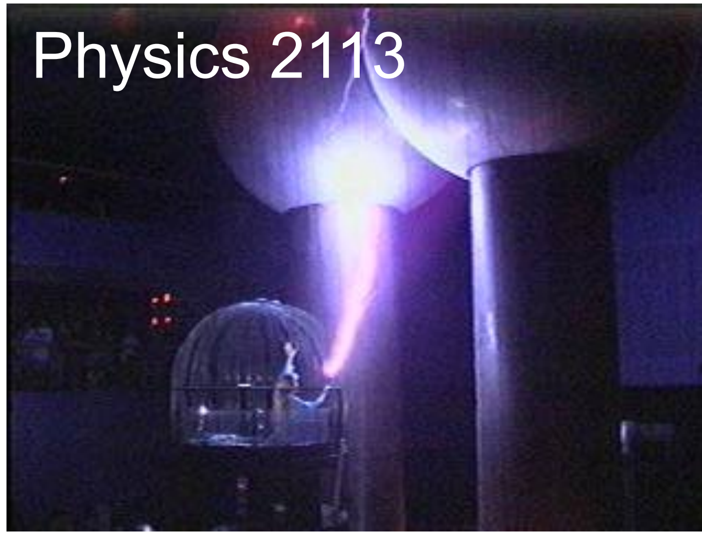
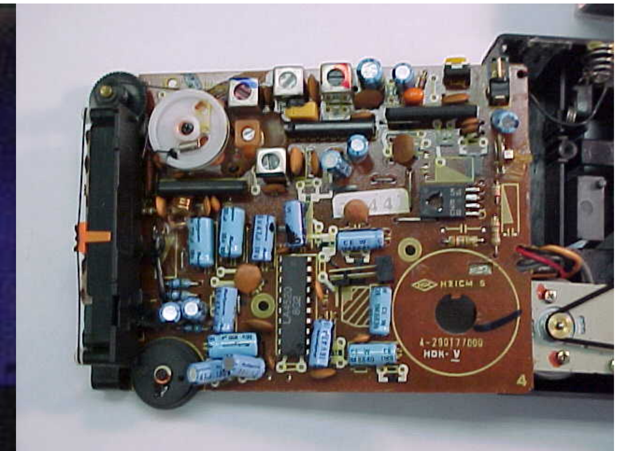
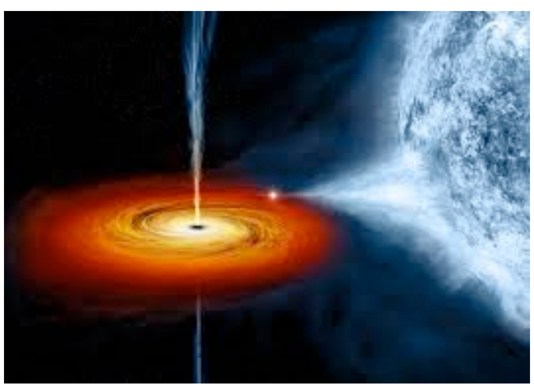




Physics 2113
Lecture 15: WED 1 OCT

CH25: Capacitance



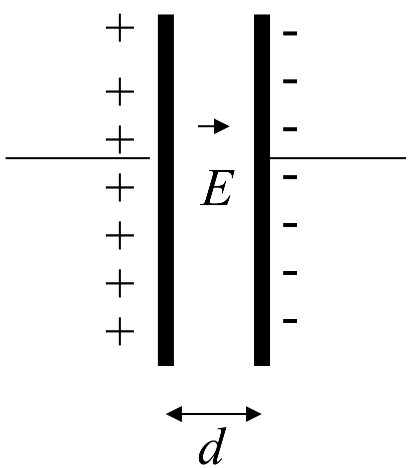
Last class:

The capacitance of a device is completely determined by the characteristics of the device (geometry, materials). It does not depend on applied potentials, charges or fields.

Computing capacitance:

Strategy: compute V as a function of q , read off the proportionality factor!

Parallel plates:



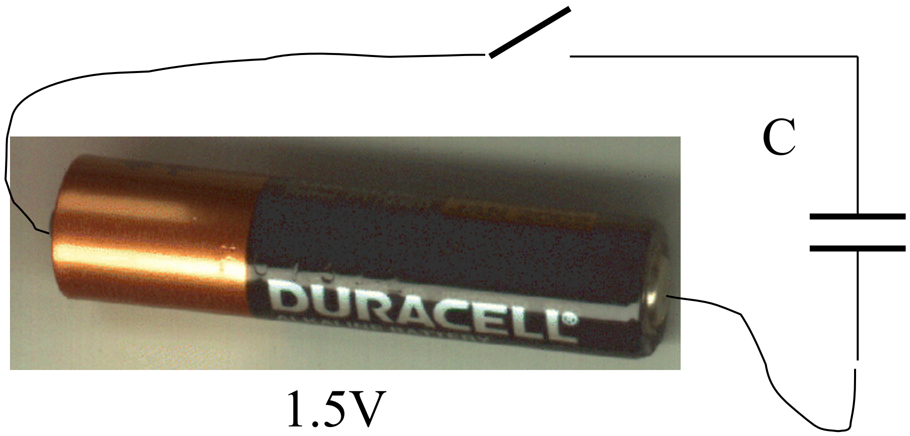
If the separation is small compared to the size of the plates we can approximate the field for that of infinite planes:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

A=area of plates.

$$V = -\int \vec{E} \cdot d\vec{s} = E\,d = \frac{Qd}{\varepsilon_0 A} \qquad V = \frac{Q}{C} \Rightarrow \boxed{C = \frac{\varepsilon_0 A}{d}}$$

Energy stored in a capacitor:



When the switch closes, electrons are moved by the battery from the top plate to the bottom one, until the potential across the capacitor equals 1.5V. Notice that in reality this is not instantaneous.

How much energy does this cost?

Let us say that at a given instant of time the charge on the capacitor is q. Then the potential across the plates is $V' = \dfrac{q}{C}$

To add an infinitesimal of charge *-dq* from the top plate to the bottom, the battery has to do an infinitesimal amount of work $dW = V'dq = \dfrac{qdq}{C}$

So to charge a capacitor with an amount of charge Q, the amount of work needed (which is equal to the energy stored in the capacitor), is,

$$\boxed{W = \int_0^Q \frac{qdq}{C} = \frac{Q^2}{2C}} \quad \text{or in terms of } V\text{, since } Q = CV, \quad \boxed{\mathrm{W} = \frac{1}{2}CV^2}$$

Where is the energy stored?

In the electric field inside the capacitor.

For a parallel-plate capacitor, the field is $E = \dfrac{V}{d}$

$$W = \frac{1}{2}CV^2 \quad = \frac{1}{2}C(Ed)^2 \quad = \frac{1}{2}\varepsilon_0 Ad\, E^2 \quad = \frac{1}{2}\varepsilon_0 \mathrm{vol}\, E^2 \quad (Ad = \text{volume of capacitor})$$

Therefore:

$$\boxed{\frac{1}{2}\varepsilon_0 E^2 \text{ is the energy density of the electric field.}}$$

True in general!

Work and energy when a dielectric is inserted into a capacitor

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference $V = 12.5$ V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

KEY IDEA

We can relate the potential energy U_i of the capacitor to the capacitance C and either the potential V (with Eq. 25-22) or the charge q (with Eq. 25-21):

$$U_i = \tfrac{1}{2}CV^2 = \frac{q^2}{2C}.$$

Calculation: Because we are given the initial potential V (= 12.5 V), we use Eq. 25-22 to find the initial stored energy:

$$U_i = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(13.5 \times 10^{-12}\ \text{F})(12.5\ \text{V})^2$$
$$= 1.055 \times 10^{-9}\ \text{J} = 1055\ \text{pJ} \approx 1100\ \text{pJ}. \qquad \text{(Answer)}$$

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

KEY IDEA

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change.

Calculations: Thus, we must now use Eq. 25-21 to write the final potential energy U_f, but now that the slab is within the capacitor, the capacitance is κC. We then have

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055\ \text{pJ}}{6.50}$$
$$= 162\ \text{pJ} \approx 160\ \text{pJ}. \qquad \text{(Answer)}$$

When the slab is introduced, the potential energy decreases by a factor of κ.

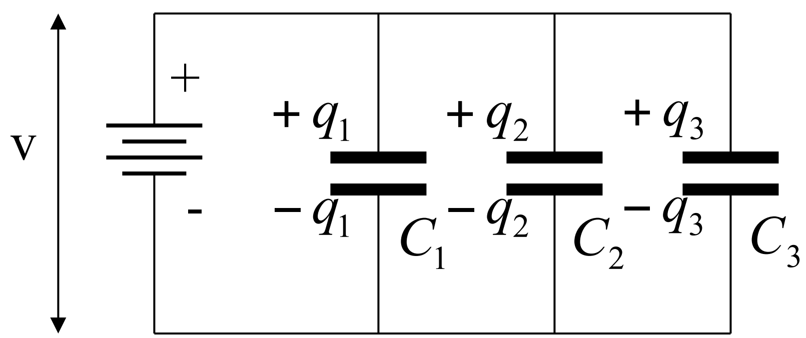
The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162)\ \text{pJ} = 893\ \text{pJ}.$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.

Hooking capacitors together:

In parallel: "side by side"



V is the same for all capacitors, therefore:

$$q_1 = C_1 V$$
$$q_2 = C_2 V$$
$$q_3 = C_3 V$$

$$q_{\text{total}} = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

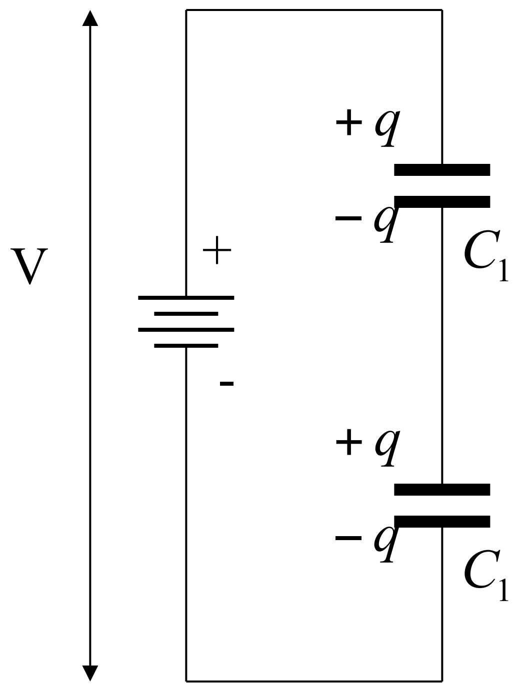
So, $q_{\text{total}} = C_{\text{total}} V$, with $C_{\text{total}} = C_1 + C_2 + C_3$

That is, the circuit behaves like a single capacitor of capacitance equal to the sum of the individual capacitors' ones.

Obviously true for any number of capacitors: $C_{\text{total}} = \sum_{i=1}^{n} C_i$

Intuitively: "More area, same separation", or "More field packed for a given V".

In series: "one after the other"



When the battery is connected, negative charges leave the top plate of C1 and move to the bottom plate of C2. The intermediate plates are disconnected from the circuit, but still get charged due to polarization.

As a consequence, the charges are all the same.

$$V = V_1 + V_2 = \frac{q}{C_1} + \frac{q}{C_2} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right) q$$

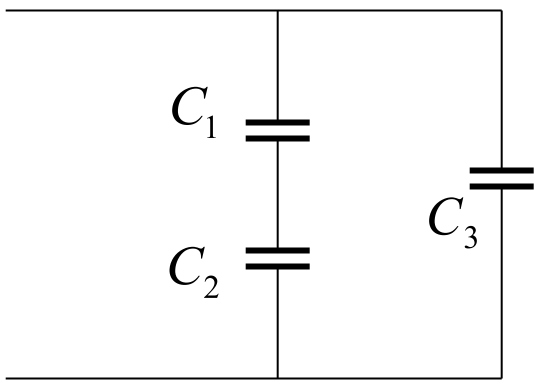
$$q = \frac{1}{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)} V \qquad q = C_{\text{total}} V \qquad \boxed{C_{\text{total}} = \frac{1}{\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}}$$

Total capacity: inverse of the sum of the inverses.

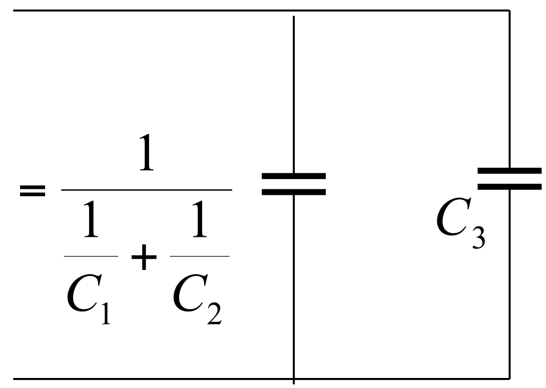
$$C_{\text{total}} = \frac{1}{\sum_{i=1}^{n} \frac{1}{C_i}}$$

Example:

Find the equivalent capacitance: $C_1 = 10\mu F,\quad C_2 = 5\mu F,\quad C_3 = 4\mu F$

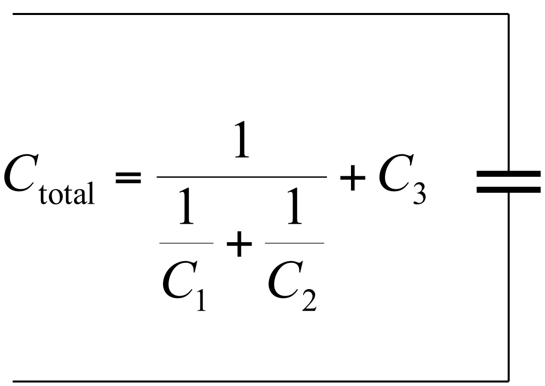


1,2 in series



$$C_{12} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

12 and 3
in parallel.



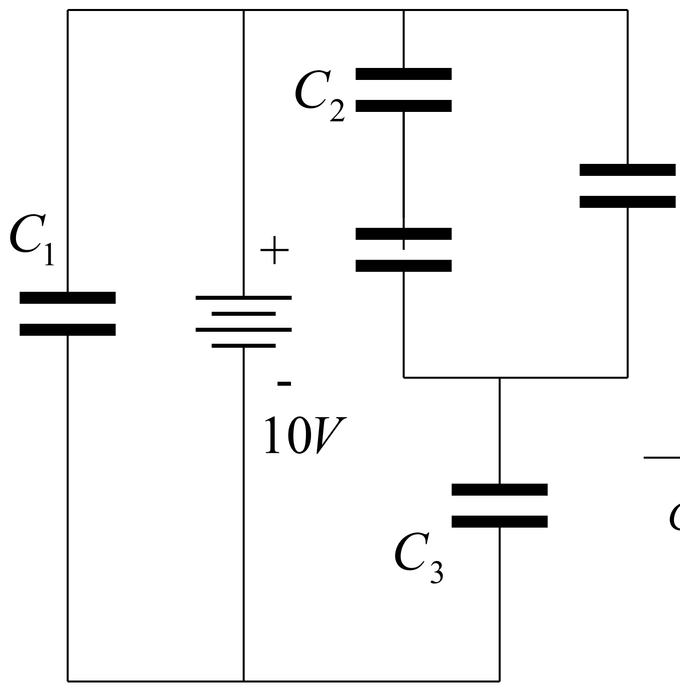
$$C_{\text{total}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} + C_3$$

$$C_{\text{total}} = 7.33\mu F$$

Example: (hard!)

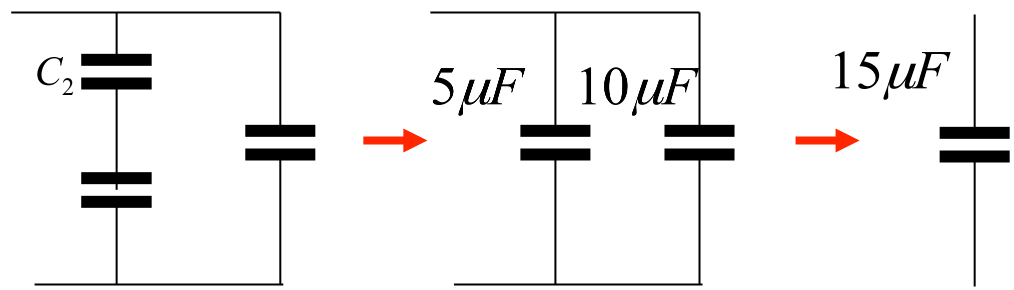
What is the charge on C_1 and C_2 ?

All $C = 10\mu F$

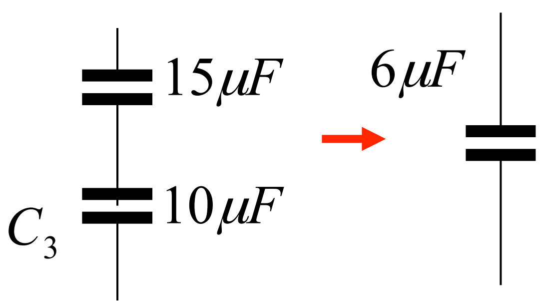


On C_1 : $q = C_1 10V \Rightarrow q = 100\mu C$

Getting the charge on C2 is more involved. We start by figuring out the capacitance of the top set.



We now combine the top set with the bottom capacitor:



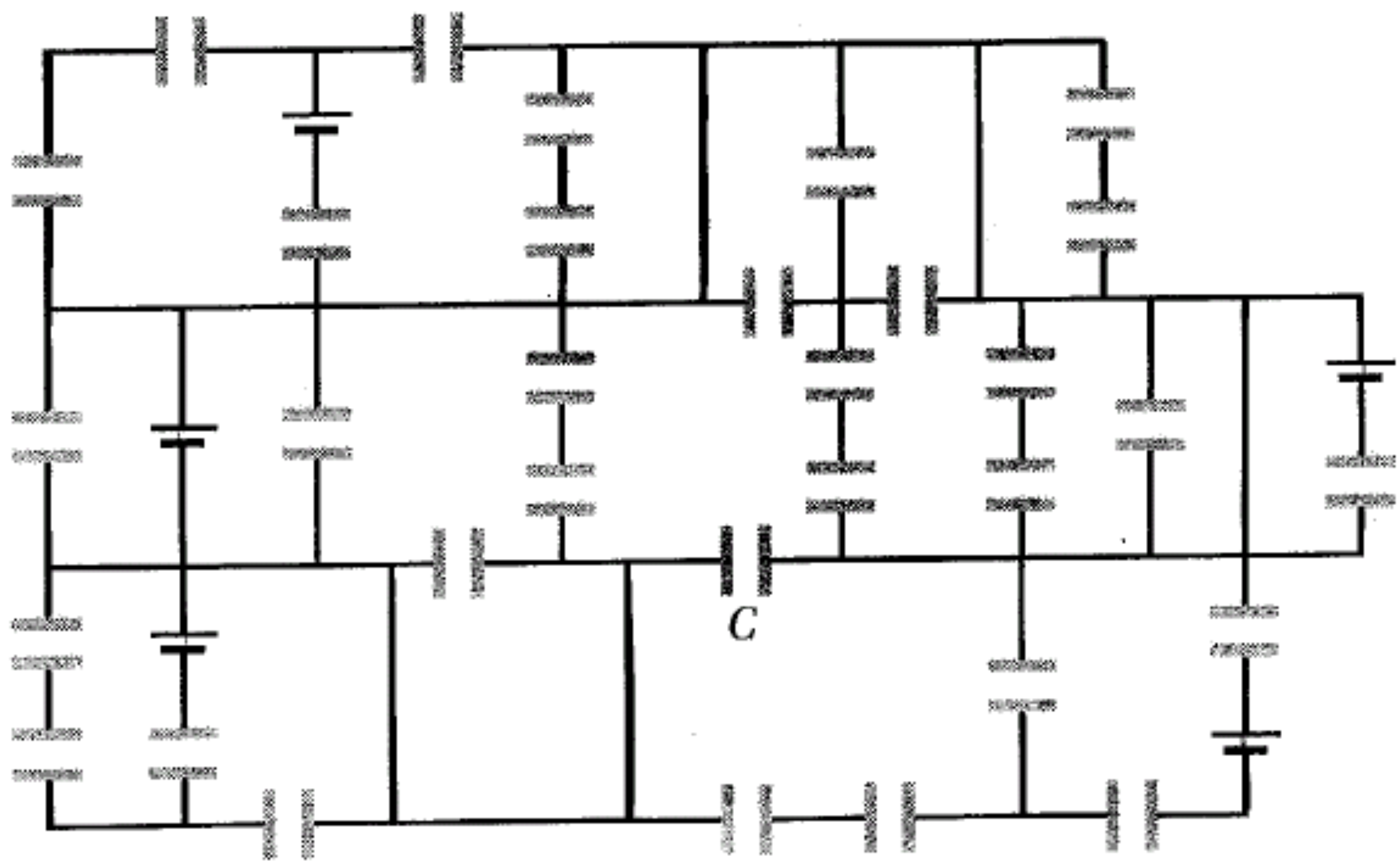
So the charge on the bottom plate of the bottom capacitor (C3) will be $q_3 = 10V\,6\mu F = 60\mu C$

Which means that $V_3 = \dfrac{q_3}{10\mu F} = 6V$

Which means that $V_2 = \dfrac{1}{2}(10V - 6V) = 2V \Rightarrow \boxed{q_2 = 20\mu C}$

Phew!

13. *Cap-monster maze.* In Fig. 28-22, all the capacitors have a capacitance of 6.0 μF, and all the batteries have an emf of 10 V. What is the charge on capacitor C? (If you can find the proper loop through this maze, you can answer the question with a few seconds of mental calculation.)



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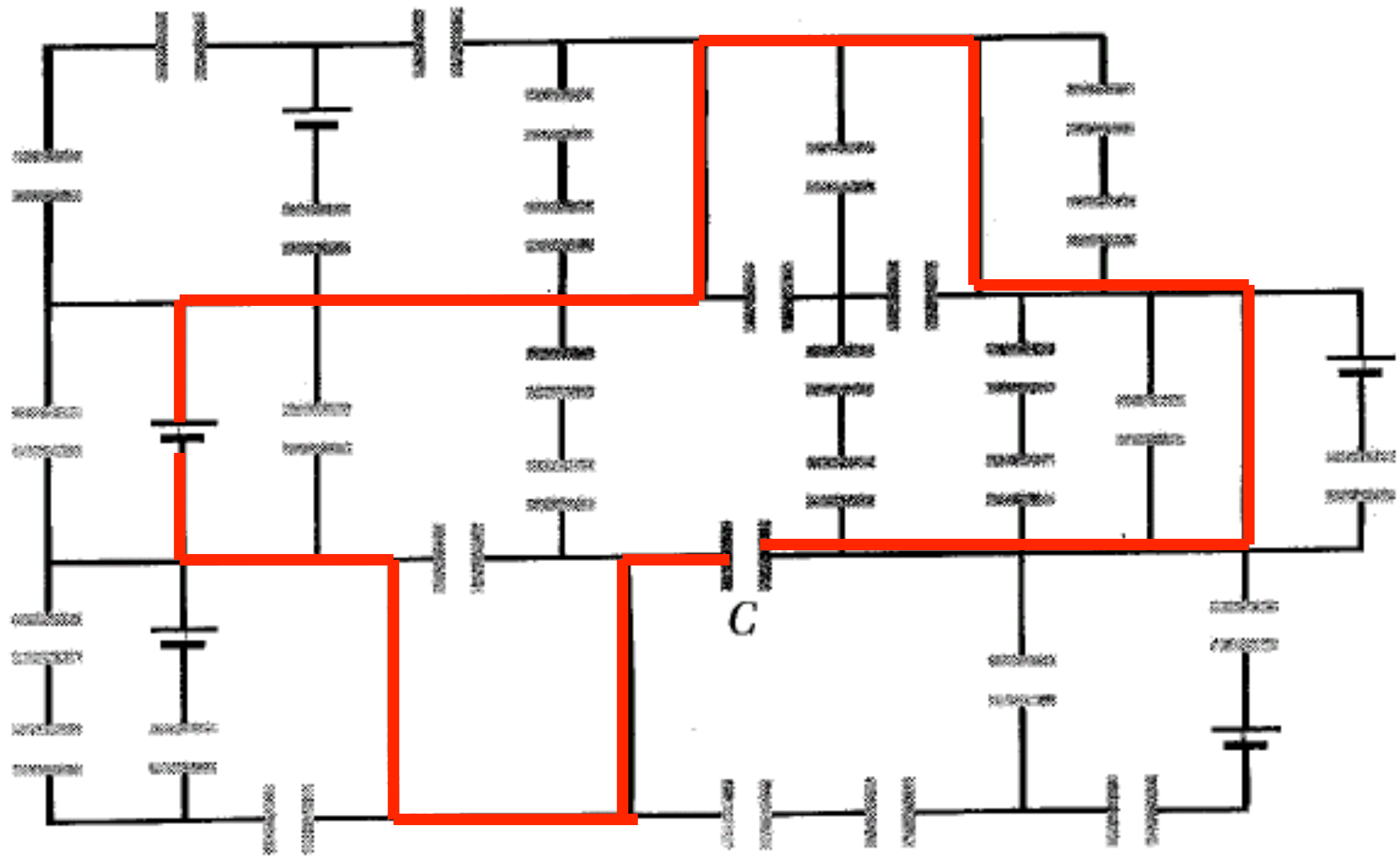
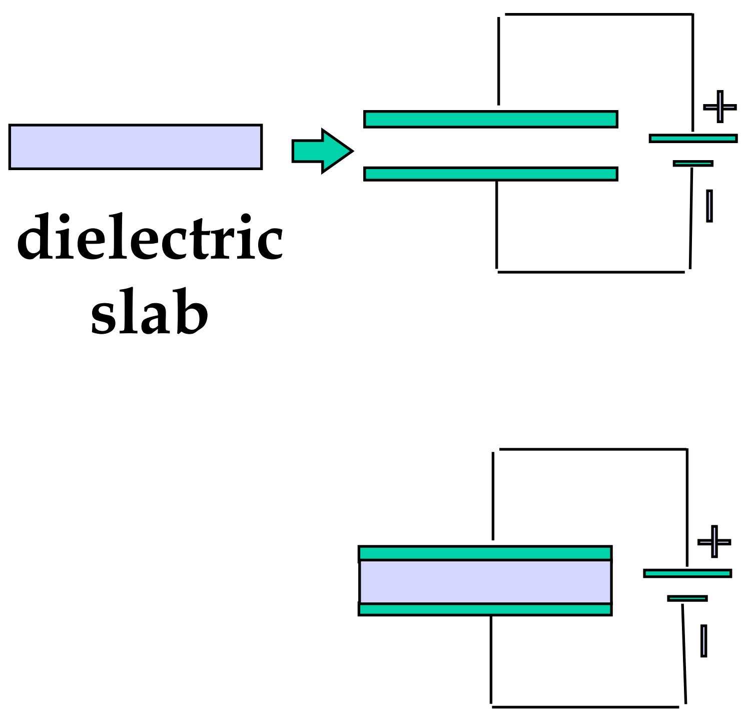


Example

- Capacitor has charge Q, voltage V
- Battery remains connected while dielectric slab is inserted.
- Do the following increase, decrease or stay the same:
  - Potential difference?
  - Capacitance?
  - Charge?
  - Electric field?



Dielectric partially filling the gap in a capacitor

Figure 25-17 shows a parallel-plate capacitor of plate area A and plate separation d. A potential difference V_0 is applied between the plates by connecting a battery between them. The battery is then disconnected, and a dielectric slab of thickness b and dielectric constant κ is placed between the plates as shown. Assume $A = 115\ \text{cm}^2$, $d = 1.24$ cm, $V_0 = 85.5$ V, $b = 0.780$ cm, and $\kappa = 2.61$.

(a) What is the capacitance C_0 before the dielectric slab is inserted?

Calculation: From Eq. 25-9 we have

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{F/m})(115 \times 10^{-4}\ \text{m}^2)}{1.24 \times 10^{-2}\ \text{m}}$$
$$= 8.21 \times 10^{-12}\ \text{F} = 8.21\ \text{pF}. \qquad \text{(Answer)}$$

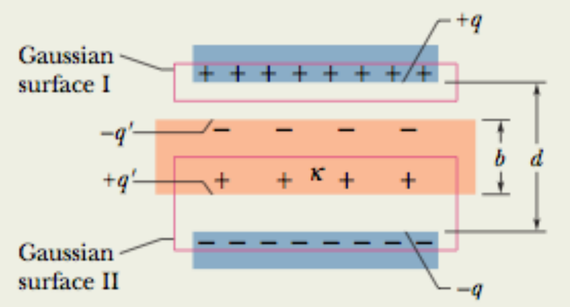
(b) What free charge appears on the plates?

Calculation: From Eq. 25-1,

$$q = C_0 V_0 = (8.21 \times 10^{-12}\ \text{F})(85.5\ \text{V})$$
$$= 7.02 \times 10^{-10}\ \text{C} = 702\ \text{pC}. \qquad \text{(Answer)}$$

Because the battery was disconnected before the slab was inserted, the free charge is unchanged.

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?



(d) What is the electric field E_1 in the dielectric slab?

(c) What is the electric field E_0 in the gaps between the plates and the dielectric slab?

Gauss' law with Gaussian Surface I and κ=1 (no dielectric):

$$\varepsilon_0 \kappa E_0 A = q,$$
$$E_0 = \frac{q}{\varepsilon_0 \kappa A}.$$

Gauss' law with Gaussian Surface II:

$$\varepsilon_0 \oint \kappa \vec{E}_1 \cdot d\vec{A} = -\varepsilon_0 \kappa E_1 A = -q.$$

$$E_1 = \frac{q}{\varepsilon_0 \kappa A} = \frac{E_0}{\kappa}$$

Main message: if you use k, then only consider real charges, not induced ones.

One capacitor charging up another capacitor

Capacitor 1, with $C_1 = 3.55\ \mu F$, is charged to a potential difference $V_0 = 6.30$ V, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95\ \mu F$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

KEY IDEAS

The situation here differs from the previous example because here an applied electric potential is *not* maintained across a combination of capacitors by a battery or some other source. Here, just after switch S is closed, the only applied electric potential is that of capacitor 1 on capacitor 2, and that potential is decreasing. Thus, the capacitors in Fig. 25-11 are not connected *in series;* and although they are drawn parallel, in this situation they are not *in parallel.*

As the electric potential across capacitor 1 decreases, that across capacitor 2 increases. Equilibrium is reached when the two potentials are equal because, with no potential difference between connected plates of the capacitors, there is no electric field within the connecting wires to move conduction electrons. The initial charge on capacitor 1 is then shared between the two capacitors.

Calculations: Initially, when capacitor 1 is connected to the battery, the charge it acquires is, from Eq. 25-1,

$$q_0 = C_1V_0 = (3.55 \times 10^{-6}\ \text{F})(6.30\ \text{V}) = 22.365 \times 10^{-6}\ \text{C}.$$

When switch S in Fig. 25-11 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until

$$V_1 = V_2 \qquad \text{(equilibrium).}$$

From Eq. 25-1, we can rewrite this as

$$\frac{q_1}{C_1} = \frac{q_2}{C_2} \qquad \text{(equilibrium).}$$

Because the total charge cannot magically change, the total after the transfer must be

$$q_1 + q_2 = q_0 \qquad \text{(charge conservation);}$$

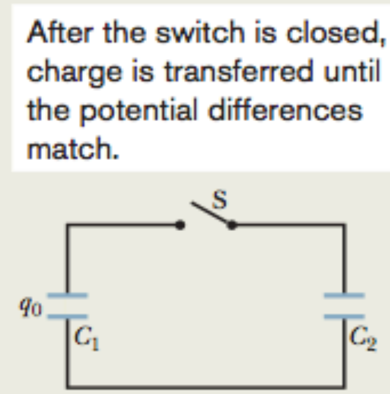


Fig. 25-11 A potential difference V_0 is applied to capacitor 1 and the charging battery is removed. Switch S is then closed so that the charge on capacitor 1 is shared with capacitor 2.

thus $$q_2 = q_0 - q_1.$$

We can now rewrite the second equilibrium equation as

$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}.$$

Solving this for q_1 and substituting given data, we find

$$q_1 = 6.35\ \mu\text{C}. \qquad \text{(Answer)}$$

The rest of the initial charge ($q_0 = 22.365\ \mu$C) must be on capacitor 2:

$$q_2 = 16.0\ \mu\text{C}. \qquad \text{(Answer)}$$

Summary:

- Capacitors store energy by means of the electric field they create.
- Hooking up capacitors “side by side” (parallel) adds their capacity up.
- Hooking them in series diminishes the capacity “adds more gaps”.