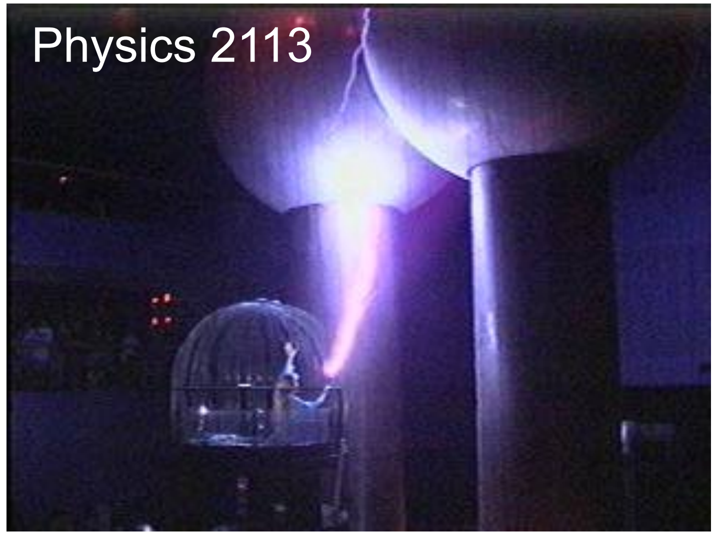
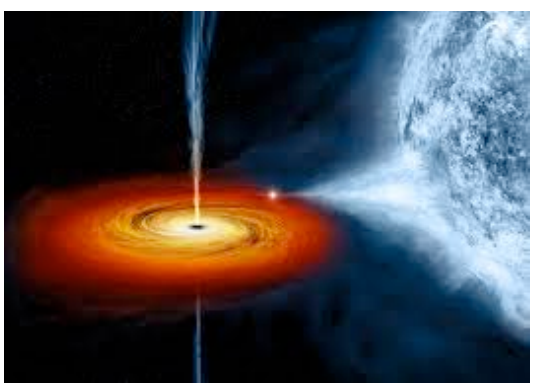


Alessandro Volta
(1745-1827)

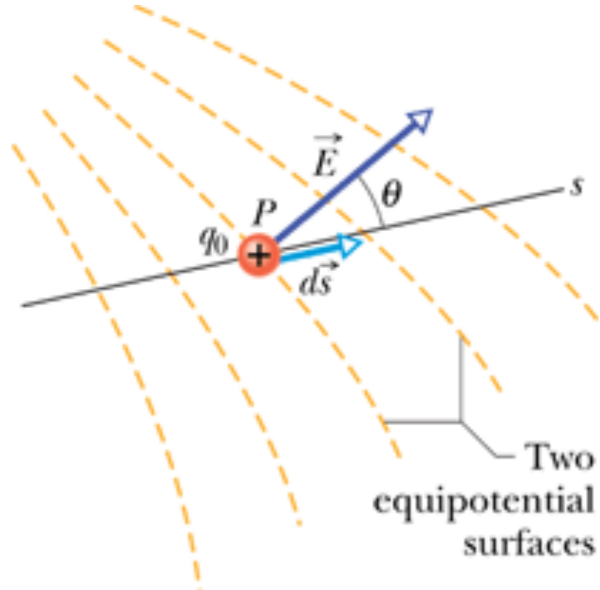
Physics 2113
Lecture 13: FRI 26 SEP

CH24: Electric Potential



Michael Faraday
(1791–1867)

Given V find E



Consider a test charge moving from one equipotential surface to another. Then the work done by the electric field on the charge is: $W = (q_0)(-dV)$

But work done is also: $W = \int \vec{F} \cdot d\vec{s}$

Hence,

$$W = q_0 \vec{E} \cdot d\vec{s} = q_0 E \cos\theta ds = q_0 E_s ds$$

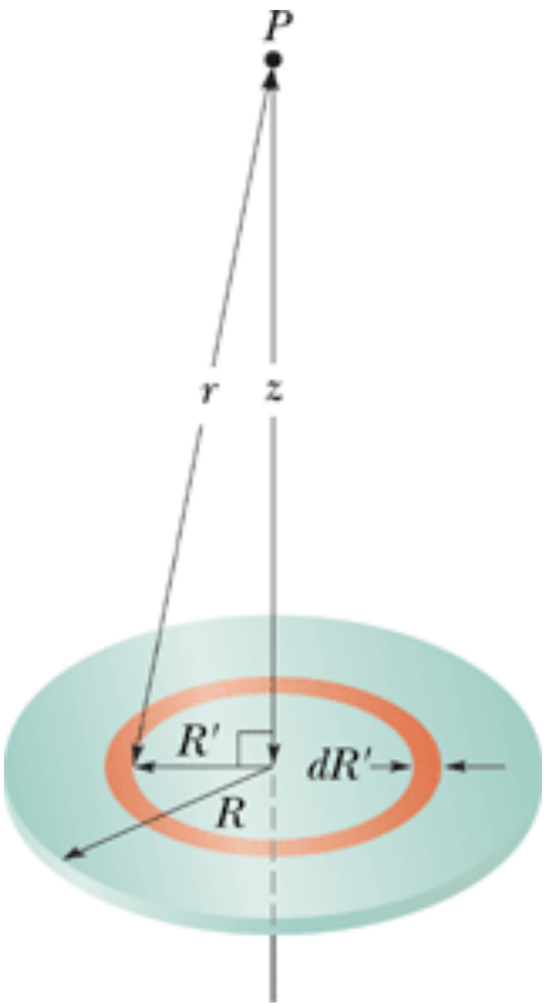
$$E_s \equiv E \cos\theta, \text{"projection of E along ds"}$$

$$\Rightarrow E_s = -\frac{dV}{ds}$$

If V depends on more than one directions:

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

Electric field due to a charged disk



We start with the computation of the potential, which we worked out last class.

We consider a differential element of radius R' and width dR', enclosing a surface area $2\pi R' dR'$

Enclosed charge: $dq = \sigma(2\pi R' dR')$

This enclosed charge leads to the potential:

$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

We can then integrate this potential from 0 to R to get the net potential due to the disk:

$$V = \int_0^R \frac{dq}{4\pi\varepsilon_0 r} = \frac{\sigma}{2\varepsilon_0}\left(\left(z^2 + R^2\right)^{1/2} - z\right)$$

Finding the field from the potential

The electric potential at any point on the central axis of a uniformly charged disk is given by Eq. 24-37,

$$V = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2 + R^2} - z).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

KEY IDEAS

We want the electric field $\vec{E}$ as a function of distance z along the axis of the disk. For any value of z, the direction of $\vec{E}$ must be along that axis because the disk has circular symmetry about that axis. Thus, we want the component E_z of $\vec{E}$ in the direction of z. This component is the negative of the rate at which the electric potential changes with distance z.

Calculation: Thus, from the last of Eqs. 24-41, we can write

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}(\sqrt{z^2 + R^2} - z)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right). \qquad \text{(Answer)}$$

This is the same expression that we derived in Section 22-7 by integration, using Coulomb's law.

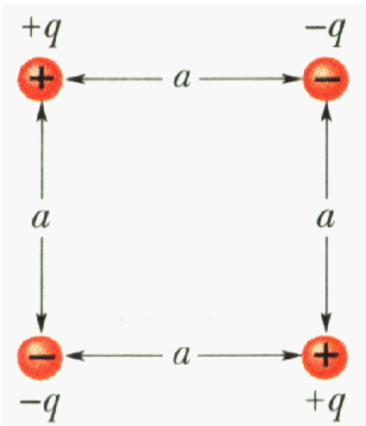
Electric potential energy of a system of charges

The electric potential energy of a system of charges is equal to the work done in assembling that system by an external observer by bringing each of those charges from infinity.

The external work done has opposite sign as the work done by the field.

For example, the work done to bring a positive charge q in the field of a positive charge Q (at a distance r) is:

$$W = qV = \frac{qQ}{4\pi\varepsilon_0 r}$$



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What is the work required to set up this configuration?

Strategy: bring in charges one at a time!

Why? Until we bring it in there is no field! Charges do not interact with their own field.

Charge #1: $\Delta W = 0$

$$\text{Charge \#2: } \Delta W = -qV_1 = -q\frac{q}{4\pi\varepsilon_0 a}$$

$$\text{Charge \#3: } \Delta W = -q(V_1 + V_2) = -q\left(\frac{q}{4\pi\varepsilon_0 a} - \frac{q}{4\pi\varepsilon_0\sqrt{2}a}\right)$$

$$\text{Charge \#4: } \Delta W = q(V_1 + V_2 + V_3) = q\left(\frac{q}{4\pi\varepsilon_0\sqrt{2}a} - \frac{2q}{4\pi\varepsilon_0 a}\right)$$

$$\Delta W = \Delta W_1 + \Delta W_2 + \Delta W_3 + \Delta W_4 = \frac{q^2}{4\pi\varepsilon_0 a}\left(-1 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 2\right) = \boxed{\frac{q^2}{4\pi\varepsilon_0 a}\left(-4 + \sqrt{2}\right)}$$

Conservation of mechanical energy with electric potential energy

An alpha particle (two protons, two neutrons) moves into a stationary gold atom (79 protons, 118 neutrons), passing through the electron region that surrounds the gold nucleus like a shell and headed directly toward the nucleus (Fig. 24-17). The alpha particle slows until it momentarily stops when its center is at radial distance $r = 9.23$ fm from the nuclear center. Then it moves back along its incoming path. (Because the gold nucleus is much more massive than the alpha particle, we can assume the gold nucleus does not move.) What was the kinetic energy K_i of the alpha particle when it was initially far away (hence external to the gold atom)? Assume that the only force acting between the alpha particle and the gold nucleus is the (electrostatic) Coulomb force.

KEY IDEA

During the entire process, the mechanical energy of the *alpha particle + gold atom* system is conserved.

Reasoning: When the alpha particle is outside the atom, the system's initial electric potential energy U_i is zero because the atom has an equal number of electrons and protons, which produce a *net* electric field of zero. However, once the alpha particle passes through the electron region surrounding the nucleus on its way to the nucleus, the electric field due to the electrons goes to zero. The reason is that the electrons act like a closed spherical shell of uniform negative charge and, as discussed in Section 23-9, such a shell produces zero electric field in the space it encloses. The alpha particle still experiences the electric field of the protons in the nucleus, which produces a repulsive force on the protons within the alpha particle.

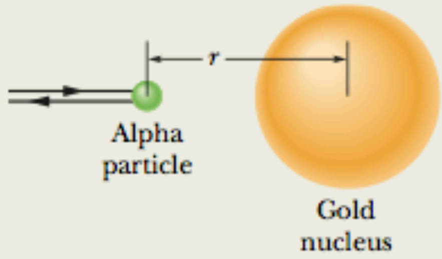


Fig. 24-17 An alpha particle, traveling head-on toward the center of a gold nucleus, comes to a momentary stop (at which time all its kinetic energy has been transferred to electric potential energy) and then reverses its path.

As the incoming alpha particle is slowed by this repulsive force, its kinetic energy is transferred to electric potential energy of the system. The transfer is complete when the alpha particle momentarily stops and the kinetic energy is $K_f = 0$.

Calculations: The principle of conservation of mechanical energy tells us that

$$K_i + U_i = K_f + U_f. \qquad (24\text{-}44)$$

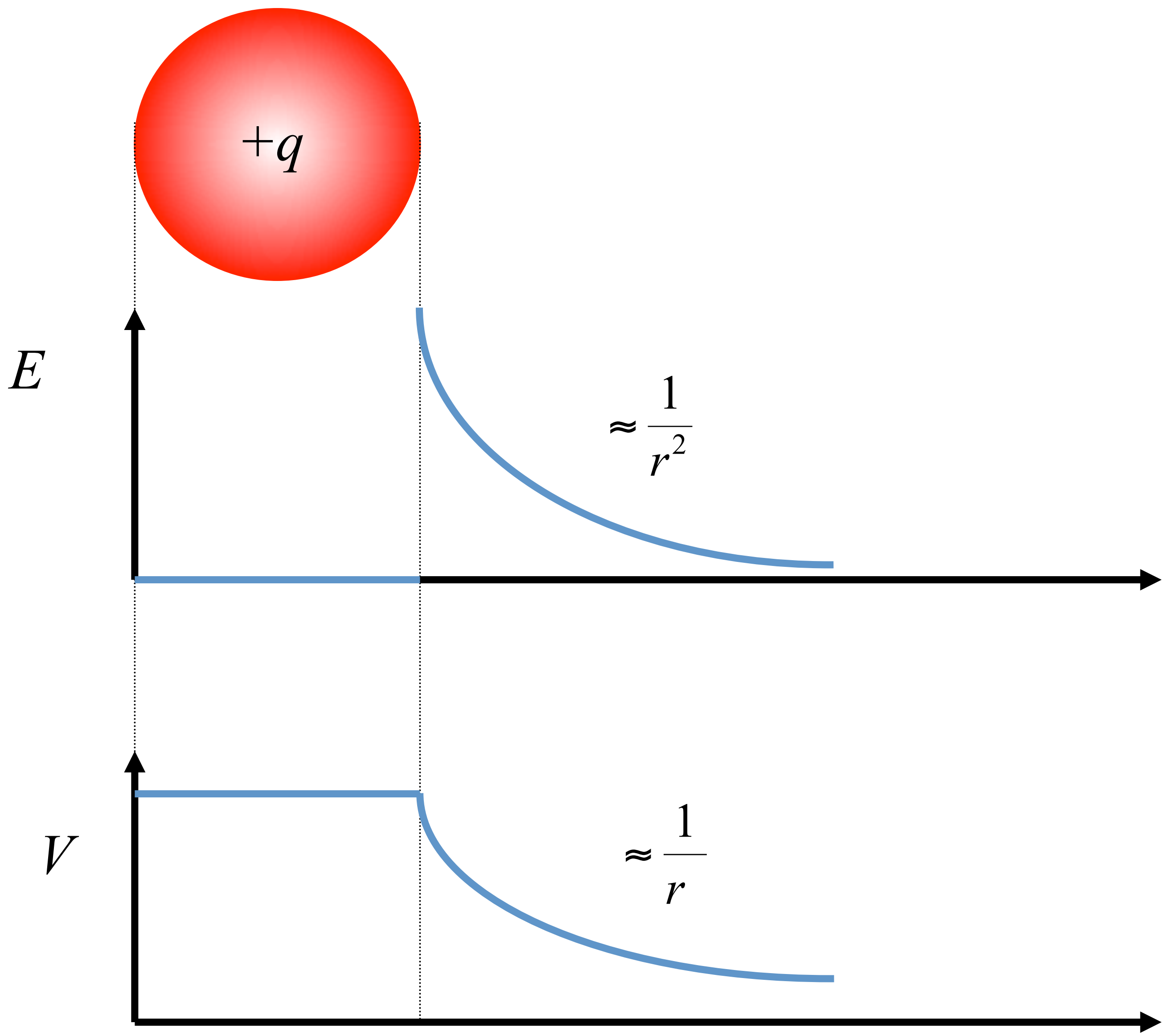
We know two values: $U_i = 0$ and $K_f = 0$. We also know that the potential energy U_f at the stopping point is given by the right side of Eq. 24-43, with $q_1 = 2e$, $q_2 = 79e$ (in which e is the elementary charge, 1.60×10^{-19} C), and $r = 9.23$ fm. Thus, we can rewrite Eq. 24-44 as

$$K_i = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(79e)}{9.23\text{ fm}}$$

$$= \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(158)(1.60 \times 10^{-19}\ \text{C})^2}{9.23 \times 10^{-15}\ \text{m}}$$

$$= 3.94 \times 10^{-12}\ \text{J} = 24.6\ \text{MeV}. \qquad \text{(Answer)}$$

Example: charged conducting sphere



Conductor:
zero field inside.

$$V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$$

Conductor:
constant
potential inside.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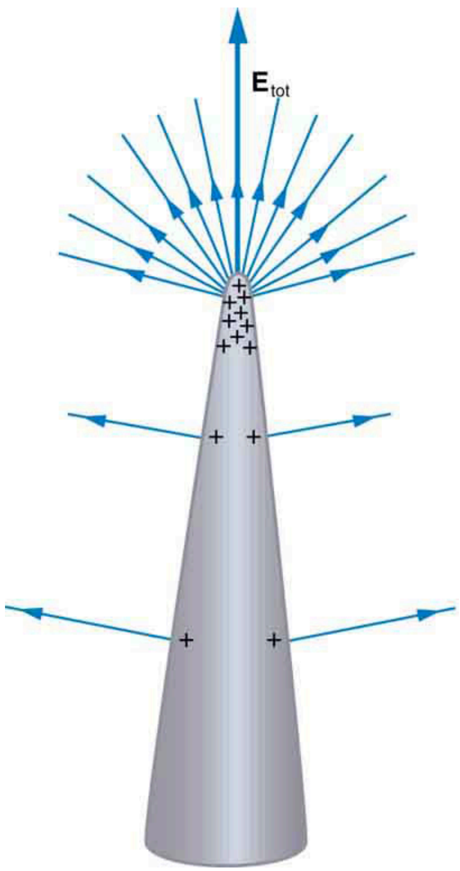
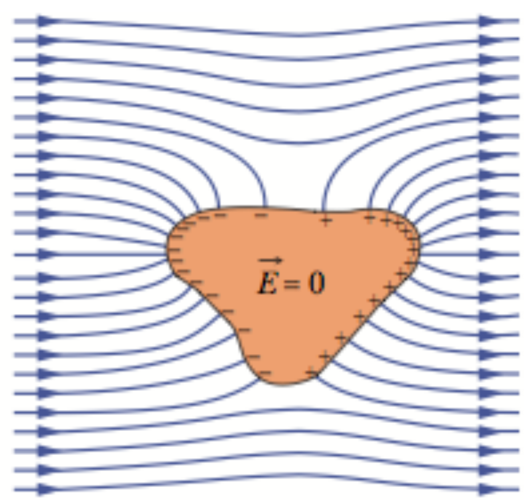
In non-spherical conductors, charges do not distribute themselves on the surface uniformly. At sharp points or edges, one can get large concentrations of charge, meaning that the electric fields can be large. Sometimes the fields are so large that air stops being an insulator and one gets an electric arc.

This is why lightning rods, have the shape of a rod. The field is maximum at the tip.



Electrons in a conductor placed in an external field arrange themselves so that the field is perpendicular to the surface of the conductor, since it is an equipotential.



Summary:

Electric Potential

- The electric potential *V* at point *P* in the electric field of a charged object:

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0},$$

Electric Potential Energy

- Electric potential energy *U* of the particle-object system:

$$U = qV.$$

- If the particle moves through potential *ΔV*:

$$\Delta U = q\,\Delta V = q(V_f - V_i).$$

Mechanical Energy

- Applying the conservation of mechanical energy gives the change in kinetic energy:

$$\Delta K = -q\,\Delta V.$$

- In case of an applied force in a particle

$$\Delta K = -q\,\Delta V + W_{\text{app}}.$$

- In a special case when *ΔK=0:*

$$W_{\text{app}} = q\,\Delta V \quad (\text{for } K_i = K_f).$$

Finding V from *E*

- The electric potential difference between two point *I* and *f* is:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s},$$

Summary:

Potential due to a Charged Particle

- due to a single charged particle at a distance r from that particle :

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

- due to a collection of charged particles

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}.$$

Potential due to an Electric Dipole

- The electric potential of the dipole is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

Potential due to a Continuous Charge Distribution

- For a continuous distribution of charge:

$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

Calculating *E* from V

- The component of ***E*** in any direction is:

$$E_s = -\frac{\partial V}{\partial s}.$$

Electric Potential Energy of a System of Charged Particle

- For two particles at separation *r*:

$$U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}.$$