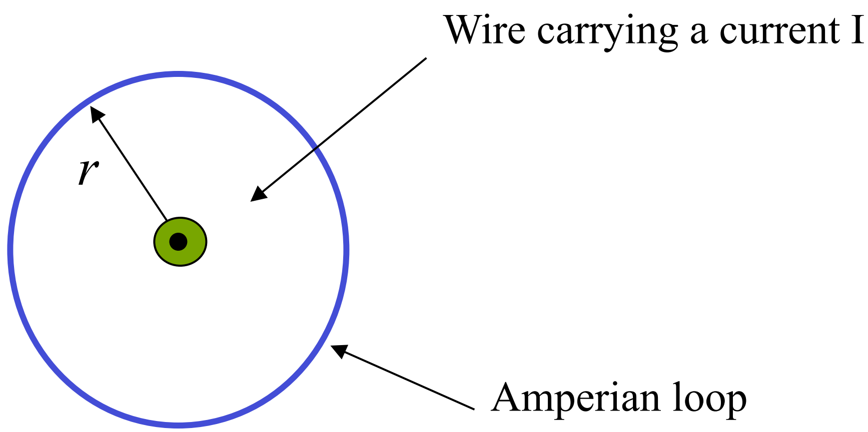
As was the case for Gauss' law, if you have a lot of symmetry, knowing the circulation of B allows you to know B.

CHECKPOINT 2

The figure here shows three equal currents i (two parallel and one antiparallel) and four Amperian loops. Rank the loops according to the magnitude of $\oint \vec{B} \cdot d\vec{s}$ along each, greatest first.



Example: a wire



By symmetry, the magnetic field is constant across the Amperian loop and is parallel to ds. Therefore,

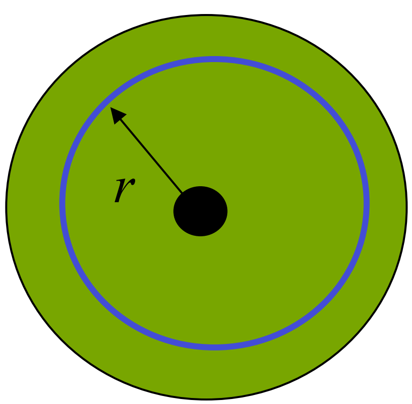
Applying Ampere's law,

$$\oint \vec{B}\cdot d\vec{s} = 2\pi rB$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

Example: field inside a thick wire



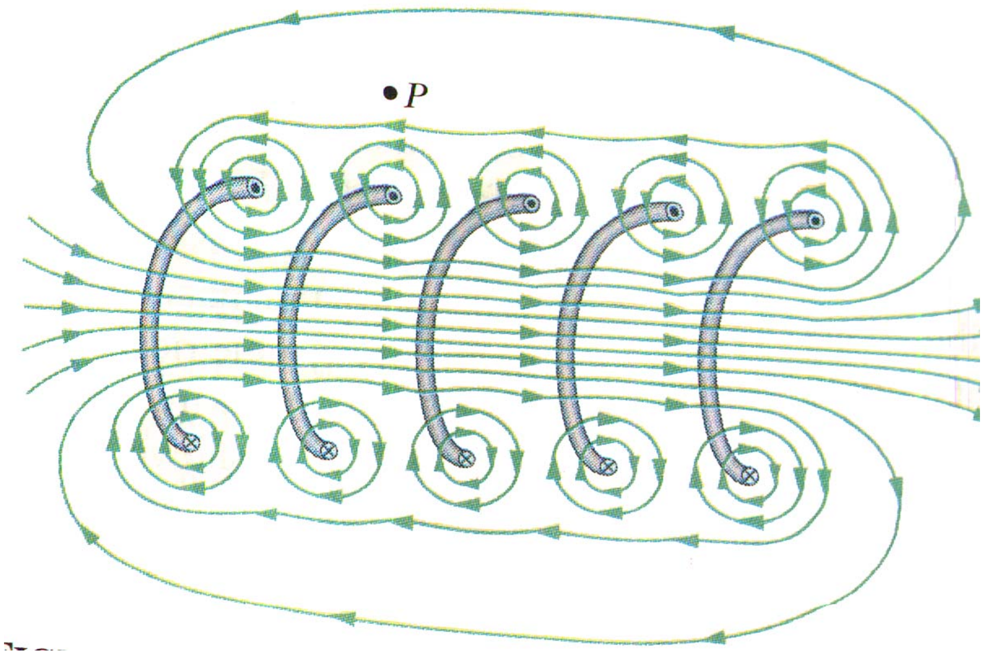
$$\oint \vec{B}\cdot d\vec{s} = 2\pi rB$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{inside}}$$

$$I_{\text{inside}} = \pi r^2 J = \pi r^2 \frac{I}{\pi R^2} = \frac{r^2 I}{R^2}$$

$$\boxed{B = \frac{rI}{2\pi R^2}}$$

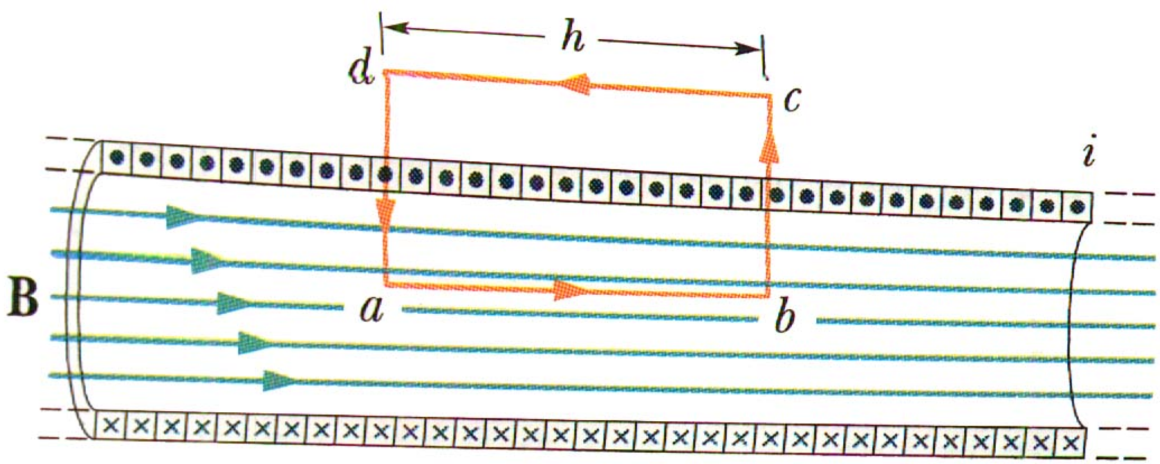
Magnetic field of a solenoid



No field outside, field concentrated inside.

Considering Amperian loop abcd,

Idealized: (infinitely tightly woven, infinite)



$$\oint \vec{B}\cdot d\vec{s} = \int_a^b \vec{B}\cdot d\vec{s} + \int_b^c \vec{B}\cdot d\vec{s} +$$

$$\int_c^d \vec{B}\cdot d\vec{s} + \int_a^b \vec{B}\cdot d\vec{s}$$

$$= Bh + 0 + 0 + 0 = Bh$$

$i_{enc} = inh,$

$$\boxed{B = \mu_0 i n}$$

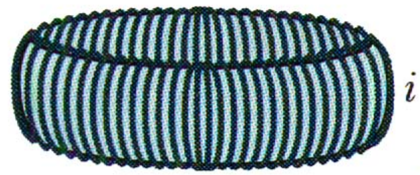
n=turns per unit length

Magnetic field of a toroid (donut):

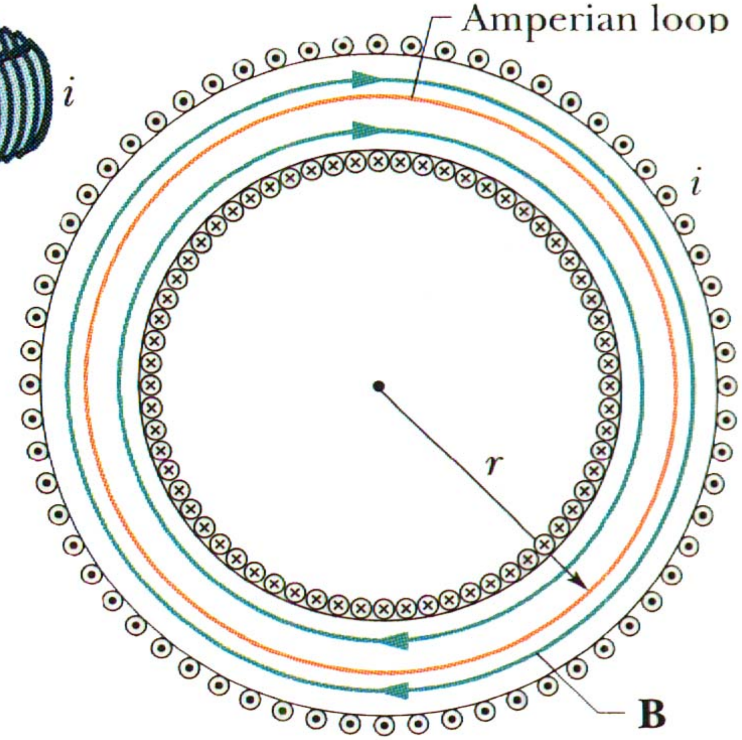
$$\oint \vec{B} \cdot d\vec{s} = 2\pi r B$$

$$\mu_0 i_{enc} = \mu_0 N i$$

$$B = \frac{\mu_0 N i}{2\pi r}$$



(a)



56 A toroid having a square cross section, 5cm on a side, an an inner radius of 15cm has 500 turns and carries a current of 0.8A. What is the magnetic field inside the toroid at

a) the inner radius,

b) the outer radius.

Picture is the same as before, only the cross-section is different.

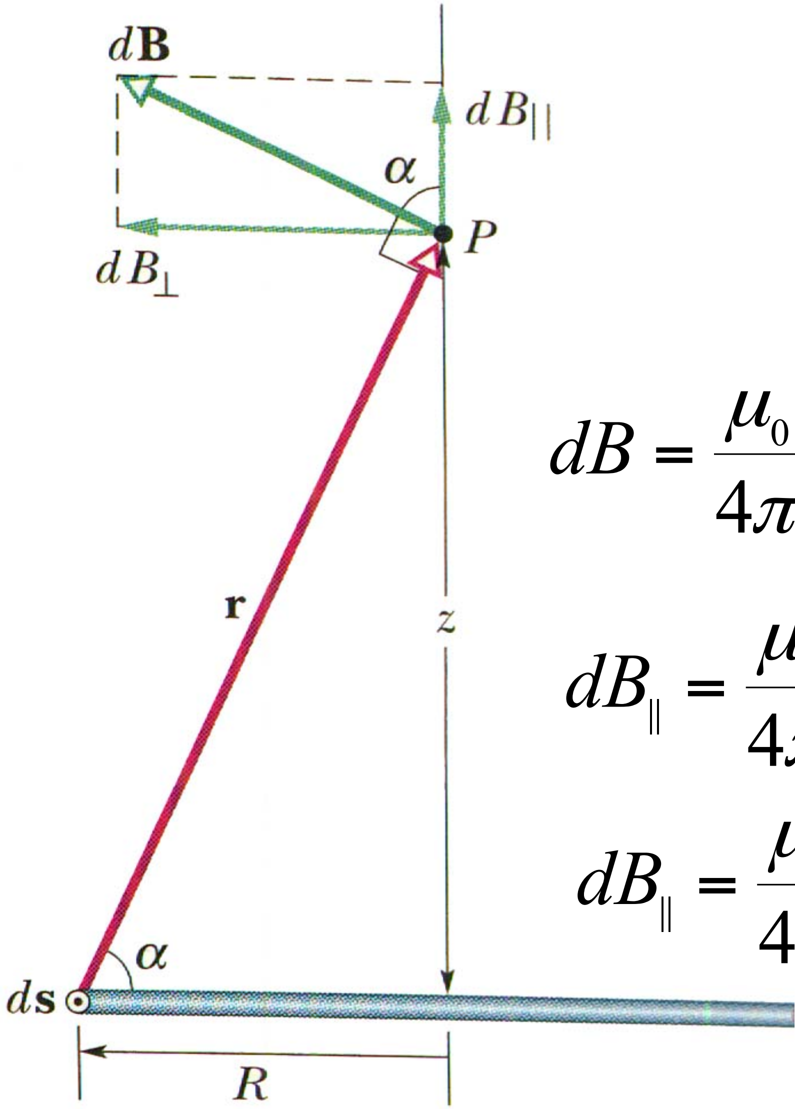
Therefore,

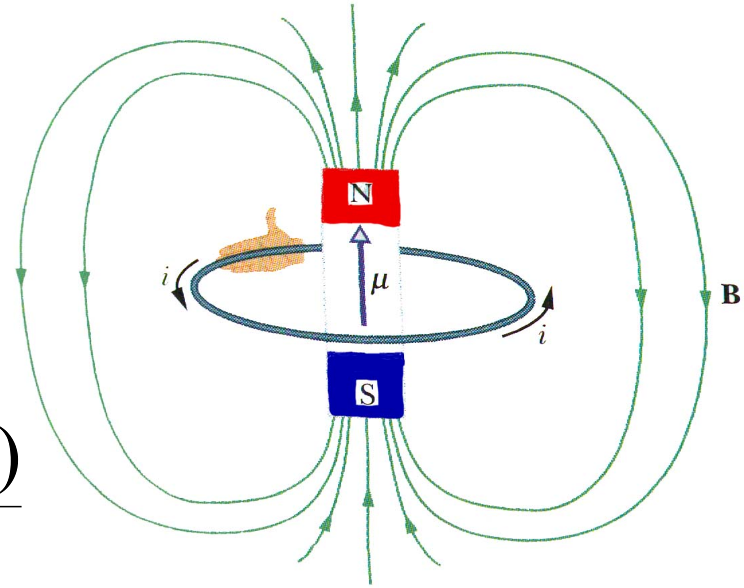
$$B = \frac{\mu_0 N i}{2\pi r}$$

For inner radius, $$B = \frac{(1.2\times10^{-6}\,T\cdot m)(0.8A)(500)}{2\pi(0.15m)} = 5.33\times10^{-4}\,T$$

For outer radius, $$B = \frac{(1.2\times10^{-6}\,T\cdot m)(0.8A)(500)}{2\pi(0.2m)} = 4\times10^{-4}\,T$$

A current carrying coil as a magnetic dipole





$$dB = \frac{\mu_0}{4\pi} \frac{i\,ds \sin(90^0)}{r^2}$$

$$dB_{\parallel} = \frac{\mu_0}{4\pi} \frac{i\,ds \cos(\alpha)}{r^2} \qquad r = \sqrt{z^2 + R^2} \qquad \cos(\alpha) = \frac{R}{r}$$

$$dB_{\parallel} = \frac{\mu_0}{4\pi} \frac{i\,ds\,R}{\left(z^2 + R^2\right)^{3/2}}$$

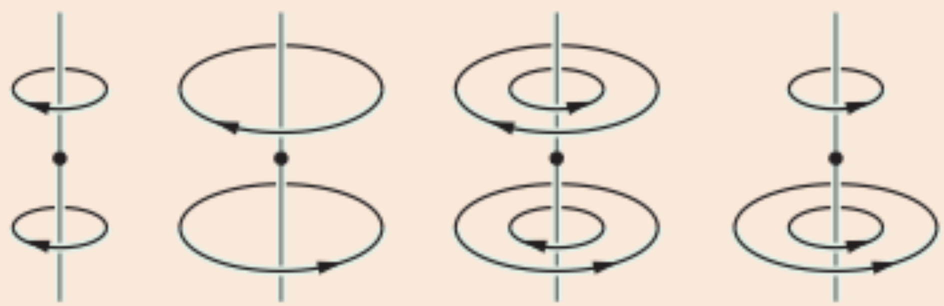
$$B = \frac{\mu_0}{2} \frac{i\,R^2}{\left(z^2 + R^2\right)^{3/2}}$$

If, $z >> R$, $\quad B = \frac{\mu_0}{2} \frac{i R^2}{z^3} = \frac{\mu_0}{2\pi} \frac{i \,\text{Area}}{z^3} = \frac{\mu_0}{2\pi} \frac{\mu}{z^3}$

μ dipole moment

CHECKPOINT 3

The figure here shows four arrangements of circular loops of radius r or $2r$, centered on vertical axes (perpendicular to the loops) and carrying identical currents in the directions indicated. Rank the arrangements according to the magnitude of the net magnetic field at the dot, midway between the loops on the central axis, greatest first.

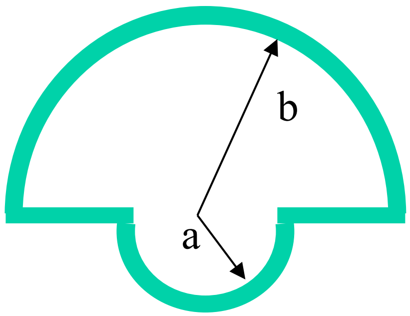


(a) (b) (c) (d)

67 A length of wire is formed into a closed loop with radii *a* and *b*, and carries a current *i*.

a) What is the value of the magnitude and direction of B at the center?

b) Find the magnetic dipole moment of the circuit.



Straight pieces do not contribute to B.

Last class, arc: $B = \frac{\mu_0}{4\pi}\frac{i\phi_0}{r} = \frac{\mu_0}{4}\frac{i}{r}$ At center, $B = \frac{\mu_0 i}{4}\left(\frac{1}{a} + \frac{1}{b}\right)$

Dipole moment, $\mu = i$ Area $= i\left(\frac{\pi a^2}{2} + \frac{\pi b^2}{2}\right) = \frac{\pi i}{2}(a^2 + b^2)$

Summary:

- Ampere's law operates like Gauss' law. One chooses an imaginary path that encircles current. The circulation of B is proportional to the net current inside.
- If one has symmetry, one can easily compute B from the circulation.
- Orientation of B is given in terms of the current using the right hand rule.