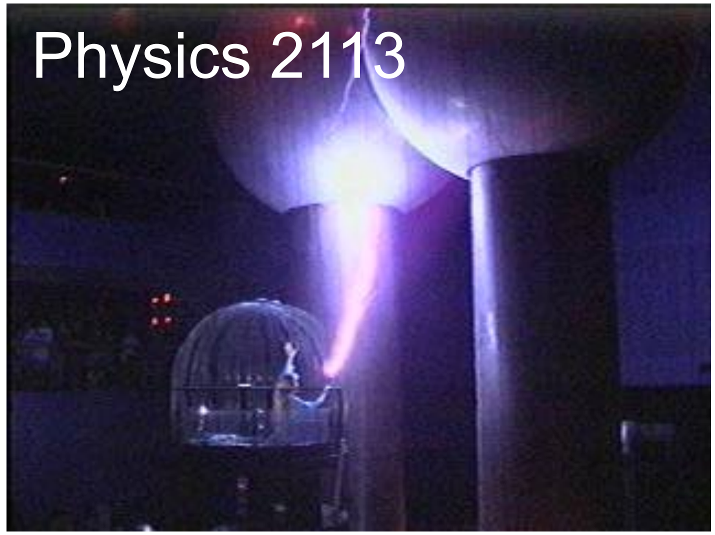
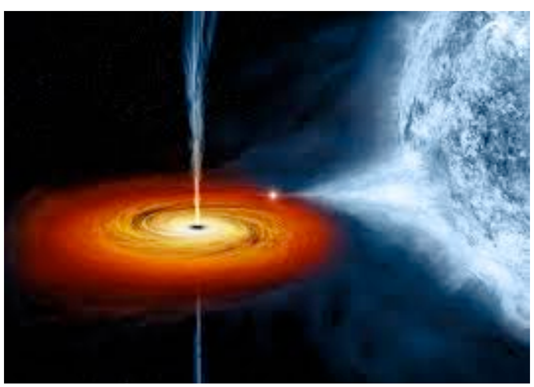


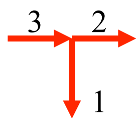
Physics 2113
Lecture 20: WED 15 OCT

CH27: Circuits

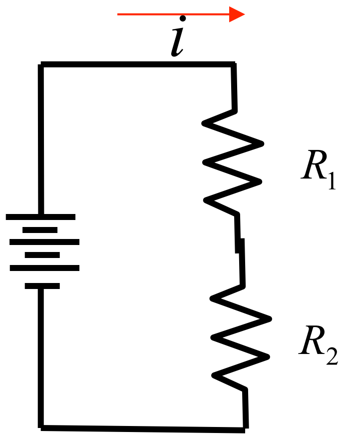


Summary of last class:

- To solve a circuit, start at any point in it and "take a walk" around it.
- Add potential every time you traverse an EMF from - to +.
- Deduct -iR potential every time you move along a resistor "with the flow".
- Take as many walks along independent loops as needed to solve the circuit.
- Treat junctions as "water pipes".



Resistors in series:



Taking a walk:

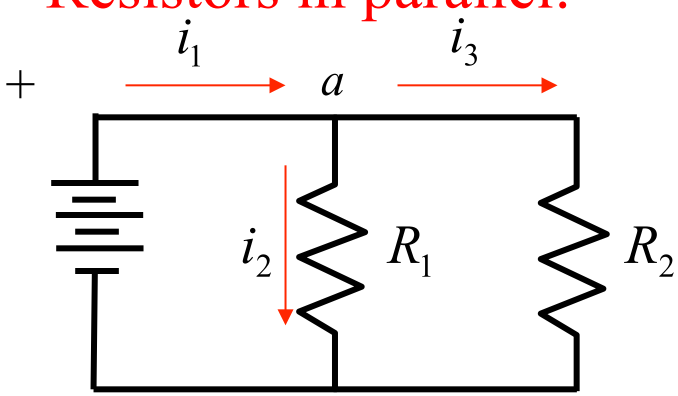
$$+\mathcal{E} - iR_1 - iR_2 = 0 \quad \Rightarrow \quad \mathcal{E} = i(R_1 + R_2) = iR_{\text{tot}}$$

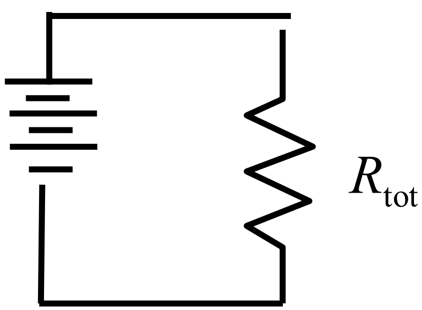
$$\boxed{R_{\text{tot}} = R_1 + R_2}$$

Behave like capacitors in parallel!

If you have n resistors in series: $R_{\text{tot}} = \sum_{i=1}^{n} R_i$

Resistors in parallel:





Node a: $i_1 = i_2 + i_3$

Left loop: $\mathcal{E} - i_2 R_1 = 0 \quad \Rightarrow \quad i_2 = \frac{\mathcal{E}}{R_1}$

Outer loop: $\mathcal{E} - i_3 R_2 = 0 \quad \Rightarrow \quad i_3 = \frac{\mathcal{E}}{R_2}$

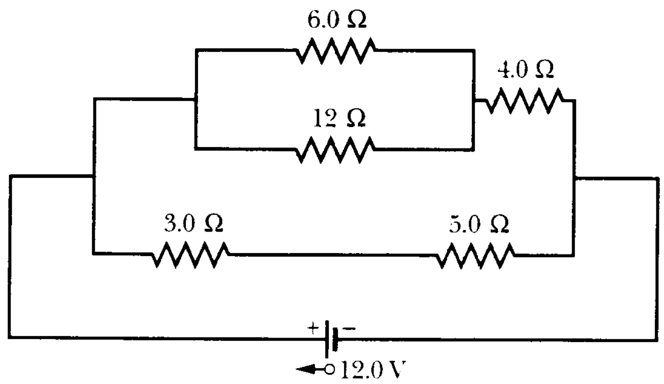
$$i_1 = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\mathcal{E}$$

$$\boxed{R_{\text{tot}} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

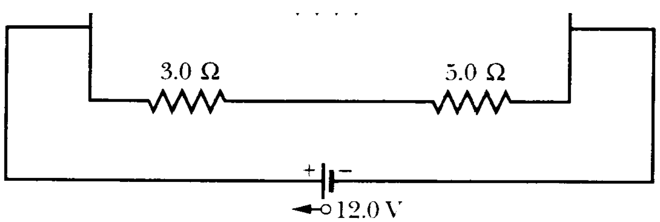
Same as capacitors in series.

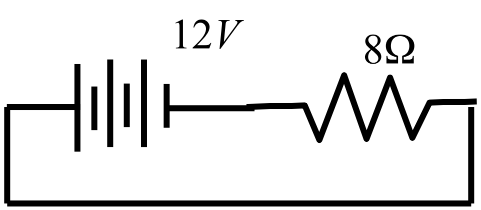
Example:

38E. A circuit containing five resistors connected to a battery with a 12.0 V emf is shown in Fig. 28-38. What is the potential difference across the 5.0 Ω resistor?



Bottom loop: (all else is irrelevant)





$$i = \frac{V}{R} = \frac{12V}{8\Omega} = 1.5A$$

Example:

48P. In Fig. 28-44, $\mathcal{E}_1 = 3.00$ V, $\mathcal{E}_2 = 1.00$ V, $R_1 = 5.00\ \Omega$, $R_2 = 2.00\ \Omega$, $R_3 = 4.00\ \Omega$, and both batteries are ideal. (a) What is the rate at which energy is dissipated in R_1? In R_2? In R_3? (b) What is the power of battery 1? Of battery 2?

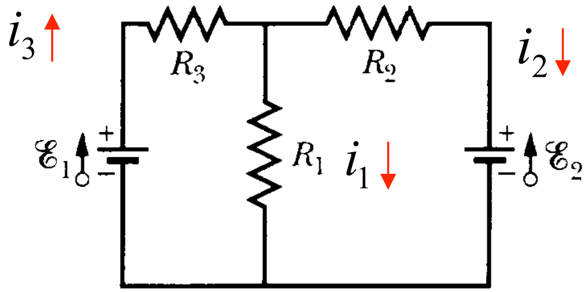
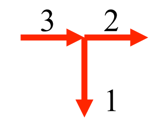


FIGURE 28-44
Problem 48.

1) Label currents, assume direction.

2) Top junction : $i_3 = i_1 + i_2$



3) Left loop : $\mathrm{E}_1 - i_3 R_3 - i_1 R_1 = 0$

4) Right loop : $-\mathrm{E}_2 + i_1 R_1 - i_2 R_2 = 0$

4) $\rightarrow \quad -1 + 5i_1 - 2i_2 = 0$

3) and 2) $\rightarrow \quad 3 - 4(i_1 + i_2) - 5i_1 = 0 \quad \Rightarrow 3 - 9i_1 - 4i_2 = 0$

$\Big\}\quad 5 - 19i_1 = 0 \Rightarrow i_1 = 0.26A$

Subst. in 4) $\rightarrow -1 + 1.31 - 2i_2 = 0 \Rightarrow i_2 = 0.16A$ Junction : $i_3 = 0.42A$

Power in R_1 : $(0.26)^2 5 = 0.34W$ in R_2 : $(0.16)^2 2 = 0.05W$ R_3 : $(0.42)^2 4 = 0.7W$

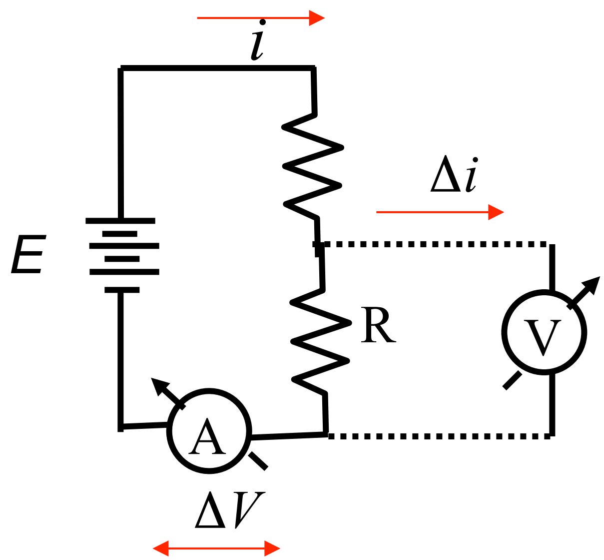
Power from battery 1 : $\mathrm{E}_1 i_3 = 1.26W$

Check : $1.26 - 0.16 = 1.10 \approx 0.34 + 0.05 + 0.7$

Power from battery 2 : $\mathrm{E}_2 i_2 = -0.16W$

Comment:

Voltmeter and ammeter "in real life".



In practice, the voltmeter will draw a (hopefully tiny) current Δi , therefore distorting the value of V across R we want to measure. Good voltmeters minimize this current in order to minimize the distortion.

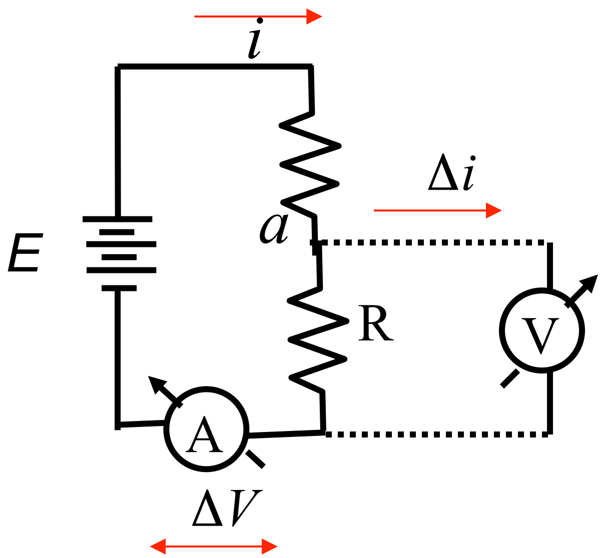
Similarly an ideal ammeter would try to allow all of the current to pass through it, opposing the least resistance. Good ammeters minimize their internal resistance.

Good voltmeter: high internal resistance, low Δi
Good ammeter: low internal resistance, low ΔV

Never plug an ammeter into a 110V outlet!

A good ammeter is essentially a "short circuit"

An application, Ohmmeters:



One is tempted to say, “I measure current A going through the resistor and voltage V across its ends, therefore using Ohm’s laws, R=V/A”.

Wrong. Voltage indeed is V.

Applying junction rule in a,

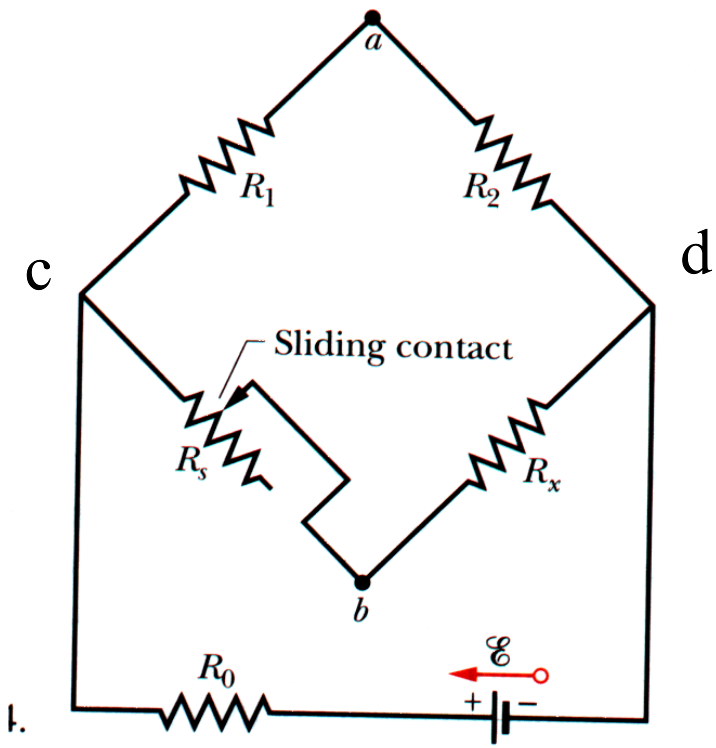
the current across the resistor

is really $i_{\text{true}} = i - \Delta i$.

Therefore, $R = \dfrac{V}{i - \Delta i}$

Real life Ohmmeters actually correct for this. The correction is harder than it looks, since Δi actually depends on R!!

Wheatstone bridge, a simple Ohmeter



One slides the contact in Rs, varying its resistance, until the potential difference between a and b vanishes.

Solving the circuit,

the current through the bottom loop is,

$$i_{\text{bottom}} = \frac{V_{cd}}{R_s + R_x},$$

and for the top is $i_{\text{top}} = \dfrac{V_{cd}}{R_1 + R_2}$

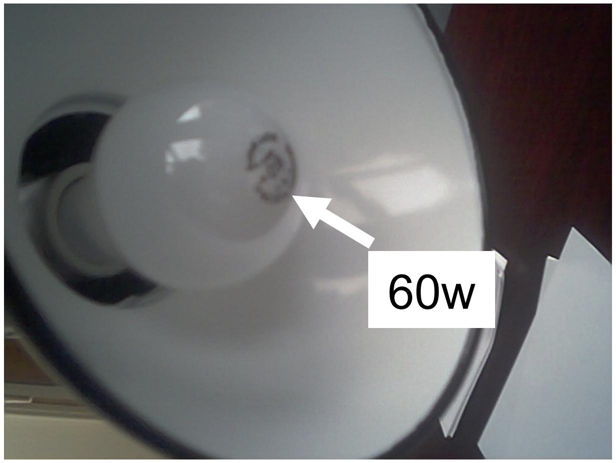
Now, $V_{ad} = i_{top} R_2$ and $V_{bd} = i_{bottom} R_x$

But $V_{ad} = V_{bd}$, therefore, $\dfrac{R_2}{R_1 + R_2} = \dfrac{R_x}{R_s + R_x} \Rightarrow \dfrac{R_x}{R_s} = \dfrac{R_2}{R_1}$

So, knowing resistances 1,2 and s, one knows resistance x.

Light bulbs in series and parallel

$$\text{Power} = \frac{V^2}{R}$$

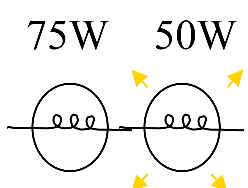


Manufacturer spec: Watts at 110V

More Watts = less resistance.

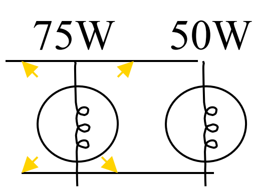
Less Watts = more resistance.

Series:



i is given. More resistance (less spec Watts), more power!

Parallel:



V is given. Less resistance (more spec Watts), more power!

Summary:

- We solve circuits involving resistors “by taking a walk”, as we discussed.
- Circuits in series share the same current, in parallel the same voltage. Beware at the time of computing powers!
- Voltmeters and Ammeters in real life behave as resistors, and therefore have non-vanishing currents and voltages across their leads respectively, that ideally would be zero.