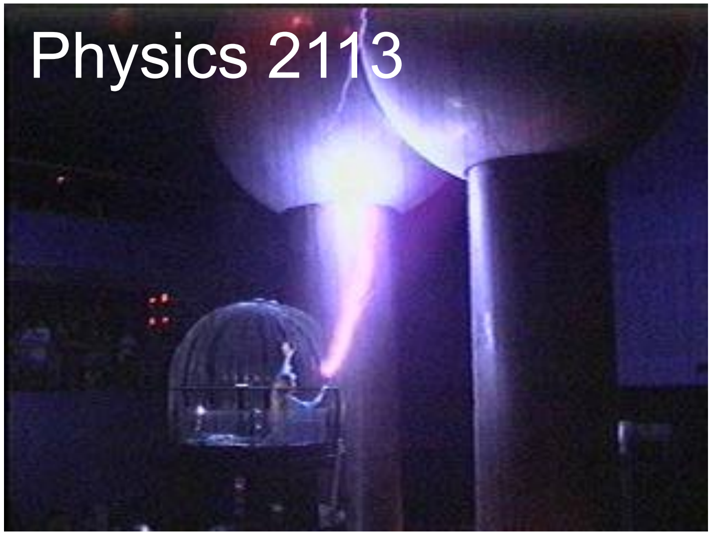
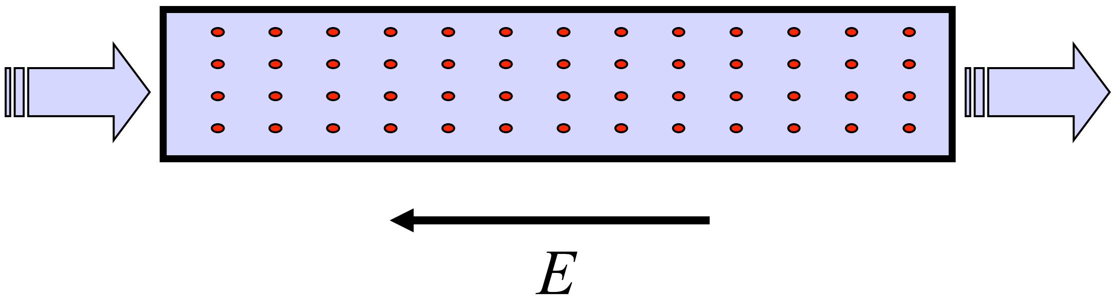



Andre-Marie
Ampere
1175-1836

# Physics 2113
# Lecture 16: MON 6 OCT

## CH26: Current and resistance

In a conductor, electrons are free to move. So if one applies an electric field, they'll move!




This motion of electrons is called electrical current. The analogy with a fluid flow is obvious.

For historical reasons, instead of thinking of currents as flows of negative charges (electrons) against the electric field, we think of them as motions of imaginary positive charges along the field directions. From a calculational point of view, both approaches yield the same results.

$$i = \frac{dq}{dt}, \qquad q = \int i\, dt \qquad \text{Units}: [\text{i}] = \frac{\text{Coulomb}}{\text{second}} \equiv \text{Ampere}$$

Andre-Marie Ampere 1175-1836

Wasn't the field supposed to be zero inside conductors?

Yes, if the situation was static. The reasoning was "electrons move till they cancel out the field". If the situation is not static, that is, if electrons are moving, then the field can be nonzero in a conductor, and the potential is not constant across it!

## Current is the rate at which charge passes a point

Water flows through a garden hose at a volume flow rate $dV/dt$ of 450 $cm^3/s$. What is the current of negative charge?

**KEY IDEAS**

The current $i$ of negative charge is due to the electrons in the water molecules moving through the hose. The current is the rate at which that negative charge passes through any plane that cuts completely across the hose.

***Calculations:*** We can write the current in terms of the number of molecules that pass through such a plane per second as

$$i = \begin{pmatrix}\text{charge}\\ \text{per}\\ \text{electron}\end{pmatrix}\begin{pmatrix}\text{electrons}\\ \text{per}\\ \text{molecule}\end{pmatrix}\begin{pmatrix}\text{molecules}\\ \text{per}\\ \text{second}\end{pmatrix}$$

or

$$i = (e)(10)\frac{dN}{dt}.$$

We substitute 10 electrons per molecule because a water ($H_2O$) molecule contains 8 electrons in the single oxygen atom and 1 electron in each of the two hydrogen atoms.

We can express the rate $dN/dt$ in terms of the given volume flow rate $dV/dt$ by first writing

$$\begin{pmatrix}\text{molecules}\\ \text{per}\\ \text{second}\end{pmatrix} = \begin{pmatrix}\text{molecules}\\ \text{per}\\ \text{mole}\end{pmatrix}\begin{pmatrix}\text{moles}\\ \text{per unit}\\ \text{mass}\end{pmatrix} \times \begin{pmatrix}\text{mass}\\ \text{per unit}\\ \text{volume}\end{pmatrix}\begin{pmatrix}\text{volume}\\ \text{per}\\ \text{second}\end{pmatrix}.$$

"Molecules per mole" is Avogadro's number $N_A$. "Moles per unit mass" is the inverse of the mass per mole, which is the molar mass $M$ of water. "Mass per unit volume" is the (mass) density $\rho_{mass}$ of water. The volume per second is the volume flow rate $dV/dt$. Thus, we have

$$\frac{dN}{dt} = N_A\left(\frac{1}{M}\right)\rho_{mass}\left(\frac{dV}{dt}\right) = \frac{N_A\rho_{mass}}{M}\frac{dV}{dt}.$$

Substituting this into the equation for $i$, we find

$$i = 10eN_AM^{-1}\rho_{mass}\frac{dV}{dt}.$$

We know that Avogadro's number $N_A$ is $6.02 \times 10^{23}$ molecules/mol, or $6.02 \times 10^{23}$ $mol^{-1}$, and from Table 15-1 we know that the density of water $\rho_{mass}$ under normal conditions is 1000 $kg/m^3$. We can get the molar mass of water from the molar masses listed in Appendix F (in grams per mole): We add the molar mass of oxygen (16 g/mol) to twice the molar mass of hydrogen (1 g/mol), obtaining 18 g/mol = 0.018 kg/mol. So, the current of negative charge due to the electrons in the water is

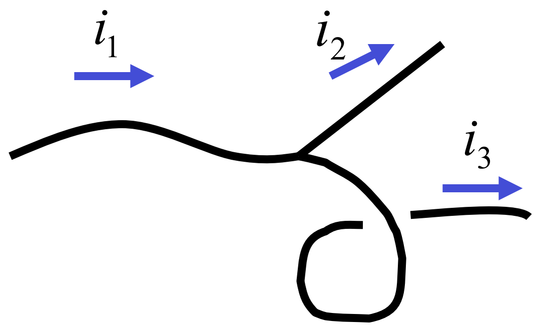
$$\begin{aligned} i &= (10)(1.6 \times 10^{-19}\ \text{C})(6.02 \times 10^{23}\ \text{mol}^{-1}) \\ &\quad \times (0.018\ \text{kg/mol})^{-1}(1000\ \text{kg/m}^3)(450 \times 10^{-6}\ \text{m}^3/\text{s}) \\ &= 2.41 \times 10^7\ \text{C/s} = 2.41 \times 10^7\ \text{A} \\ &= 24.1\ \text{MA}. \end{aligned}$$

(Answer)

This current of negative charge is exactly compensated by a current of positive charge associated with the nuclei of the three atoms that make up the water molecule. Thus, there is no net flow of charge through the hose.

Current is a scalar, although we use arrows to indicate direction of propagation. It is conserved.

Concretely, at a juncture one must make sure that “what goes in, comes out” (just like for water pipes).

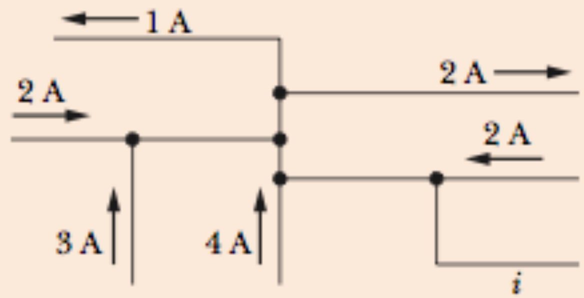


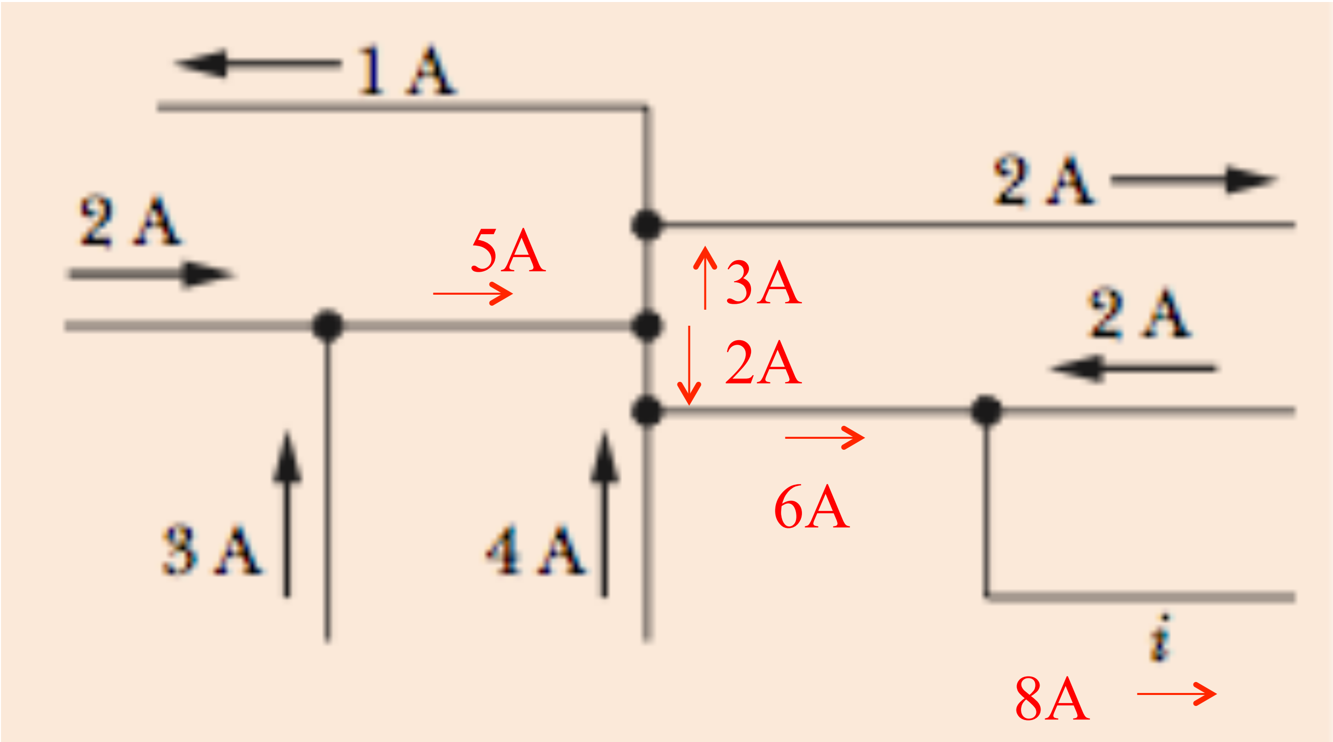

$$i_1 = i_2 + i_3$$

In reality, electrons are not “completely free to move” in a conductor. In fact, electrons move erratically, colliding with the nuclei all the time. Therefore “there is a price to be paid” to establish a current in a conductor. This leads to the concept of resistance.

## CHECKPOINT 1

The figure here shows a portion of a circuit. What are the magnitude and direction of the current $i$ in the lower right-hand wire?



# Resistance:

How much potential do I need to apply to a device to drive a given current through it?

**Ohm's laws**

$$R \equiv \frac{V}{i} \quad \text{and therefore}: \; i = \frac{V}{R} \quad \text{and} \quad V = iR$$

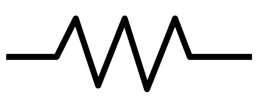
Georg Simon Ohm
(1789-1854)

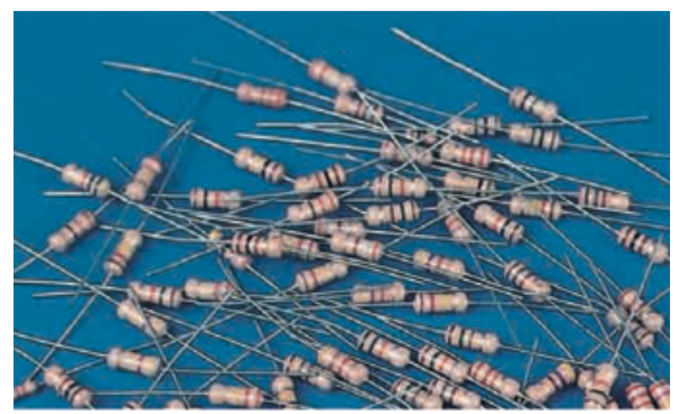
$$\text{Units}: [\mathrm{R}] = \frac{\text{Volt}}{\text{Ampere}} \equiv \text{Ohm (abbr. } \Omega)$$

"a professor who preaches such heresies is unworthy to teach science." Prussian minister of education 1830

For many materials, R remains a constant for a wide range of values of current and potential.

Devices specifically designed to have a constant value of R are called resistors, and symbolized by



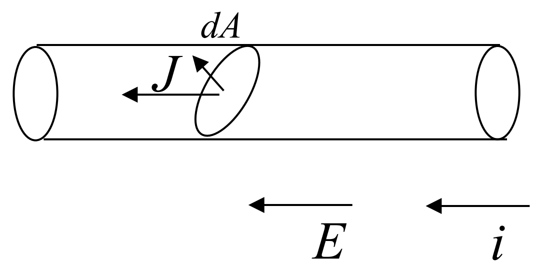


The definition of current we introduced refers to a “bulk” property of a conducting device. If we wish to refer in more detail to what happens inside it, the concept of current density is useful.

Vector: $\vec{J}$ Same direction as $\vec{E}$ such that $i = \int \vec{J} \cdot d\vec{A}$

If surface is perpendicular to a constant electric field, then i=JA, or J=i/A

Units: $[J] = \dfrac{\text{Ampere}}{\text{m}^2}$

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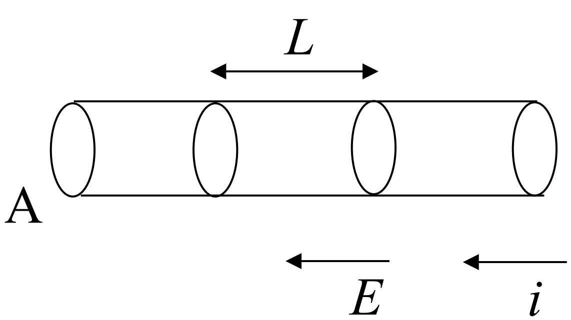
Drift speed: $v_d$

Velocity at which electrons move in order to establish a current.




Charge $q$ in the length $L$ of conductor: $q = (n\,A\,L)e$

$n$ =density of electrons, $e$ =electric charge

$$t = \frac{L}{v_d} \qquad i = \frac{q}{t} = \frac{n\,A\,L\,e}{\frac{L}{v_d}} = n\,A\,e\,v_d$$

$$v_d = \frac{i}{n\,A\,e} = \frac{J}{n\,e}$$

$$\vec{J} = n\,e\,\vec{v}_d$$

## Current density, uniform and nonuniform

(a) The current density in a cylindrical wire of radius $R = 2.0$ mm is uniform across a cross section of the wire and is $J = 2.0 \times 10^5$ A/m$^2$. What is the current through the outer portion of the wire between radial distances $R/2$ and $R$ (Fig. 26-6*a*)?

**KEY IDEA**

Because the current density is uniform across the cross section, the current density $J$, the current $i$, and the cross-sectional area $A$ are related by Eq. 26-5 ($J = i/A$).

***Calculations:*** We want only the current through a reduced cross-sectional area $A'$ of the wire (rather than the entire area), where

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \pi\left(\frac{3R^2}{4}\right)$$

$$= \frac{3\pi}{4}(0.0020 \text{ m})^2 = 9.424 \times 10^{-6} \text{ m}^2.$$

So, we rewrite Eq. 26-5 as

$$i = JA'$$

and then substitute the data to find

$$i = (2.0 \times 10^5 \text{ A/m}^2)(9.424 \times 10^{-6} \text{ m}^2)$$

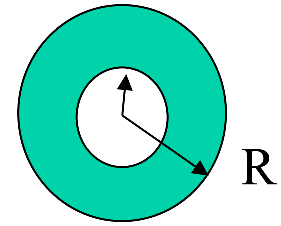
$$= 1.9 \text{ A.} \quad \text{(Answer)}$$

(b) Suppose, instead, that the current density through a cross section varies with radial distance $r$ as $J = ar^2$, in which $a = 3.0 \times 10^{11}$ A/m$^4$ and $r$ is in meters. What now is the current through the same outer portion of the wire?

**KEY IDEA**

Because the current density is not uniform across a cross section of the wire, we must resort to Eq. 26-4 ($i = \int \vec{J} \cdot d\vec{A}$) and integrate the current density over the portion of the wire from $r = R/2$ to $r = R$.

## Part b

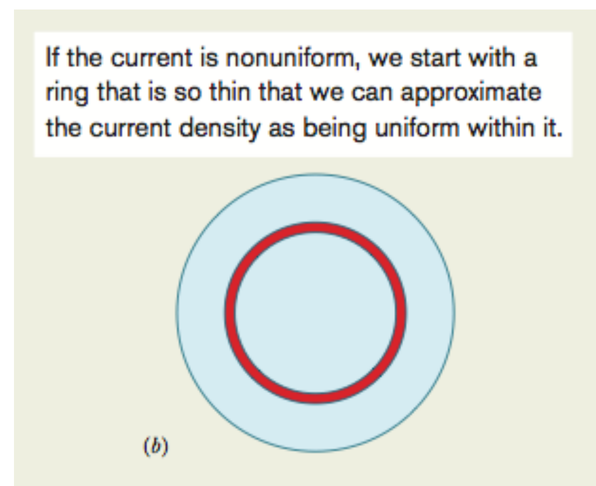



Suppose that the current density through a cross section of a cylindrical wire varies with radial distance r as $J = ar^2$. What is the current in the outer portion of the wire between $R/2$ and R?

$i = \int \vec{J} \cdot d\vec{A}$ Current is perpendicular to section : $di = JdA$

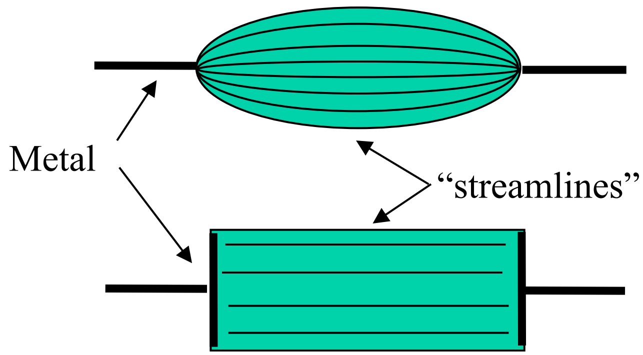
Break the integral into rings of infinitesimal width $dr$.

$$i = \int_{R/2}^{R} ar^2 \underbrace{2\pi r dr}_{dA} = 2\pi a \int_{R/2}^{R} r^3 dr = \frac{15}{32}\pi a R^4 = 7.1A$$



# Resistivity:

"Resistance at a point"




These two devices could have the same resistance R, when measured on the outgoing metal leads. However, it is obvious that inside of them go on different things.

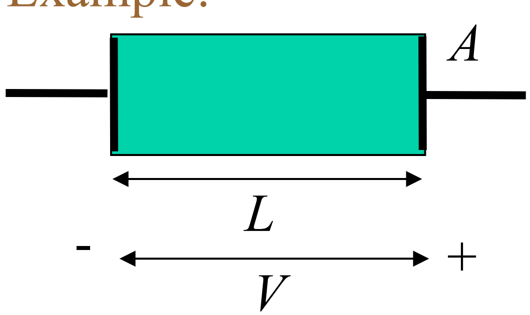
In order to quantify this, we introduce the concept of resistivity:

$$\rho = \frac{E}{J} \text{ or, as vectors, } \vec{E} = \rho \vec{J}$$

$$\text{Conductivity}: \ \sigma = \frac{1}{\rho}$$

Resistivity is associated with a material, resistance with respect to a device constructed with the material.

Example:




$$E = \frac{V}{L}, \quad J = \frac{i}{A} \quad \rho = \frac{V/L}{i/A} = R\frac{A}{L}$$

$$\boxed{R = \rho\frac{L}{A}}$$

Makes sense!

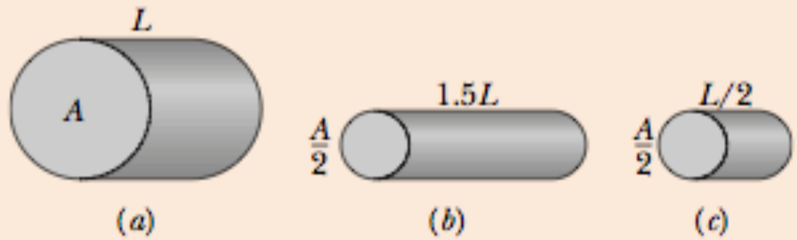
For a given material:

Longer → More resistance

Thicker → Less resistance

## CHECKPOINT 3

The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference $V$ is placed across their lengths.




$$R = \rho \frac{L}{A}$$

Ohm's laws

$R \equiv \dfrac{V}{i}$ and therefore $i = \dfrac{V}{R}$ and $V = iR$

# Summary:

- We saw that charges moving through conductors experience "resistance" to their motion.
- Next class we will study how this "resistance" gets larger with temperature, and see that "moving against it" costs energy.