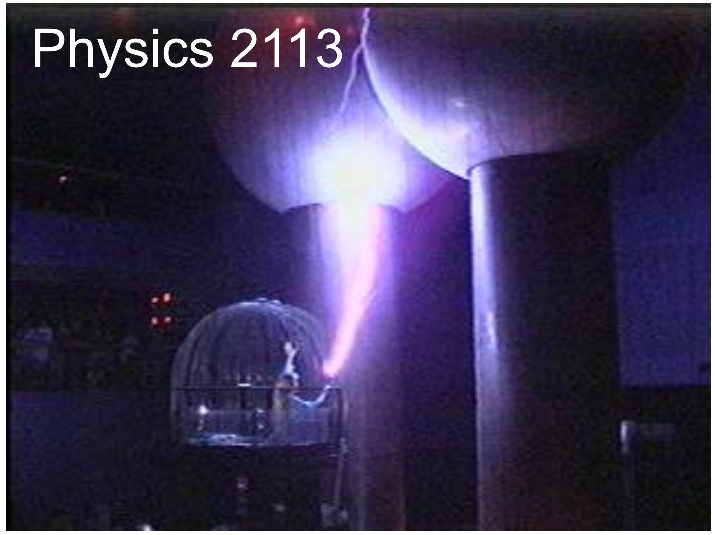
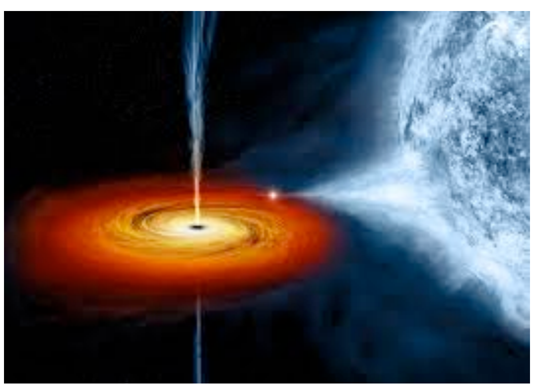


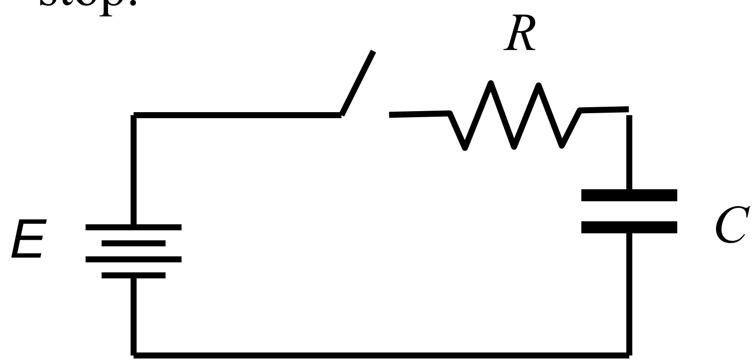
Physics 2113
Lecture 21: FRI 17 OCT

CH27: Circuits



In circuits involving only resistors and batteries, current flows "forever" (or until batteries run out…).

Capacitors, on the other hand, are not conductors of current. Current can flow into them to "build up charge" on the plates, but it will eventually stop.



For the capacitor: $V=q/C$.

"Taking a walk"

$$\mathcal{E} - iR - \frac{q}{C} = 0 \qquad \text{Now, } i = \frac{dq}{dt}$$

$$\text{Therefore: } \mathcal{E} - \frac{dq}{dt}R - \frac{q}{C} = 0$$

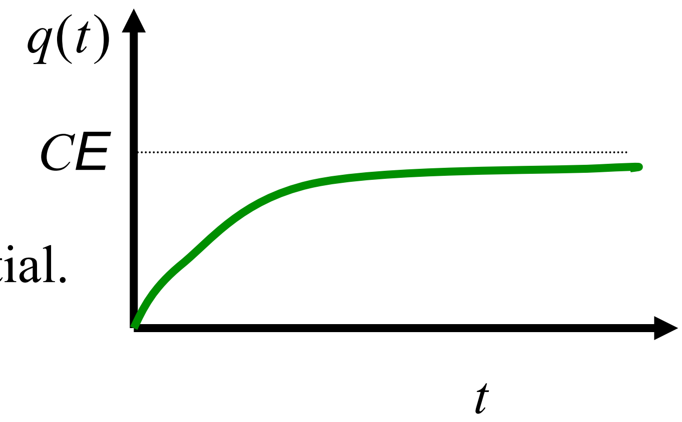
The solution is given by solving a differential equation: a function $q(t)$ such that $q=0$ at $t=0$.

$$\text{One can check that: } q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right) \qquad \text{Where } e = 2.718281\ldots$$

Is a solution.

$$q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

The capacitor charges as an exponential.



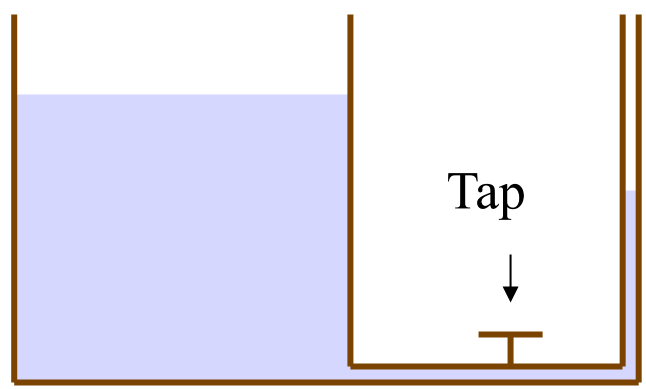
Proof that *q(t)* is a solution:

$$i = \frac{dq}{dt} = C\mathcal{E}\,\frac{e^{-\frac{t}{RC}}}{RC} = \mathcal{E}\,\frac{e^{-\frac{t}{RC}}}{R} \qquad Ri = \mathcal{E}\,e^{-\frac{t}{RC}} \qquad \frac{q}{C} = \mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

So, $\mathcal{E} - iR - \frac{q}{C} = 0$

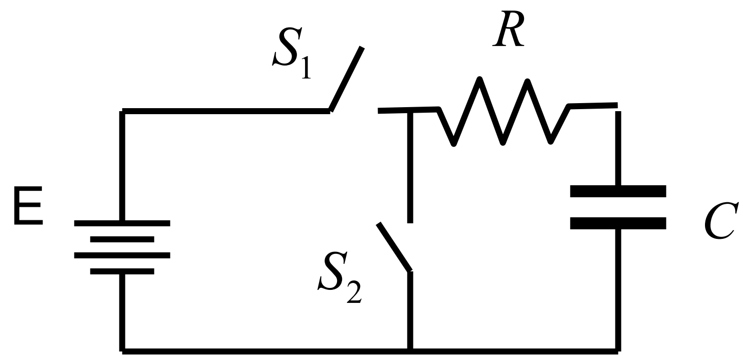
Analogy:

Water tank



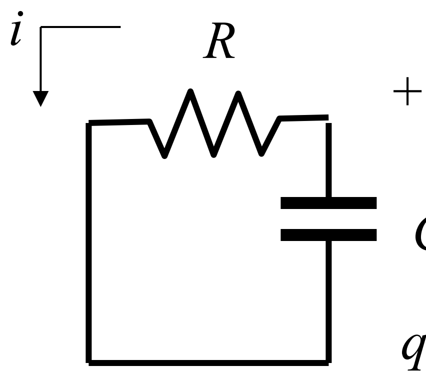
When tap is opened, the water level in the thin pipe will rise, until it equals the level of the big tank. The approach is actually exponential as well.

Discharge:



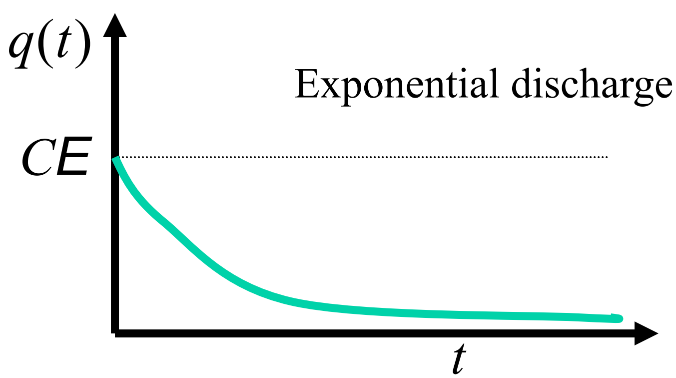
Suppose someone closes S1, and waits for a long time. The capacitor will become fully charged, the potential difference across the capacitor is E. The switch S1 is then opened, and the capacitor remains charged.

Now let us close the switch S2. The battery is disconnected from the circuit. The resulting relevant loop is,

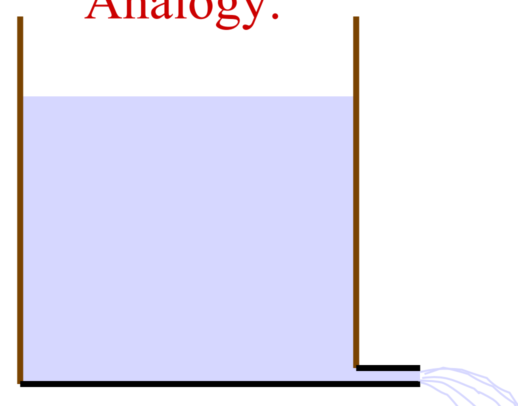


Going around the circuit: $-Ri - \frac{q}{C} = 0 \Rightarrow R\frac{dq}{dt} + \frac{q}{C} = 0$

Solution: $q(t) = C\mathrm{E}e^{-\frac{t}{RC}}$



Analogy:



Time constant:

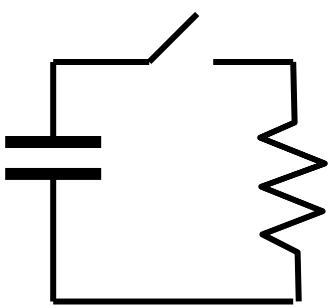
$q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$ Sometimes written as: $q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{\tau}}\right)$

Where $\tau = RC$ is called the "time constant" (it has units of time, seconds).

It represents the time that it takes the potential V to be equal to 63% of $\mathcal{E}$

$$\tau = 1 \Rightarrow \left(1 - e^{-1}\right) \approx 0.63$$

Example:



The capacitor starts charged, with a potential of 100V.

At $t=0$, close switch (on). At $t=10s$, potential$=1V$

What is τ? Potential at 17s?

$$V = V_0 e^{-\frac{t}{\tau}} \Rightarrow 1V = 100Ve^{-\frac{10s}{\tau}} \Rightarrow \ln(0.01) = -\frac{10s}{\tau} \Rightarrow \tau = 2.17s$$

$$V = 100Ve^{-\frac{17}{2.17}} = 0.0396V$$

76P. The circuit of Fig. 28-56 shows a capacitor C, two ideal batteries, two resistors, and a switch S. Initially S has been open for a long time. If it is then closed for a long time, by how much does the charge on the capacitor change? Assume $C = 10\ \mu F$, $\mathcal{E}_1 = 1.0$ V, $\mathcal{E}_2 = 3.0$ V, $R_1 = 0.20\ \Omega$, and $R_2 = 0.40\ \Omega$.

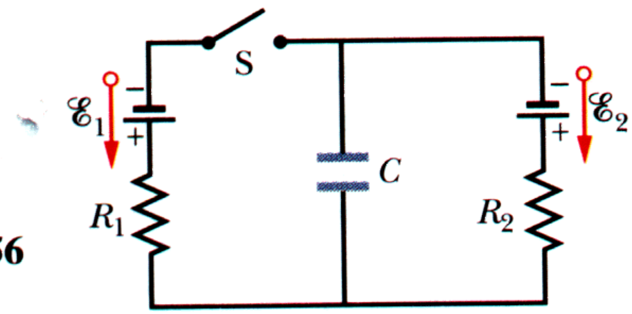
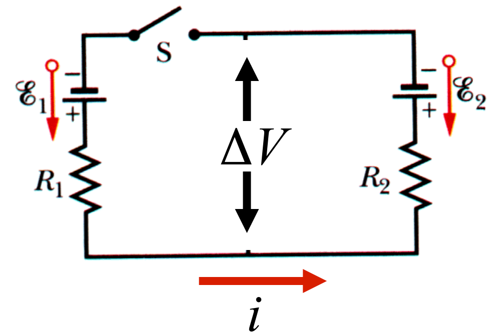


FIGURE 28-56 Problem 76.

S initially open for a long time, then ΔV across C equals E_2, and therefore $q_{\text{initial}} = CE_2 = 30\mu C$



Closed for a long time, therefore fully charged at the potential difference between top and bottom,

"Taking a walk", $E_1 - iR_1 - iR_2 - E_2 = 0$

Therefore $i = \dfrac{E_1 - E_2}{R_1 + R_2} = -3.33A$, and $\Delta V = E_2 + iR_2 = 3 - 3.33 \times 0.4 = 1.66V$

And thus, $q = 1.67V \times 10\mu F = 16.7\mu C$,

and $q = q_{\text{final}} - q_{\text{initial}} = -13.3\mu C$

••63 SSM WWW In the circuit of Fig. 27-65, $\mathscr{E} = 1.2$ kV, $C = 6.5\ \mu$F, $R_1 = R_2 = R_3 = 0.73$ MΩ. With C completely uncharged, switch S is suddenly closed (at $t = 0$). At $t = 0$, what are (a) current i_1 in resistor 1, (b) current i_2 in resistor 2, and (c) current i_3 in resistor 3? At $t = \infty$ (that is, after many time constants), what are (d) i_1, (e) i_2, and (f) i_3? What is the potential difference V_2 across resistor 2 at (g) $t = 0$ and (h) $t = \infty$? (i) Sketch V_2 versus t between these two extreme times.

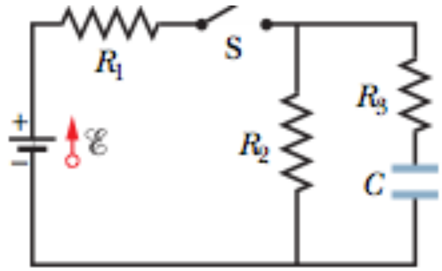
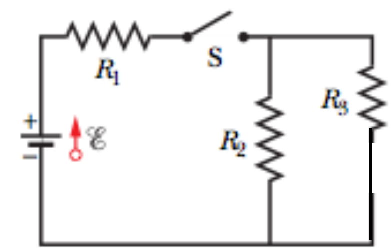


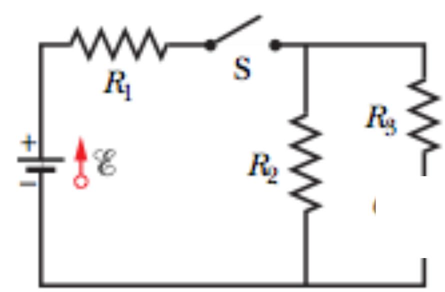
Fig. 27-65 Problem 63.

Sketch of the solution:

a,b,c) At t=0 capacitor is discharged, so it behaves as a wire: Resistors 2,3 in parallel with each other and the resulting resistor in series with R1.



d,e,f) At t=infinity, capacitor fully charged, behaves as if the circuit is open. So there is no current in R3, and R1, R2 are in series.



Summary:

- Questions concerning circuits with capacitors deal with two kinds of questions: what happens while they are getting charged or discharged.
- Charge or discharge is an exponential process, the "time constant" in the exponent being RC.
- Final charge is treated via the formula $q=CV$.