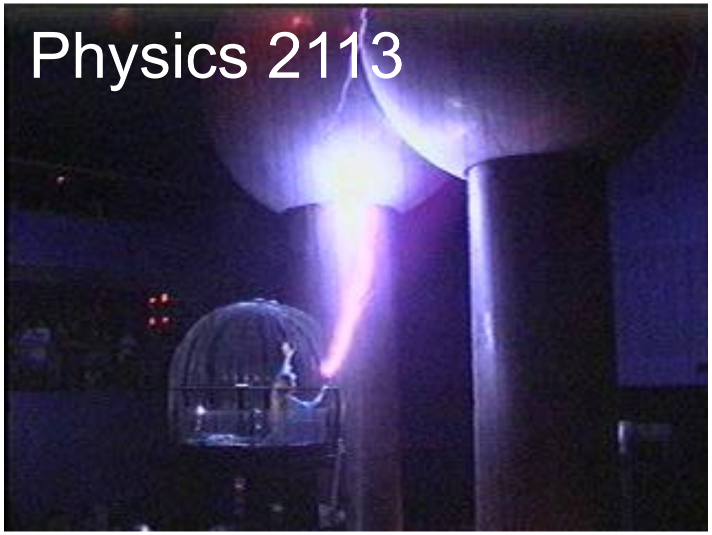


Physics 2113
Lecture 25: MON 27 OCT

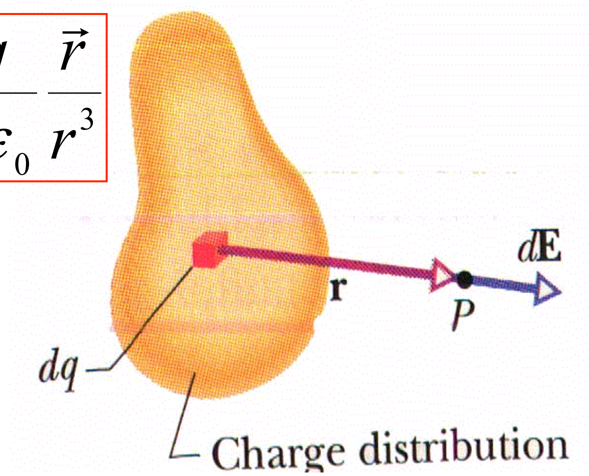
CH28: Magnetic fields due to currents

When we computed the electric field due to charges we used Coulomb's law. If one had a large irregular object, one broke it into infinitesimal pieces and computed,

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

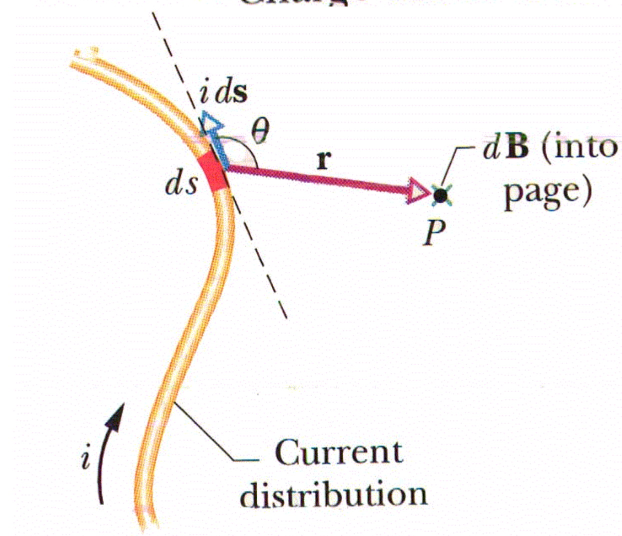
Which we sometimes wrote as,

$$\boxed{d\vec{E} = \frac{dq}{4\pi\varepsilon_0}\frac{\vec{r}}{r^3}}$$



If you wish to compute the magnetic field due to a current in a wire, you use the law of Biot and Savart,

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3}} \qquad dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin(\theta)}{r^2}$$



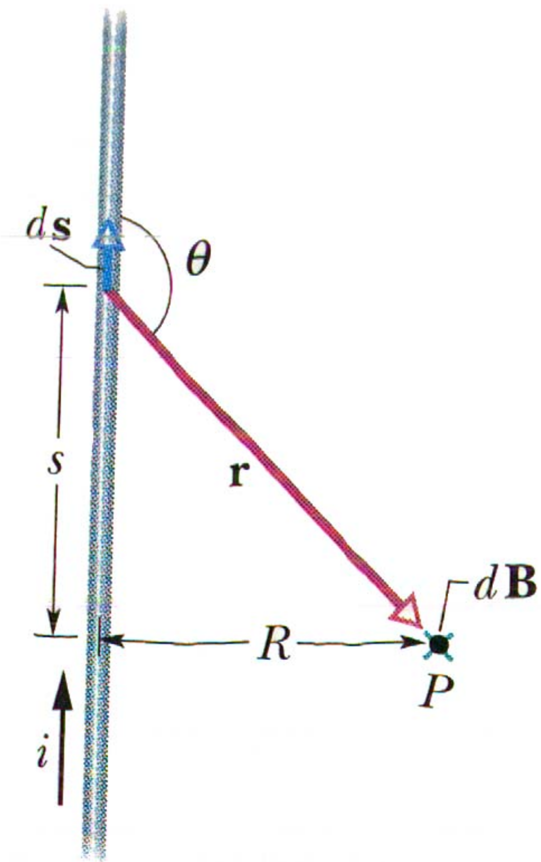
$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3}}$$

$$\mu_0 = 4\pi\times 10^{-7}\,\frac{Tm}{A} \approx 1.2\times 10^{-6}\,\frac{Tm}{A}$$

Permeability of vacuum

Jean-Baptiste Biot (1774-1862)

Example: magnetic field due to a straight wire



$$r = \sqrt{R^2 + s^2}$$

$$\sin(\theta) = \sin(\pi - \theta) = \frac{R}{\sqrt{R^2 + s^2}}$$

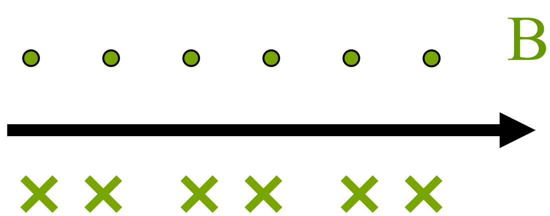
$$B = 2\int_0^\infty dB = 2\int_0^\infty \frac{\mu_0}{4\pi}\frac{i\,ds\sin(\theta)}{r^2}$$

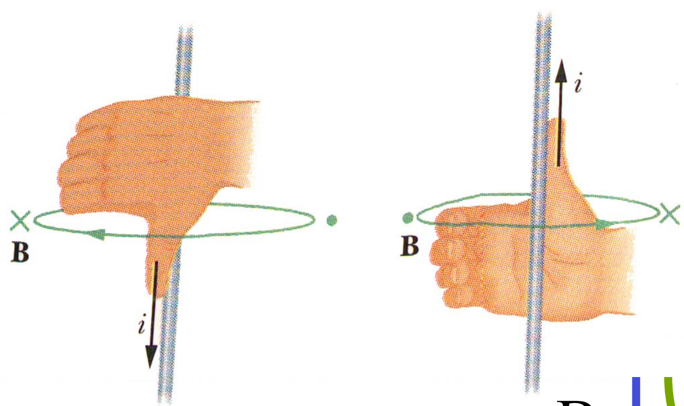
$$= \frac{\mu_0 i}{2\pi}\int_0^\infty ds\,\frac{R}{\sqrt{R^2+s^2}}\,\frac{1}{R^2+s^2} = \frac{\mu_0 i}{2\pi}\int_0^\infty ds\,\frac{R}{(R^2+s^2)^{3/2}}$$

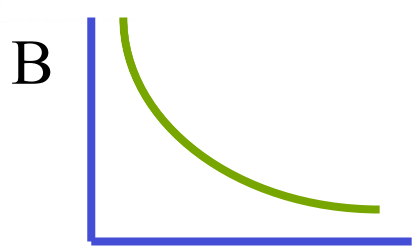
$$= \frac{\mu_0 i}{2\pi R}\left[\frac{s}{(R^2+s^2)^{1/2}}\right]_0^\infty = \boxed{\frac{\mu_0 i}{2\pi R}}$$

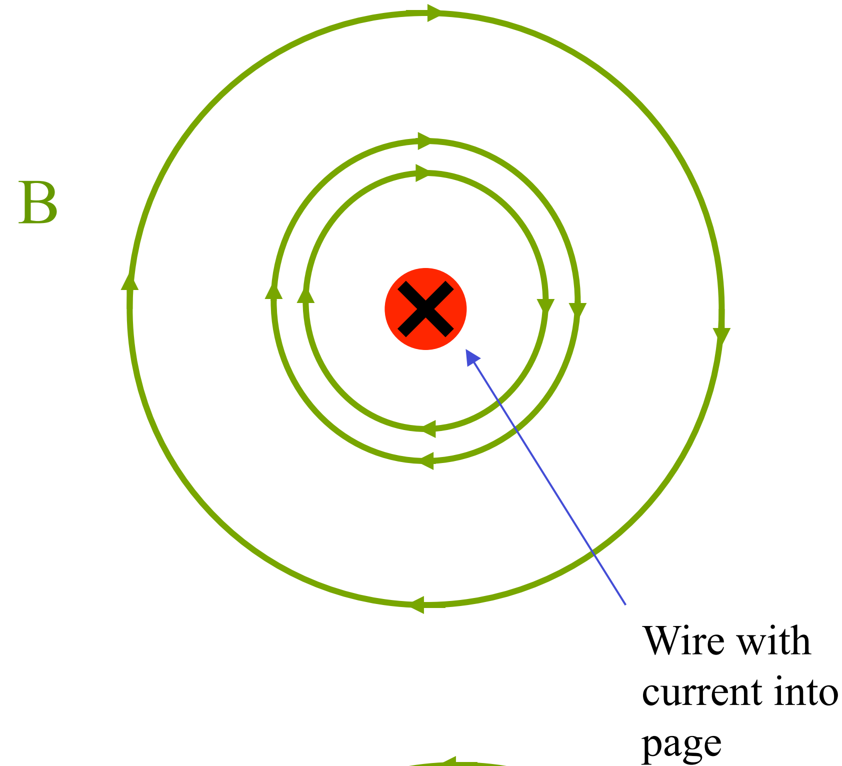
Therefore for a wire,

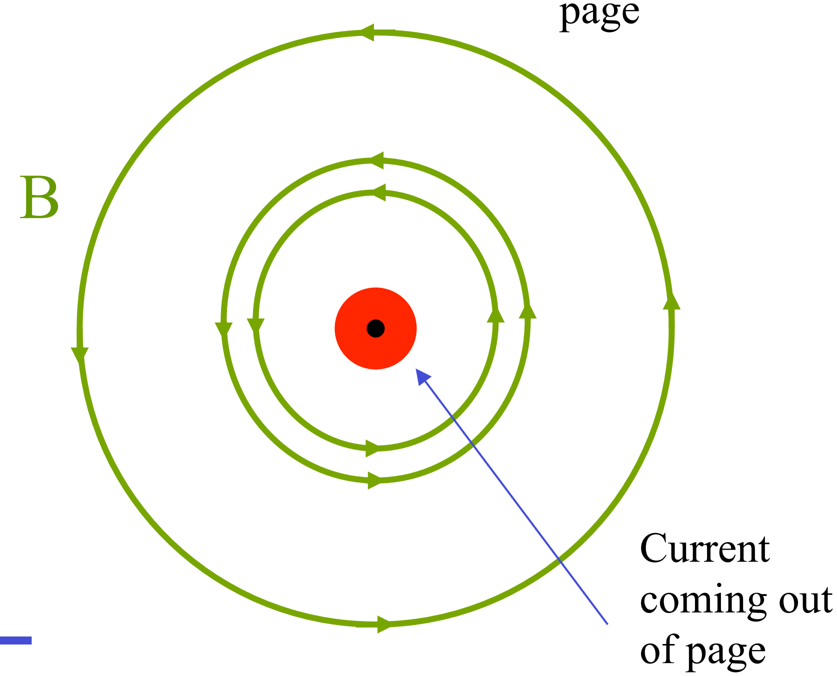
$$\boxed{B = \frac{\mu_0 i}{2\pi R}}$$



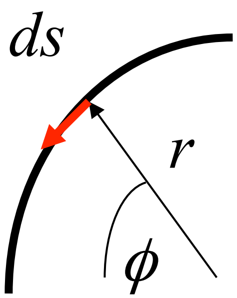








Magnetic field due to a circular arc



$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin(\theta)}{r^2}$$

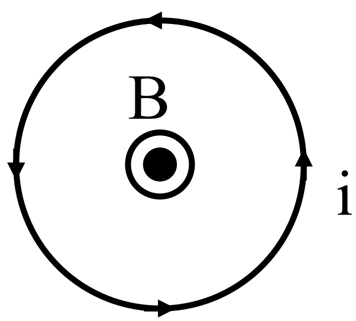
But θ, the angle between ds and r, is 90 degrees.

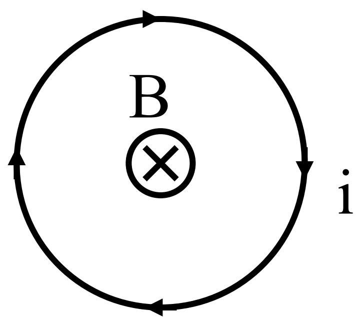
$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds}{r^2}$$

$$B = \int_0^{\phi_0} \frac{\mu_0}{4\pi}\frac{i\,r d\phi}{r^2}$$

$$ds = r d\phi$$

$$B = \frac{\mu_0}{4\pi}\frac{i\phi_0}{r}$$

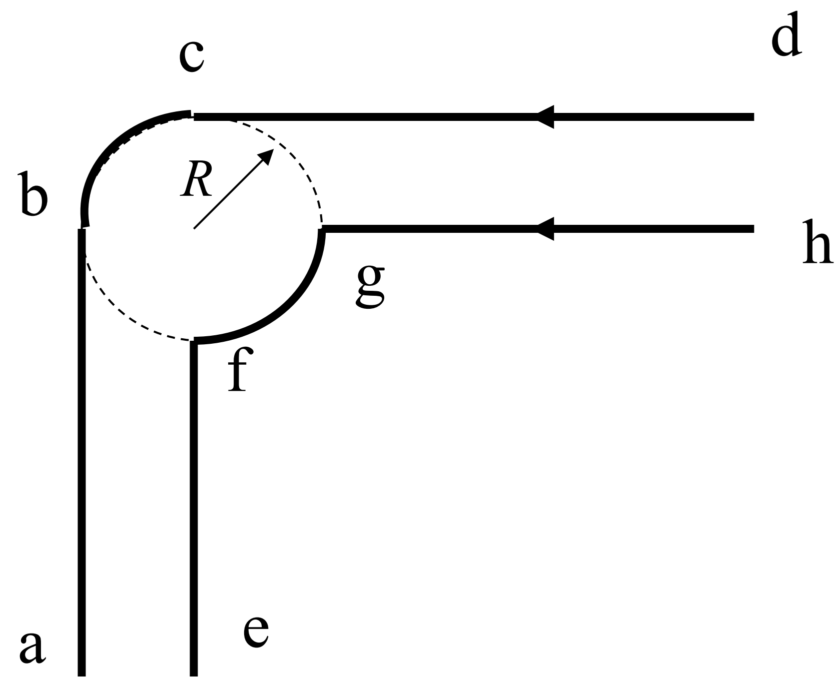




Problem

Magnetic field at center of circle?

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\vec{r}}{r^3}}$$

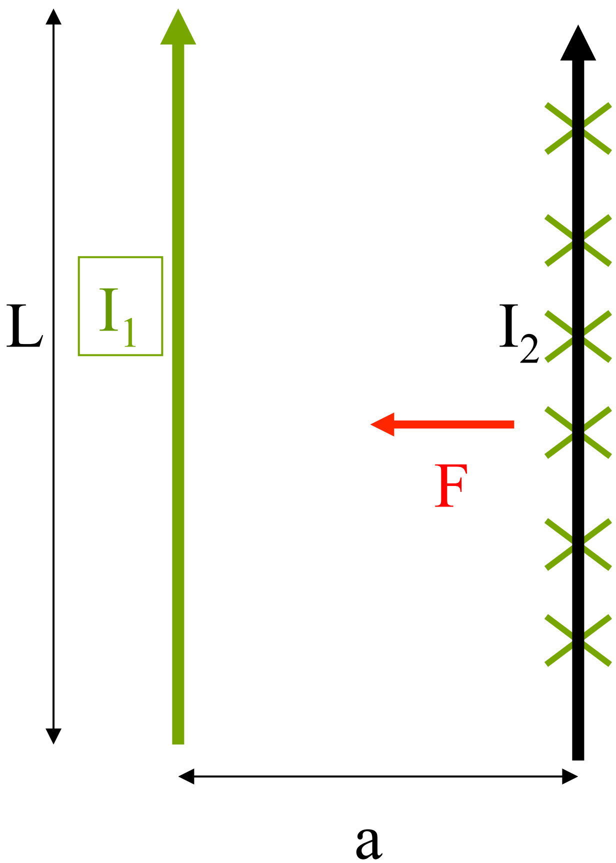


Sections gh and fe do not contribute to the magnetic field at the center (in the Biot-Savart law, ds and r are parallel).

Sections fg and bc produce equal and opposite fields at the center of the circle.

Finally, rotate section ab in such a way that it forms a straight line with cd. Its contribution to the field at the origin of the circle goes unchanged by such motion. Therefore, the field is that of a straight wire running from a to d horizontally.

Force between parallel wires carrying current

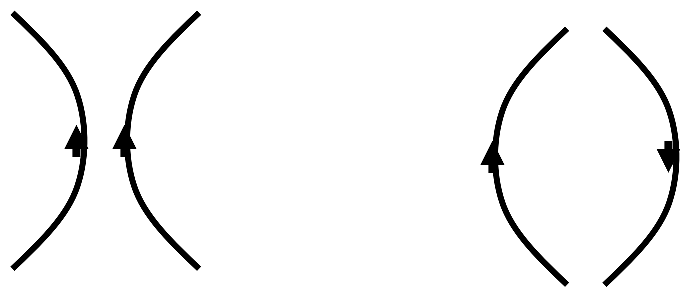


Magnetic field due to wire 1
where the wire 2 is,

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

Force on wire 2 due to this field,

$$F_{21} = L\,I_2\,B_1 = \frac{\mu_0 L I_1 I_2}{2\pi a}$$



Magnetic field at the center of a circular arc of current

The wire in Fig. 29-7a carries a current i and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field $\vec{B}$ (magnitude and direction) does the current produce at C?

KEY IDEAS

We can find the magnetic field $\vec{B}$ at point C by applying the Biot–Savart law of Eq. 29-3 to the wire, point by point along the full length of the wire. However, the application of Eq. 29-3 can be simplified by evaluating $\vec{B}$ separately for the three distinguishable sections of the wire—namely, (1) the straight section at the left, (2) the straight section at the right, and (3) the circular arc.

Straight sections: For any current-length element in section 1, the angle θ between $d\vec{s}$ and $\hat{r}$ is zero (Fig. 29-7b); so Eq. 29-1 gives us

$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at C:

$$B_1 = 0.$$

The same situation prevails in straight section 2, where the angle θ between $d\vec{s}$ and $\hat{r}$ for any current-length element is 180°. Thus,

$$B_2 = 0.$$

Circular arc: Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc leads to Eq. 29-9 ($B = \mu_0 i\phi/4\pi R$). Here the central angle ϕ of the arc is $\pi/2$ rad. Thus from Eq. 29-9, the magnitude of the magnetic field $\vec{B}_3$ at the arc's center C is

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

To find the direction of $\vec{B}_3$, we apply the right-hand rule displayed in Fig. 29-4. Mentally grasp the circular arc with your right hand as in Fig. 29-7c, with your thumb in the direction of the current. The direction in which your fingers curl around the wire indicates the direction of the magnetic field lines around the wire. They form circles around the wire, coming out of the page above the arc and going into the page inside the arc. In the region of point C (inside the arc), your fingertips point *into the plane* of the page. Thus, $\vec{B}_3$ is directed into that plane.

Net field: Generally, when we must combine two or more magnetic fields to find the net magnetic field, we must combine the fields as vectors and not simply add their magnitudes. Here, however, only the circular arc produces a magnetic field at point C. Thus, we can write the magnitude of the net field $\vec{B}$ as

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}. \qquad \text{(Answer)}$$

The direction of $\vec{B}$ is the direction of $\vec{B}_3$—namely, into the plane of Fig. 29-7.

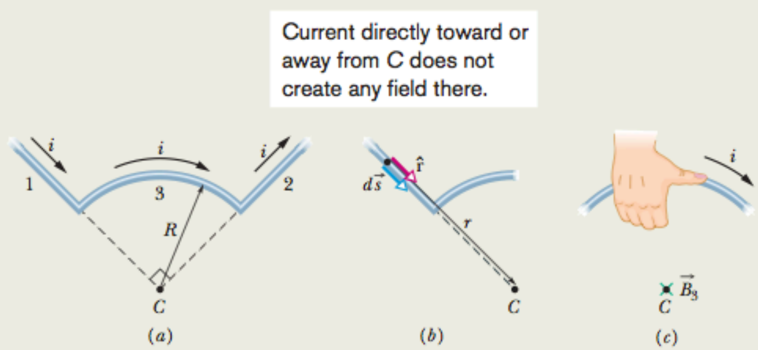


Fig. 29-7 (a) A wire consists of two straight sections (1 and 2) and a circular arc (3), and carries current i. (b) For a current-length element in section 1, the angle between $d\vec{s}$ and $\hat{r}$ is zero. (c) Determining the direction of magnetic field $\vec{B}_3$ at C due to the current in the circular arc; the field is into the page there.

Magnetic field off to the side of two long straight currents

Figure 29-8*a* shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and $d = 5.3$ cm.

KEY IDEAS

(1) The net magnetic field $\vec{B}$ at point P is the vector sum of the magnetic fields due to the currents in the two wires. (2) We can find the magnetic field due to any current by applying the Biot–Savart law to the current. For points near the current in a long straight wire, that law leads to Eq. 29-4.

Finding the vectors: In Fig. 29-8*a*, point P is distance R from both currents i_1 and i_2. Thus, Eq. 29-4 tells us that at point P those currents produce magnetic fields $\vec{B}_1$ and $\vec{B}_2$ with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29-8*a*, note that the base angles (between sides R and d) are both 45°. This allows us to write $\cos 45° = R/d$ and replace R with $d \cos 45°$. Then the field magnitudes B_1 and B_2 become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45°} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45°}.$$

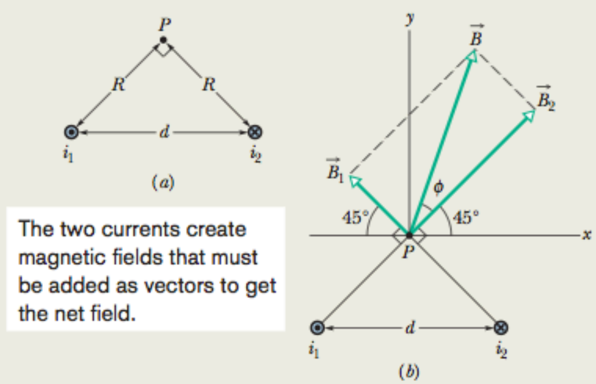


Fig. 29-8 (*a*) Two wires carry currents i_1 and i_2 in opposite directions (out of and into the page). Note the right angle at P. (*b*) The separate fields $\vec{B}_1$ and $\vec{B}_2$ are combined vectorially to yield the net field $\vec{B}$.

We want to combine $\vec{B}_1$ and $\vec{B}_2$ to find their vector sum, which is the net field $\vec{B}$ at P. To find the directions of $\vec{B}_1$ and $\vec{B}_2$, we apply the right-hand rule of Fig. 29-4 to each current in Fig. 29-8*a*. For wire 1, with current out of the page, we mentally grasp the wire with the right hand, with the thumb pointing out of the page. Then the curled fingers indicate that the field lines run counterclockwise. In particular, in the region of point P, they are directed upward to the left. Recall that the magnetic field at a point near a long, straight current-carrying wire must be directed perpendicular to a radial line between the point and the current. Thus, $\vec{B}_1$ must be directed upward to the left as drawn in Fig. 29-8*b*. (Note carefully the perpendicular symbol between vector $\vec{B}_1$ and the line connecting point P and wire 1.)

Repeating this analysis for the current in wire 2, we find that $\vec{B}_2$ is directed upward to the right as drawn in Fig. 29-8*b*. (Note the perpendicular symbol between vector $\vec{B}_2$ and the line connecting point P and wire 2.)

Adding the vectors: We can now vectorially add $\vec{B}_1$ and $\vec{B}_2$ to find the net magnetic field $\vec{B}$ at point P, either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of $\vec{B}$. However, in Fig. 29-8*b*, there is a third method: Because $\vec{B}_1$ and $\vec{B}_2$ are perpendicular to each other, they form the legs of a right triangle, with $\vec{B}$ as the hypotenuse. The Pythagorean theorem then gives us

$$\begin{aligned} B &= \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45°)}\sqrt{i_1^2 + i_2^2} \\ &= \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})\sqrt{(15\,\mathrm{A})^2 + (32\,\mathrm{A})^2}}{(2\pi)(5.3 \times 10^{-2}\,\mathrm{m})(\cos 45°)} \\ &= 1.89 \times 10^{-4}\,\mathrm{T} \approx 190\,\mu\mathrm{T}. \qquad \text{(Answer)} \end{aligned}$$

The angle ϕ between the directions of $\vec{B}$ and $\vec{B}_2$ in Fig. 29-8*b* follows from

$$\phi = \tan^{-1}\frac{B_1}{B_2},$$

which, with B_1 and B_2 as given above, yields

$$\phi = \tan^{-1}\frac{i_1}{i_2} = \tan^{-1}\frac{15\,\mathrm{A}}{32\,\mathrm{A}} = 25°.$$

The angle between the direction of $\vec{B}$ and the x axis shown in Fig. 29-8*b* is then

$$\phi + 45° = 25° + 45° = 70°. \qquad \text{(Answer)}$$

Summary:

- Currents create magnetic fields, following a law somewhat similar to Coulomb’s law for charges, but more complicated due to the vector product in it (Biot-Savart law).
- Wires carrying currents interact magnetically, like charged wires do electrically, but equals attract.
- Next time we will introduce a math trick to simplify calculations involving Biot-Savart’s law.