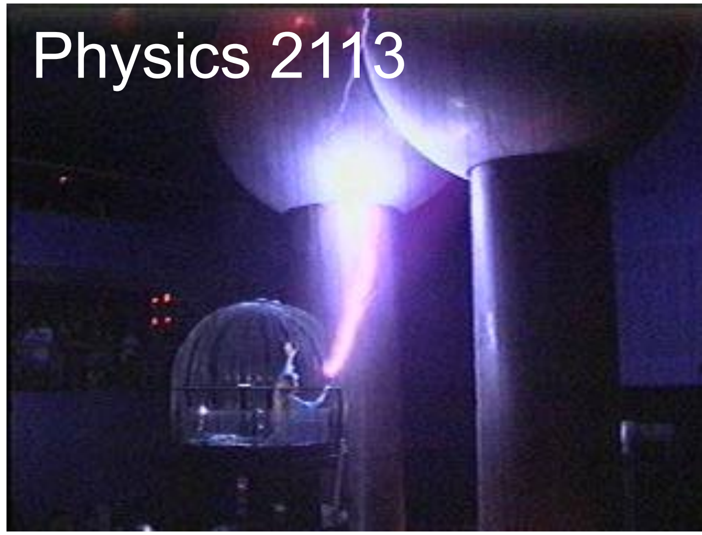
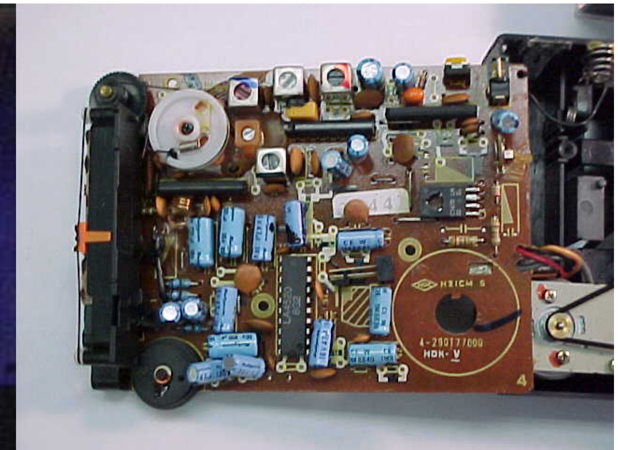
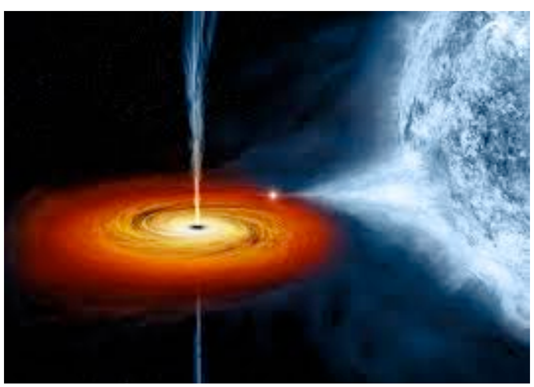




Physics 2113
Lecture 14: MON 29 SEP

CH25: Capacitance



CHAPTER V

VON KLEIST AND THE LEYDEN JAR

"I would not take a second shock for the Kingdom of France."—MUSCHENBROECK, *in a letter to Reaumer.*

TOWARD the close of 1745, Von Kleist, Bishop of Pomerania, desiring to isolate electricity, conceived the idea of leading a charge from an electric machine into a glass bottle, arguing, in all probability, that he might in this manner be able to

Von Kleist's invention of the Leyden jar.

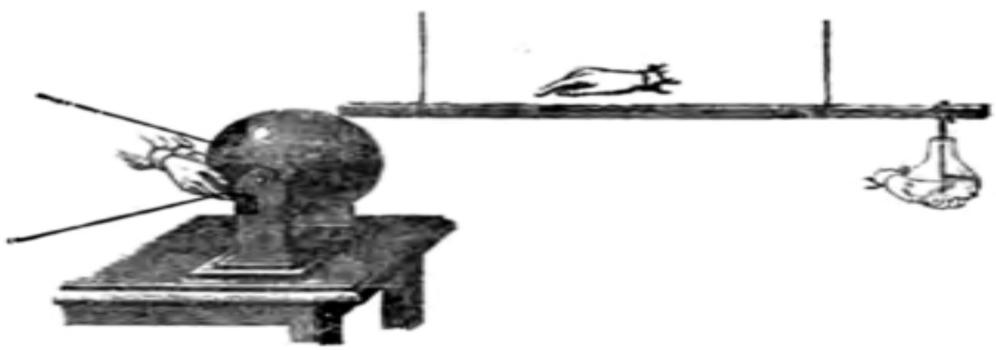


FIG. 26.—Von Kleist's discovery of the Leyden jar. Note the objectionable sharp angles and edges of the prime conductor. This was before the discharging power of points or sharp edges was known.

fill the bottle with electricity, since he imagined the electricity would not be able to escape on account of the non-conducting property of the glass. With this end in view, he partially filled a small glass bottle with water, and, holding it in one hand, con-

Von Kleist was able to store electricity in the jar. Unknowingly, he had actually invented a novel device to store potential difference.

The water in the jar formed the inner coating and his hands formed the outer coating of this device, with glass as the medium isolating two conductors (his body and water).

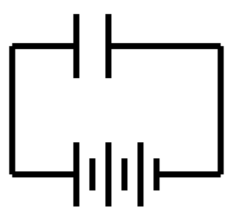
Von Kleist switched on his generator, charge flowed to the glass jar, and he decided to remove the glass jar from the machine.

Surprisingly, he received an electric shock!

Why?

Potential directly leads to a work. If a device can store the potential difference, we can use it to do necessary work.

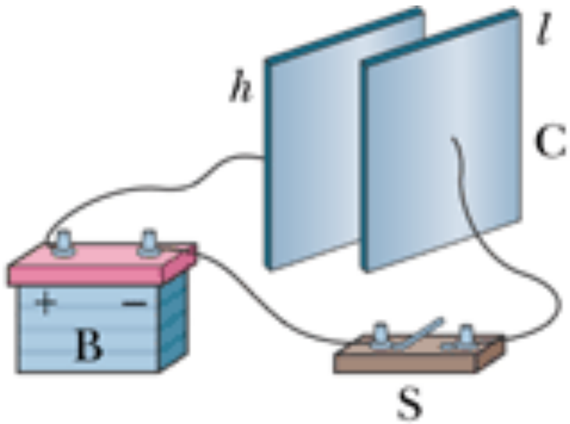
Capacitors (or condensers as Volta called them) are examples of such devices. Potential difference generated at isolated charged surfaces, leads to generation of a static electric field between the surfaces, which stores the energy.



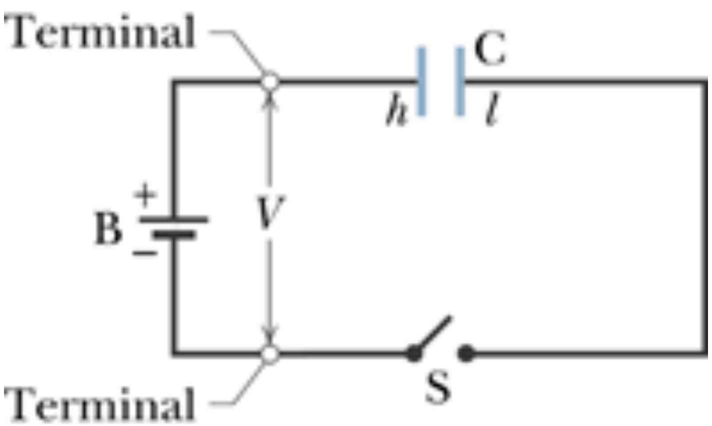
A battery is a device that maintains a constant potential difference across its ends (through chemical means).

A circuit is a path along which charge flows.

A switch is a device which if open does not allow flow of charge. Then the circuit is said to be open. If a switch is closed, circuit is closed.



(a)

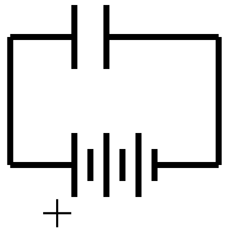


(b)

When the switch is open, the circuit is incomplete and there is no flow of charge.

When the switch is closed, the circuit is complete and electrons from plate h are attracted to the positive terminal of battery, making plate h positively charged. Similarly, plate l becomes negatively charged.

After some time, when the plates are fully charged, the potential difference between the plates is same as V. There is no more flow of charge.



If the parallel plates were uncharged and later the battery was connected to them, it will take electrons from the + plate towards the - plate.

The parallel plates are the simplest example of capacitor. They pack an electric field among them. Appropriately constructed, the field can be very intense.

The charge on each plate is proportional to the potential among the plates:

$$q = CV$$

Whenever these quantities are proportional, the constant C is defined as the capacitance of the device in question.

Units: $[C] = \frac{\text{Coulomb}}{\text{Volt}} \equiv \text{Farad}$

This unit is typically too large. Most common are:

$$\text{MicroFarad} = \mu\text{F} = 10^{-6}\,F$$

$$\text{PicoFarad} = \text{pF} = 10^{-12}\,F \quad \text{(MicroMicroF)}$$

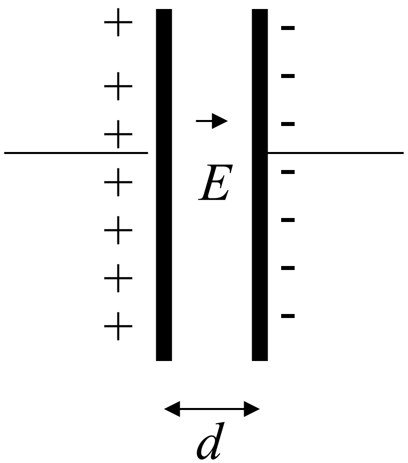
$$\text{NanoFarad} = \text{nF} = 10^{-9}\,F$$

The capacitance of a device is completely determined by the characteristics of the device (geometry, materials). It does not depend on applied potentials, charges or fields.

Computing capacitance:

Strategy: compute V as a function of q , read off the proportionality factor!

Parallel plates:



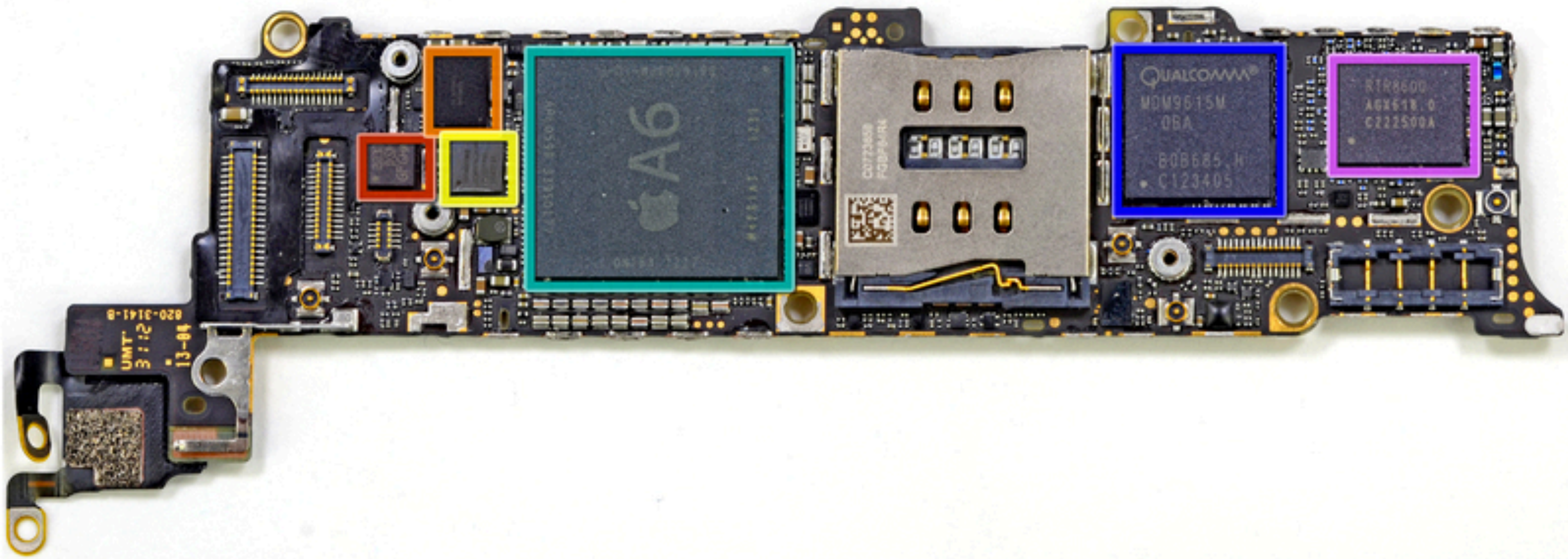
If the separation is small compared to the size of the plates we can approximate the field for that of infinite planes:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

A=area of plates.

$$V = -\int \vec{E} \cdot d\vec{s} = E\,d = \frac{Qd}{\varepsilon_0 A} \qquad V = \frac{Q}{C} \Rightarrow \boxed{C = \frac{\varepsilon_0 A}{d}}$$

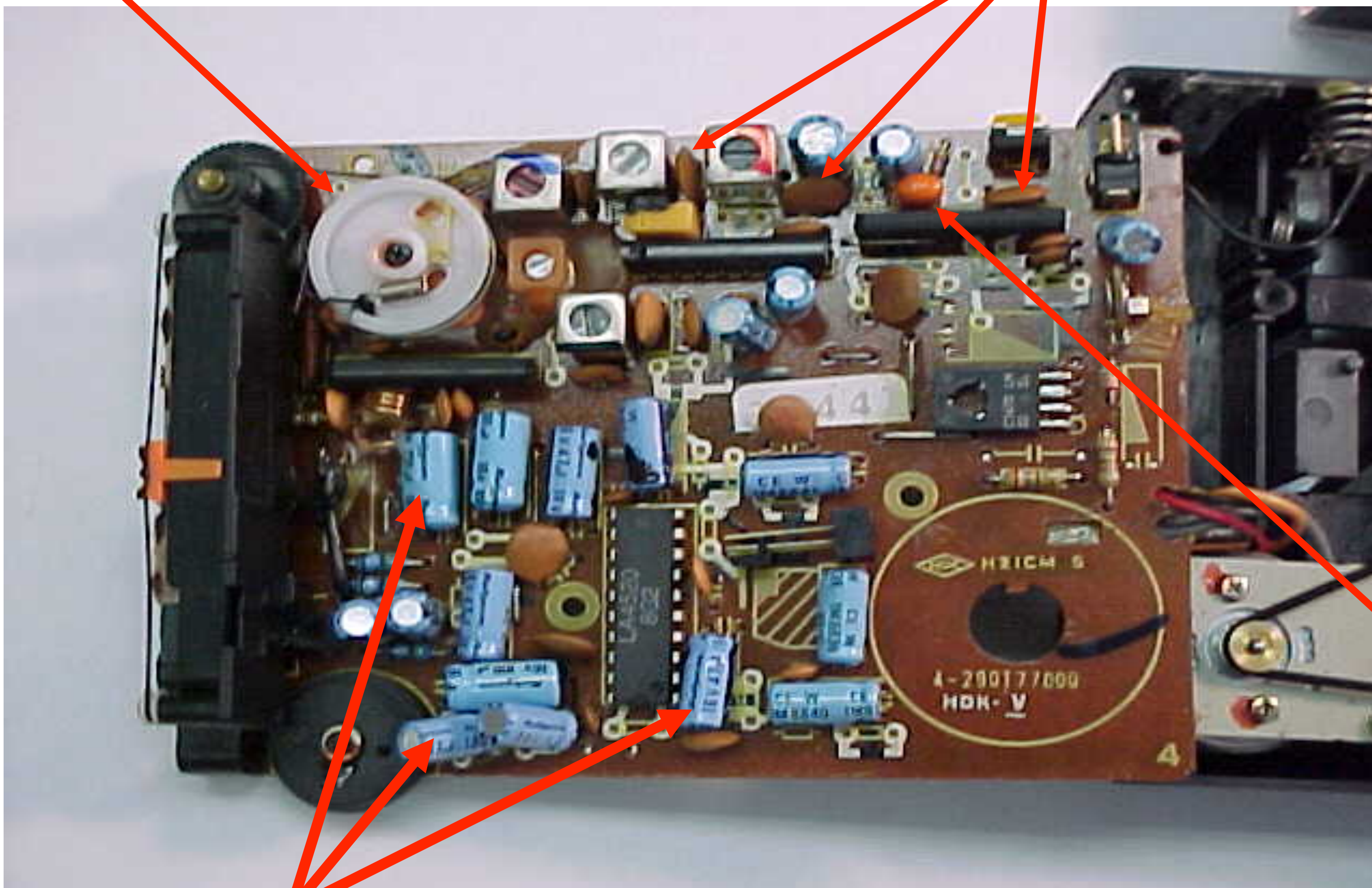
Motherboard of Iphone S



Interior of a 1989 Sanyo Walkman

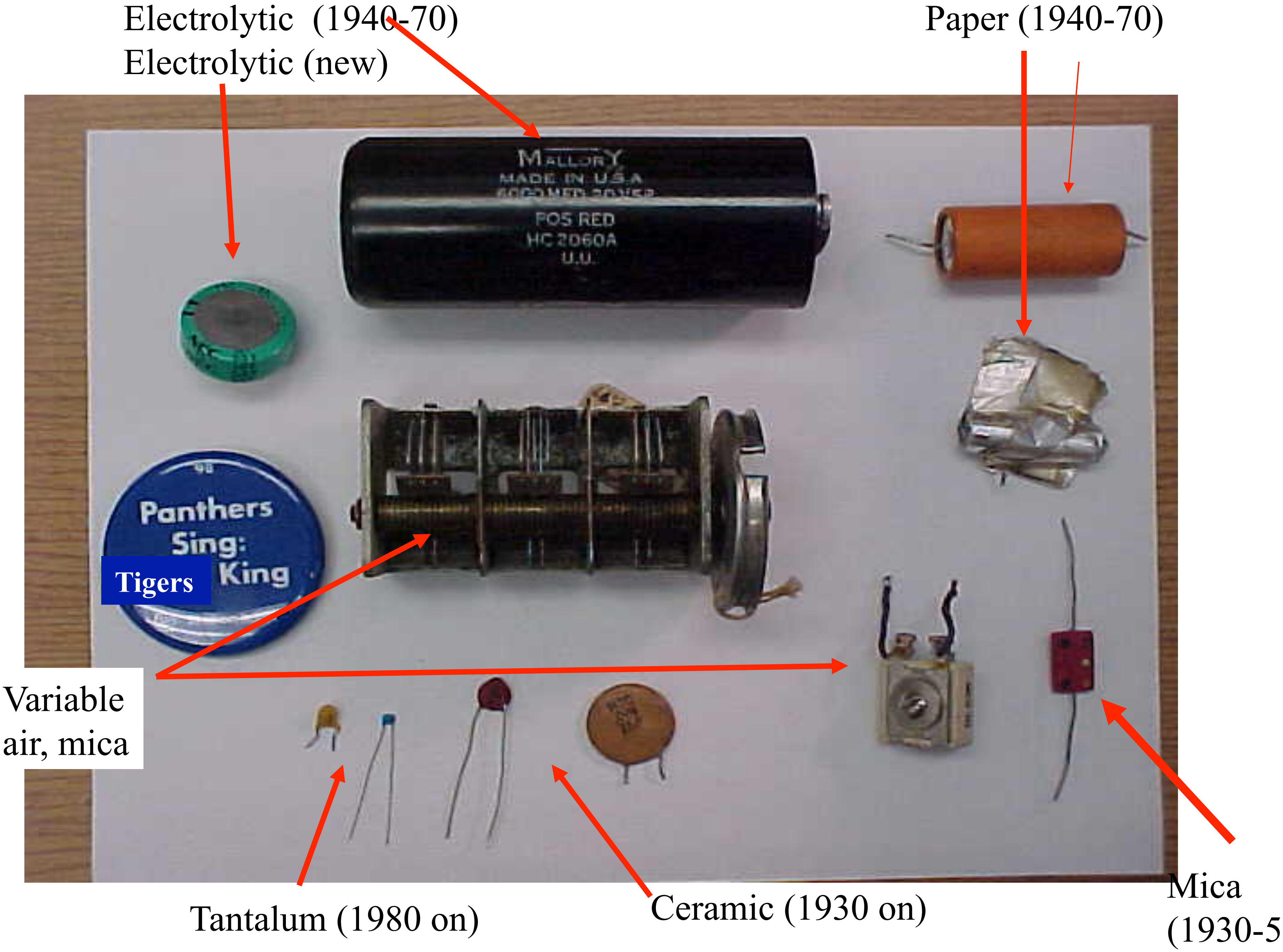
Ceramic capacitors

Variable capacitor (tuner)

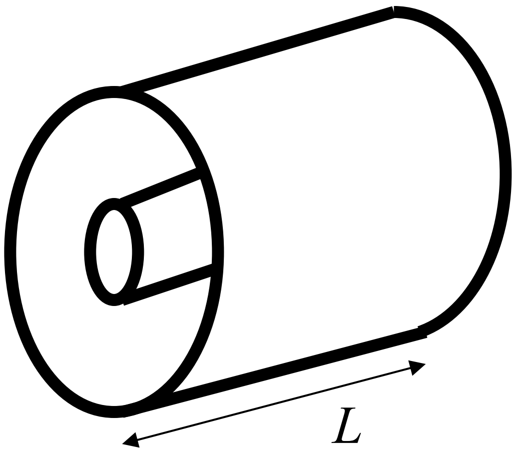


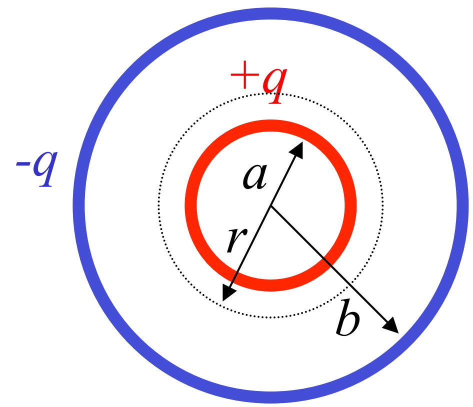
Tantalum Capacitor

Electrolytic capacitors


Electrolytic (1940-70)
Electrolytic (new)
Paper (1940-70)
MALLORY
MADE IN U.S.A
POS RED
HC 2060A
U.U.
Panthers
Sing:
Tigers
King
Variable
air, mica
Tantalum (1980 on)
Ceramic (1930 on)
Mica
(1930-5

Cylindrical plates:





Again, we approximate the field for that of an infinite cylinder. To compute this, we surround the interior cylinder with a cylindrical Gaussian surface.

$$\int \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} \Rightarrow 2\pi rLE = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi rL\varepsilon_0}$$

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l} = -\int_a^b E dr = -\frac{Q}{2\pi L\varepsilon_0}\int_a^b \frac{dr}{r} = -\frac{Q}{2\pi L\varepsilon_0}\left[\ln(b) - \ln(a)\right] =$$

$$-\frac{Q}{2\pi L\varepsilon_0}\left[\ln\left(\frac{b}{a}\right)\right]$$

$$V = \frac{Q}{2\pi L\varepsilon_0}\left[\ln\left(\frac{b}{a}\right)\right]$$

so

$$C = \frac{2\pi L\varepsilon_0}{\ln\left(\frac{b}{a}\right)}$$

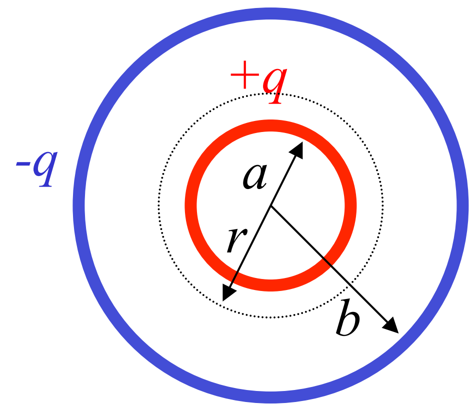
Cylindrical capacitor

If separation is small, and radius large, $b - a = d$

$$\ln\left(\frac{b}{a}\right) = \ln\left(\frac{b-a+a}{a}\right) = \ln\left(\frac{d+a}{a}\right) = \ln\left(1+\frac{d}{a}\right) \approx \frac{d}{a}$$ for large a.

$$C \to \frac{2\pi L\varepsilon_0}{\left(\frac{d}{a}\right)} = \frac{A\varepsilon_0}{d}$$ like a plane capacitor.

Spherical capacitor:



Similar to cylinder,

$$\int \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0} \quad \Rightarrow 4\pi r^2 E = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi r^2 \varepsilon_0}$$

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l} = -\int_a^b E dr = -\frac{Q}{4\pi\varepsilon_0}\int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_a^b = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{b}-\frac{1}{a}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{a-b}{ab}\right]$$

Now $V = V_a - V_b$, therefore

$$C = 4\pi\varepsilon_0\left[\frac{ab}{b-a}\right]$$

Limit cases:

If $a \approx b \rightarrow R$, noting that $A = 4\pi R^2$,

we have, $C = \dfrac{4\pi R^2 \varepsilon_0}{d} = \dfrac{A\varepsilon_0}{d}$ Planar

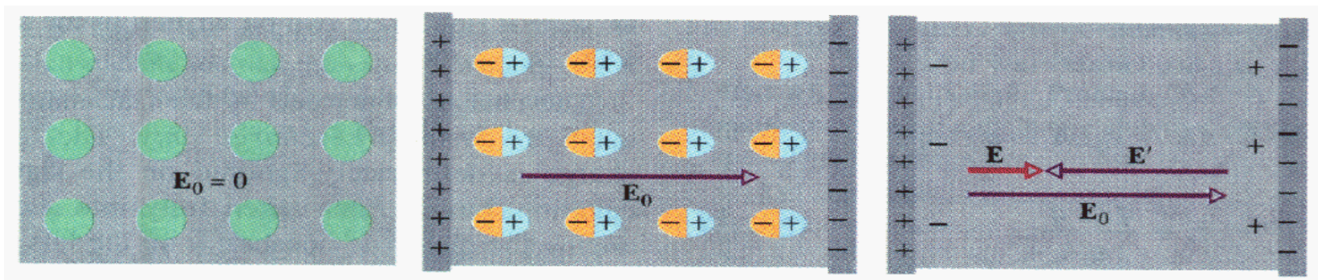
If $b \rightarrow \infty, \quad C \rightarrow 4\pi\varepsilon_0 a$

Capacity of a conducting sphere

Dielectrics:

We know that in insulators, electrons cannot move freely as in conductors.

However, they do exhibit a limited range of motion. Molecules are not spherically symmetric and they "tilt" in response to an applied electric field.



As all polarization effects, it has the consequence of diminishing the field inside the insulator (extreme case: conductor, no field).

That is, if one inserts an insulator between the plates of a capacitor, due to polarization effects the field (and the potential) will be smaller for a given charge, than if one had vacuum among the plates.

$q = CV$ Implies that q is fixed, V is smaller, therefore C is bigger!

Inserting a dielectric is a practical way of increasing the capacitance of a capacitor. It also allows to tolerate a higher V among plates without sparks flying between them (some dielectrics "break down" less than air).

Different types of Dielectrics:

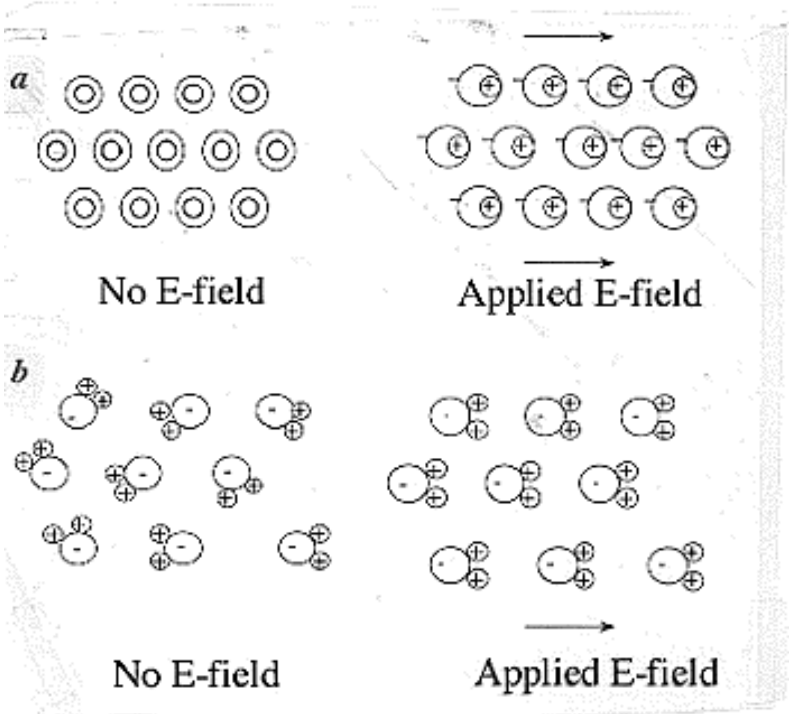


Figure 1. Electric polarization effects in simple models of *a*, non-polar material, and *b*, polar substance, e.g. H_2O.

Polar dielectrics: The molecules of these materials have permanent electric dipole moment. These electric dipoles align along the electric field when it is externally applied on the material.

Non-Polar dielectrics: Molecules acquire dipole moment when places in the electric field. It is caused because of stretching of the molecules.

Given two identical capacitors, one with dielectric and the other in vacuum, we write, $C_{\text{diel}} = \kappa C_{\text{vacuum}}$ for a parallel plate capacitor : $C_{\text{diel}} = \dfrac{\kappa\varepsilon_0 A}{d}$

So we can absorb the constant κ in the definition of $\varepsilon = \kappa\varepsilon_0$

All equations we derived up to now are correct in the presence of dielectrics if one replaces ε in place of ε_0!

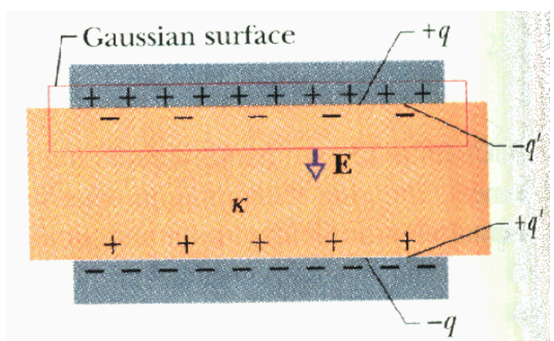
Some values: $\kappa_{\text{vacuum}} = 1$ $\kappa_{\text{paper}} = 35$

$\kappa_{\text{air}} = 1.00054$ $\kappa_{\text{water}} = 80$

Electric field of a point charge in a dielectric $E = \dfrac{q}{4\pi\varepsilon r^2}$

Gauss' law:

$$\int \vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon}$$



$$\frac{q}{\varepsilon} = \frac{q-q'}{\varepsilon_0}$$

If one uses ε, there is no need to take into account explicitly induced charges.

Summary:

- We’ve seen what a capacitor is: a device that packs electric field in a confined region.
- Next class we will learn what happens when you connect more than one such device, and how they actually store energy.