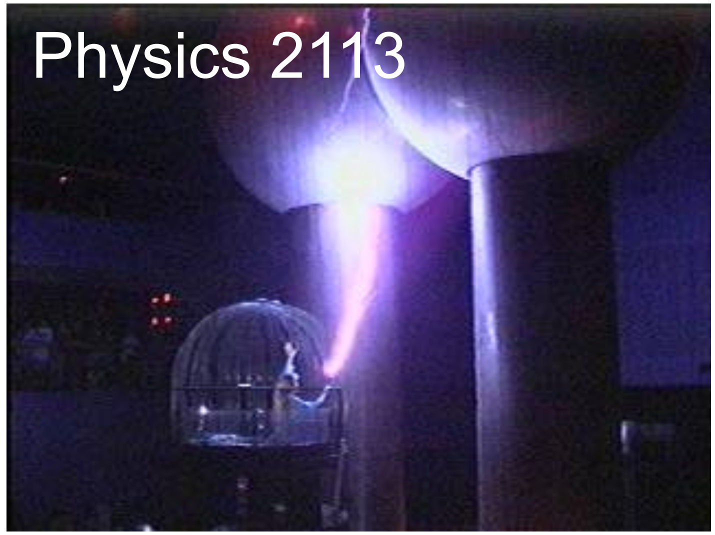
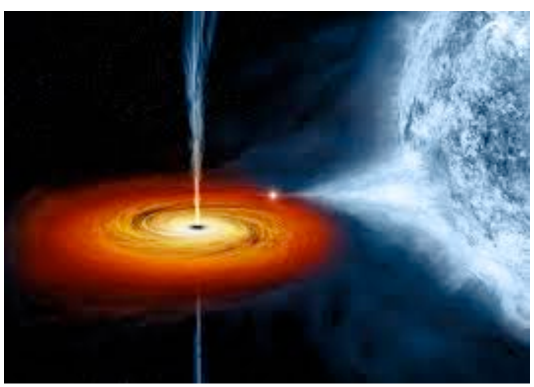


Andre-Marie
Ampere
1175-1836

Physics 2113
Lecture 17: WED 8 OCT

CH26: Current and resistance



Last class:

Resistance:

Ohm's laws

$$R \equiv \frac{V}{i} \qquad \text{and therefore}: \; i = \frac{V}{R} \quad \text{and} \quad V = iR$$

Georg Simon Ohm
(1789-1854)

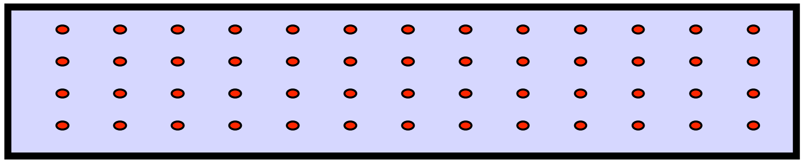
CHECKPOINT 4

The following table gives the current i (in amperes) through two devices for several values of potential difference V (in volts). From these data, determine which device does not obey Ohm's law.

Device 1		Device 2	
V	i	V	i
2.00	4.50	2.00	1.50
3.00	6.75	3.00	2.20
4.00	9.00	4.00	2.80

Variation of resistance (resistivity) with temperature

R, ρ change with temperature in a complicated, material-dependent way.



Why does it change? Nuclei vibrate due to thermal agitation, and scatter electrons as they pass.

For many conductors, it can be approximated by a linear temperature dependence (for a small range of temperatures),

$$\frac{\rho - \rho_0}{\rho_0} = \alpha (T - T_0)$$

With α determined empirically and listed in tables.

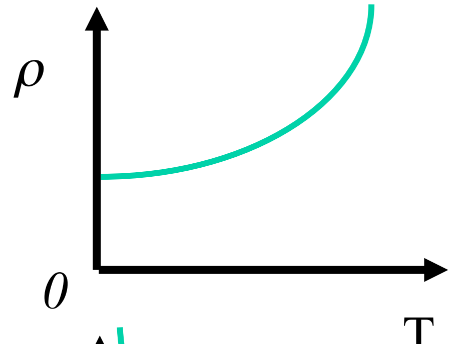
Trivia: why do light bulbs mostly die at the moment of switch-on?

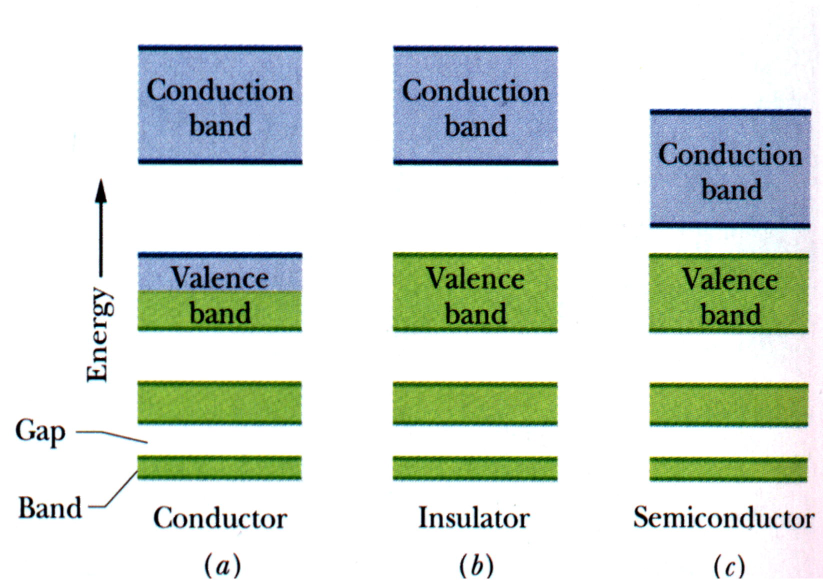
Answer: when the filament is cold it has less resistance, therefore it is the moment when the current is maximum.

Various materials and temperature:

Conductors:

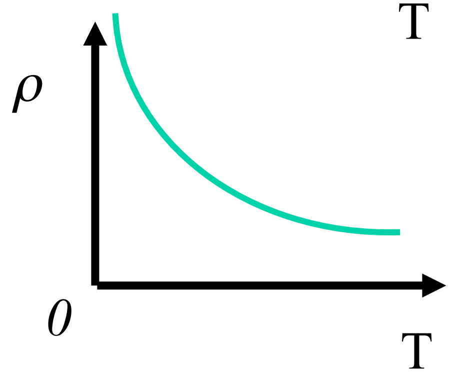
E.g.: copper, aluminum, iron.



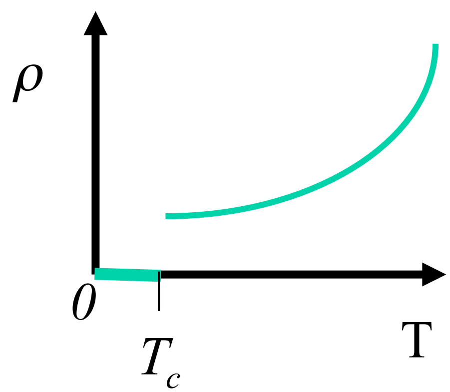


Semiconductors

E.g.: Germanium, silicon.



Superconductors

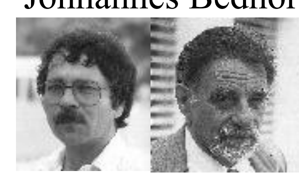


T_C : 0.1 to 20K for "usual" materials, e.g. mercury (1911)

30 to 180K for "high temperature" materials, e.g. YBaCu (1986)

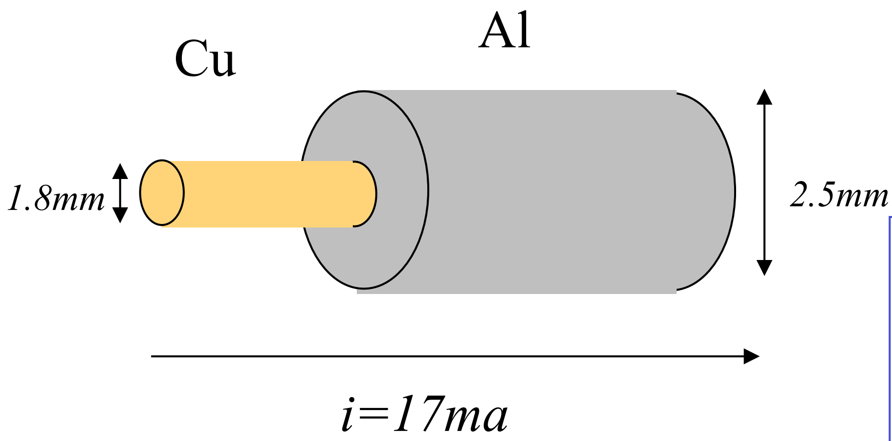
Heike Kammerlingh-Onnes

Karl Mueller Johannes Bednorz



Example: HRW Sample problem

a) J? b) v_d



$$A_{\mathrm{Al}} = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{4}\left(2.5\times10^{-3}m\right)^2 = 4.4\times10^{-6}m^2$$

$$\boxed{\begin{aligned} J_{\mathrm{Al}} &= \frac{i}{A_{\mathrm{Al}}} = \frac{17\times10^{-3}A}{4.4\times10^{-6}m^2} = 3.5\times10^3 A/m^2 \\ J_{\mathrm{Cu}} &= \frac{i}{A_{\mathrm{Cu}}} = \frac{17\times10^{-3}A}{2.54\times10^{-6}m^2} = 6.7\times10^3 A/m^2 \end{aligned}}$$

Part b)

Assume one electron per atom:

$$n = \frac{\#\,\mathrm{electrons}}{\mathrm{volume}} = \frac{N_A \dfrac{\mathrm{mass}}{\mathrm{Atomic\ mass}}}{\mathrm{volume}} = \frac{N_A\,\mathrm{density}}{\mathrm{Atomic\ mass}} = \frac{6\times10^{23}\ \mathrm{mol}^{-1}\ \ 9\times10^3 kg/m^3}{64\times10^{-2}kg/\mathrm{mol}}$$

$$= 8.5\times10^{23}\frac{e}{m^3}$$

Having n, we can compute the drift velocity:

$$v_d = \frac{6.7\times10^3\, A/m^3}{8.5\times10^{28}\, e/m^3 \quad 1.6\times10^{-19}\, C/e} = 4.9\times10^{-7}\, m/s = 1.8\, mm/h$$

How could this be?

When I flick a switch, a lamp that is yards away goes on immediately!

Surfer analogy: a surfer can move fast whereas a person floating in the same water is barely moved by a wave.

Similarly, disturbances in the electron distribution in a conductor travel essentially at the speed of light, whereas the electrons themselves move very slowly.

Microscopic view of Ohm's law

We argued that electrons are pretty free to move in conductors, but that they may "bounce" with nuclei. This, in turn explained why resistivity depended on temperature.

When an electric field is applied, an electron free to move will acquire an acceleration,

$$a = \frac{F}{m} = \frac{eE}{m}$$

We can do a rough model of the motion by assuming that after a collision, the electron loses all memory of the acceleration a it had acquired. In the average time between collisions τ, the electron will acquire a drift velocity $v_d = a\,\tau = e\,E\,\tau/m$.

Combining this result with the expression we worked out last class for the current density in terms of the drift velocity, $\vec{J} = ne\vec{v}_d$ we get,

$$v_d = \frac{J}{ne} = \frac{eE\tau}{m}$$

Which we can write as $E = \left(\frac{m}{e^2 n\tau}\right) J$ and therefore $\boxed{\rho = \frac{m}{e^2 n\tau}}$

Mean free time and mean free distance

(a) What is the mean free time τ between collisions for the conduction electrons in copper?

KEY IDEAS

The mean free time τ of copper is approximately constant, and in particular does not depend on any electric field that might be applied to a sample of the copper. Thus, we need not consider any particular value of applied electric field. However, because the resistivity ρ displayed by copper under an electric field depends on τ, we can find the mean free time τ from Eq. 26-22 ($\rho = m/e^2 n\tau$).

Calculations: That equation gives us

$$\tau = \frac{m}{ne^2\rho}. \qquad (26\text{-}23)$$

The number of conduction electrons per unit volume in copper is $8.49 \times 10^{28}\ \text{m}^{-3}$. We take the value of ρ from Table 26-1. The denominator then becomes

$$(8.49 \times 10^{28}\ \text{m}^{-3})(1.6 \times 10^{-19}\ \text{C})^2(1.69 \times 10^{-8}\ \Omega\cdot\text{m}) = 3.67 \times 10^{-17}\ \text{C}^2\cdot\Omega/\text{m}^2 = 3.67 \times 10^{-17}\ \text{kg/s},$$

where we converted units as

$$\frac{\text{C}^2\cdot\Omega}{\text{m}^2} = \frac{\text{C}^2\cdot\text{V}}{\text{m}^2\cdot\text{A}} = \frac{\text{C}^2\cdot\text{J/C}}{\text{m}^2\cdot\text{C/s}} = \frac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{m}^2/\text{s}} = \frac{\text{kg}}{\text{s}}.$$

Using these results and substituting for the electron mass m, we then have

$$\tau = \frac{9.1 \times 10^{-31}\ \text{kg}}{3.67 \times 10^{-17}\ \text{kg/s}} = 2.5 \times 10^{-14}\ \text{s}. \qquad \text{(Answer)}$$

(b) The mean free path λ of the conduction electrons in a conductor is the average distance traveled by an electron between collisions. (This definition parallels that in Section 19-6 for the mean free path of molecules in a gas.) What is λ for the conduction electrons in copper, assuming that their effective speed v_{eff} is 1.6×10^6 m/s?

KEY IDEA

The distance d any particle travels in a certain time t at a constant speed v is $d = vt$.

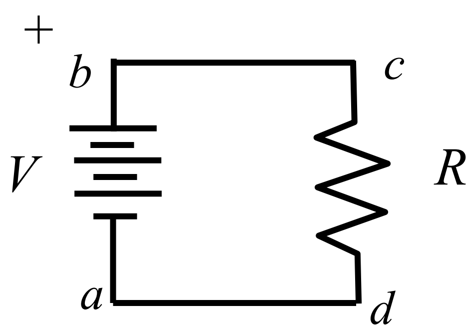
Calculation: For the electrons in copper, this gives us

$$\begin{aligned}\lambda &= v_{\text{eff}}\tau && (26\text{-}24)\\ &= (1.6 \times 10^6\ \text{m/s})(2.5 \times 10^{-14}\ \text{s})\\ &= 4.0 \times 10^{-8}\ \text{m} = 40\ \text{nm}. && \text{(Answer)}\end{aligned}$$

This is about 150 times the distance between nearest-neighbor atoms in a copper lattice. Thus, on the average, each conduction electron passes many copper atoms before finally hitting one.

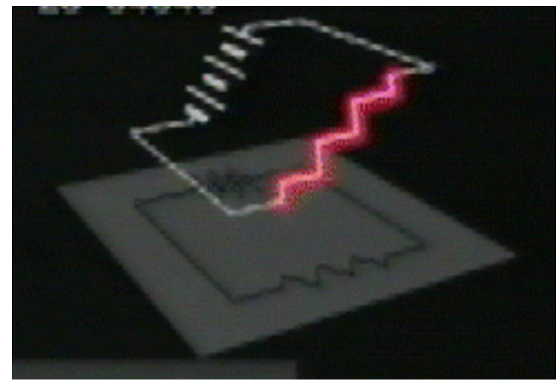
Power dissipation:

Resistance was a measure of the "cost" of establishing a current in a realistic conductor. The "cost" can be characterized in terms of the energy one needs to constantly input to a conductor in order to keep a current going.



Let us follow an amount of charge dq as it moves through the circuit, starting at a.

From a to b, through the battery, its potential energy is increased by Vdq.
From b to c its potential is constant, similarly from d to a.



When it is back at a, its potential energy should be the same as when it started. Therefore there must have been a loss of potential energy of amount $-Vdq$ when moving through the resistance.

$$dU = Vdq = V\,i\,dt \qquad \Rightarrow \qquad \boxed{\text{Power} = \frac{dU}{dt} = V\,i}$$

Units: Watt

$$\text{Applying Ohm's laws:} \quad \text{Power} = (i\,R)\,i = \boxed{i^2 R} \qquad \text{Power} = V\left(\frac{V}{R}\right) = \boxed{\frac{V^2}{R}}$$

Rate of energy dissipation in a wire carrying current

You are given a length of uniform heating wire made of a nickel–chromium–iron alloy called Nichrome; it has a resistance R of 72 Ω. At what rate is energy dissipated in each of the following situations? (1) A potential difference of 120 V is applied across the full length of the wire. (2) The wire is cut in half, and a potential difference of 120 V is applied across the length of each half.

KEY IDEA

Current in a resistive material produces a transfer of mechanical energy to thermal energy; the rate of transfer (dissipation) is given by Eqs. 26-26 to 26-28.

Calculations: Because we know the potential V and resistance R, we use Eq. 26-28, which yields, for situation 1,

$$P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{72\ \Omega} = 200\ \text{W}. \qquad \text{(Answer)}$$

In situation 2, the resistance of each half of the wire is $(72\ \Omega)/2$, or 36 Ω. Thus, the dissipation rate for each half is

$$P' = \frac{(120\ \text{V})^2}{36\ \Omega} = 400\ \text{W},$$

and that for the two halves is

$$P = 2P' = 800\ \text{W}. \qquad \text{(Answer)}$$

This is four times the dissipation rate of the full length of wire. Thus, you might conclude that you could buy a heating coil, cut it in half, and reconnect it to obtain four times the heat output. Why is this unwise? (What would happen to the amount of current in the coil?)

Summary

Current

- The electric current i in a conductor is defined by

$$i = \frac{dq}{dt}.$$

Current Density

- Current is related to current density $i = \int \vec{J} \cdot d\vec{A},$

Drift Speed of the Charge Carriers

- Drift speed of the charge carriers in an applied electric field is related to current density by $\vec{J} = (ne)\vec{v}_d,$

Resistance of a Conductor

- Resistance R of a conductor is defined by

$$R = \frac{V}{i}$$

- Similarly the resistivity and conductivity of a material is defined by $\rho = \frac{1}{\sigma} = \frac{E}{J}$

- Resistance of a conducting wire of length L and uniform cross section is

$$R = \rho \frac{L}{A}$$

Change of ρ with Temperature

- The resistivity of most material changes with temperature and is given as

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0).$$

Summary

Ohm's Law

- A given device (conductor, resistor, or any other electrical device) obeys Ohm's law if its resistance R (defined by **Eq. 26-8** as V/i) is independent of the applied potential difference V.

Resistivity of a Metal

- By assuming that the conduction electrons in a metal are free to move like the molecules of a gas, it is possible to derive an expression for the resistivity of a metal:

$$\rho = \frac{m}{e^2 n \tau}.$$

Power

- The power P, or rate of energy transfer, in an electrical device across which a potential difference V is maintained is

$$P = iV$$

- If the device is a resistor, we can write

$$P = i^2 R = \frac{V^2}{R}$$