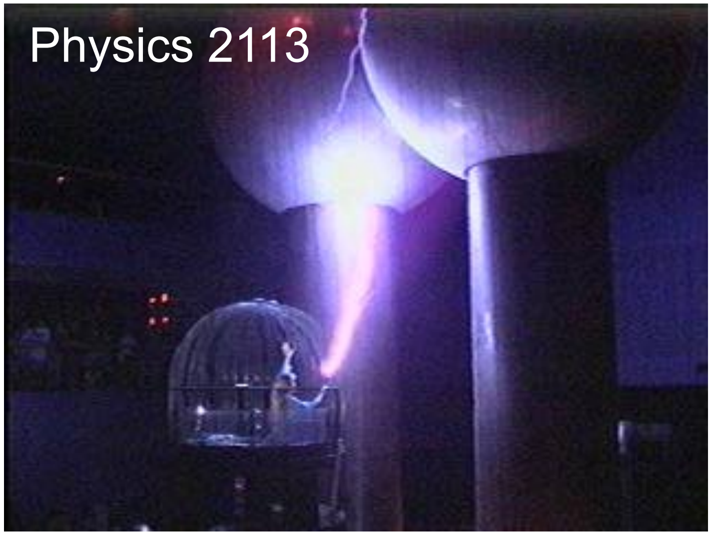

Alessandro Volta
(1745-1827)

Physics 2113
Lecture 12: WED 24 SEP

CH24: Electric Potential

Michael Faraday
(1791–1867)

Definition of electric potential:

Potential energy of a system per unit charge $V = \dfrac{U}{q}$

Units… Units...

$$V_f - V_i = \frac{U_f - U_i}{q} = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$\text{Units: } [\mathrm{V}] = \frac{\text{Joule}}{\text{Coulomb}} \equiv \text{Volt}$$

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$$[\mathrm{Volt}] = \left[\frac{\mathrm{N}}{\mathrm{C}}\right][\mathrm{m}] \Rightarrow \left[\frac{\mathrm{N}}{\mathrm{C}}\right] = \left[\frac{\mathrm{V}}{\mathrm{m}}\right]$$

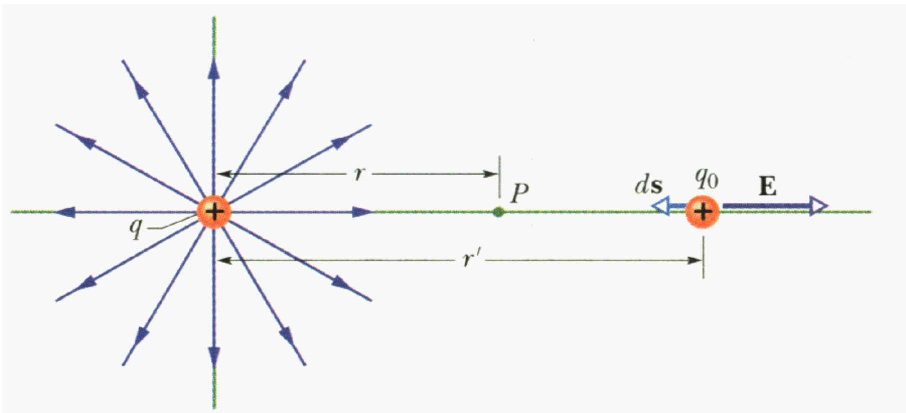
Unit most commonly used for electric fields

$$\Delta V = \frac{\Delta U}{q} \Rightarrow \Delta U = q\Delta V$$

eV=electron-volt, the energy that an electron acquires when placed in an electric potential of *1V*

$$1\, eV = (1.6\times 10^{-19}\, C)V = 1.6\times 10^{-19}\, J$$

Potential due to a point charge:



Change in potential in bringing q0 from infinity to a point P.

$$\vec{E}\cdot d\vec{s} = E\,ds\cos 180^0 = -E\,ds$$

$$ds = -dr'$$

$$\Delta V = -\int_A^B \vec{E}\cdot d\vec{s} = -\int_\infty^r E\,dr' = -\int_\infty^r \frac{q}{4\pi\varepsilon_0 r'^2}dr' = -\frac{q}{4\pi\varepsilon_0}\int_\infty^r \frac{1}{r'^2}dr' = -\frac{q}{4\pi\varepsilon_0}\left[-\frac{1}{r'}\right]_\infty^r = \frac{q}{4\pi\varepsilon_0 r}$$

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

- If charge is negative, then potential is negative.
- At infinity, potential is zero, as expected for isolated sources.
- For several charges, potentials are simply superposed:

$$V = \sum_i V_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

As was the case with electric fields, the potential outside a charged sphere or charged shell coincides with the potential of a point charge at the origin.

Potential is not a vector, orientation is irrelevant

(a) In Fig. 24-9*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?

KEY IDEAS

(1) The electric potential V at C is the algebraic sum of the electric potentials contributed by all the electrons. (Because electric potential is a scalar, the orientations of the electrons do not matter.) (2) The electric field at C is a vector quantity and thus the orientation of the electrons *is* important.

Calculations: Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, Eq. 24-27 gives us

$$V = -12 \frac{1}{4\pi\varepsilon_0} \frac{e}{R}. \qquad \text{(Answer)} \quad (24\text{-}28)$$

Because of the symmetry of the arrangement in Fig. 24-9*a*, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C,

$$\vec{E} = 0. \qquad \text{(Answer)}$$

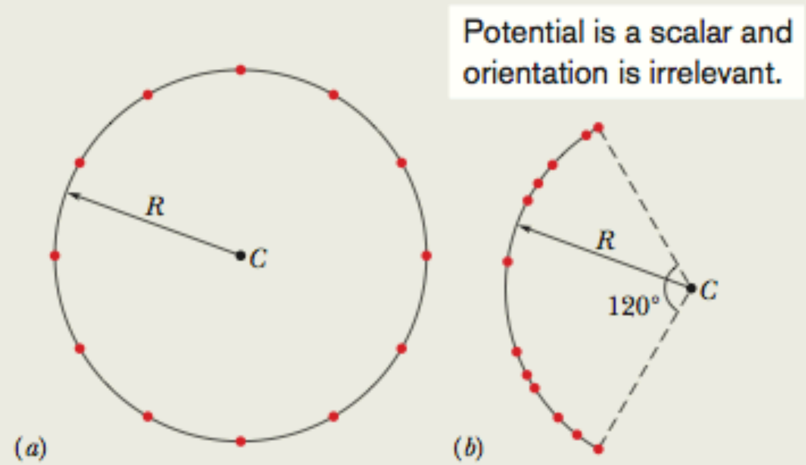
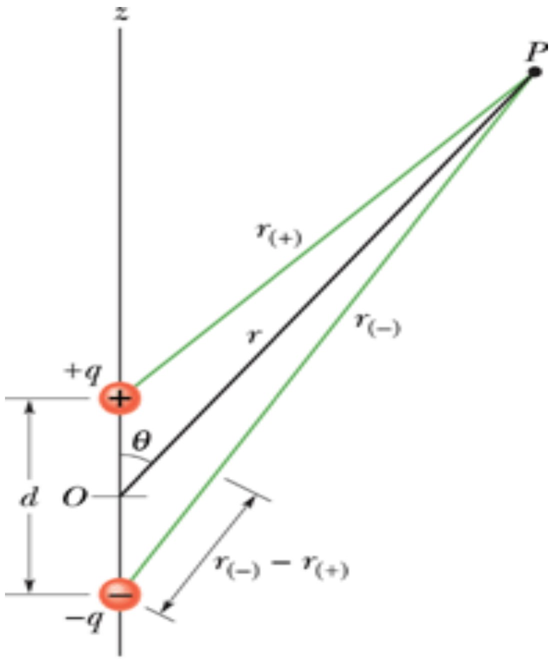


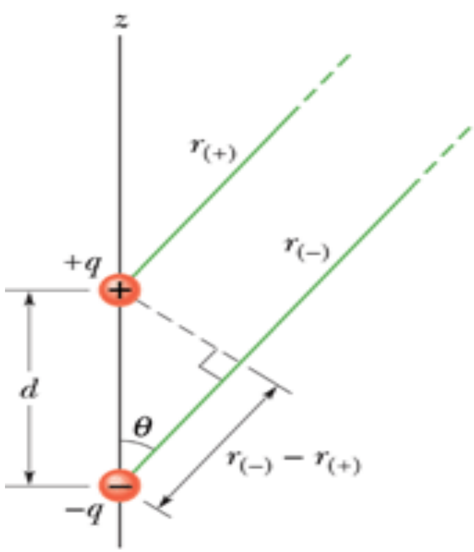
Fig. 24-9 (*a*) Twelve electrons uniformly spaced around a circle. (*b*) The electrons nonuniformly spaced along an arc of the original circle.

(b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 24-9*b*), what then is the potential at C? How does the electric field at C change (if at all)?

Reasoning: The potential is still given by Eq. 24-28, because the distance between C and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.



(a)



(b)

Potential due to a Dipole

At point P, the total potential is due to that of +q and -q

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+} - \frac{q}{r_-}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_- - r_+}{r_- r_+}\right)$$

If point P is at “infinity” or r >> d, then in this approximation we can consider fig (b):

$$r_- - r_+ = d\cos\theta \qquad \text{and} \qquad r_- r_+ \approx r^2$$

Then,

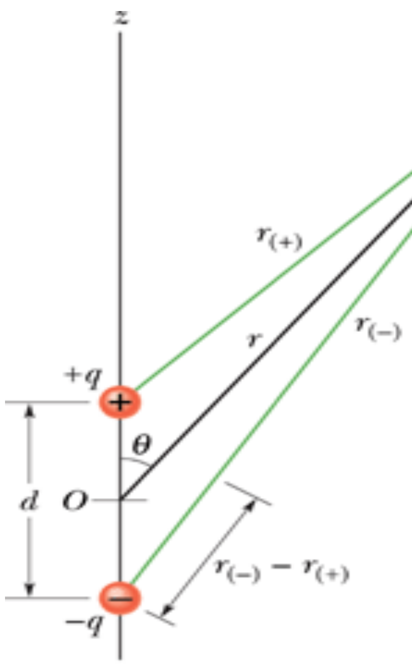
$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{d\cos\theta}{r^2}\right)$$

Electric dipole: defined as p = d q

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

CHECKPOINT 5

Suppose that three points are set at equal (large) distances r from the center of the dipole in Fig. 24-10: Point a is on the dipole axis above the positive charge, point b is on the axis below the negative charge, and point c is on a perpendicular bisector through the line connecting the two charges. Rank the points according to the electric potential of the dipole there, greatest (most positive) first.



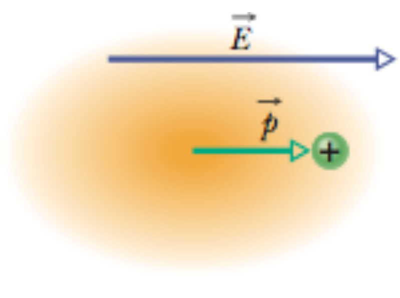
(a)

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

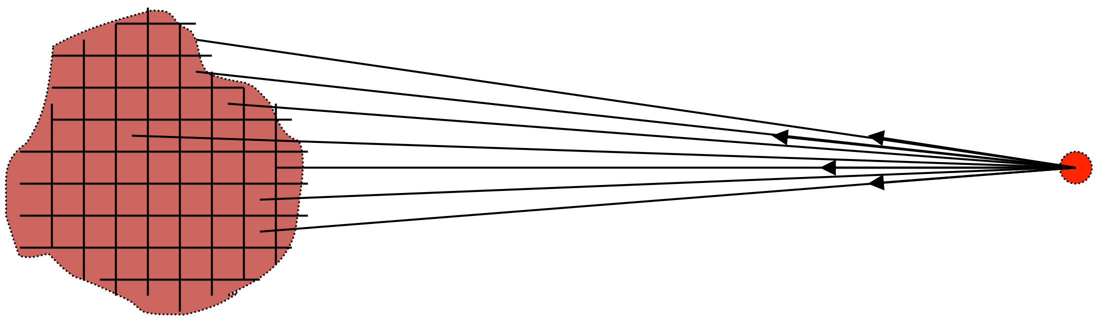
Induced dipole

As we discussed, some molecules (H20) have a permanent dipolar nature. Others do not, the distribution of electrons is spherical and its center coincides with the center of the nucleus.

But when a field is applied, a dipole moment is induced



Potential due to continuous distributions of charge



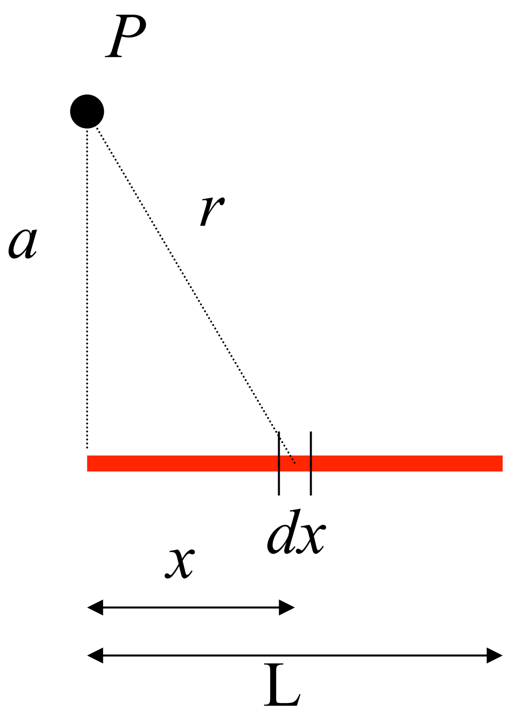
Like for electric fields, you break it up into small pieces, treat each little piece like a point charge, and add up the resulting potentials. Unlike
electric fields, you superpose the potentials as scalars, not vectors.

So it is messy, but a bit simpler.

Potential due to continuous distributions of charge

Strategy: same as for field calculations, break up into infinitesimal pieces, integrate. It is easier than for the field, since the potential is a scalar.

Example: charged rod



$$\lambda = q / L \qquad dq = \lambda dx$$

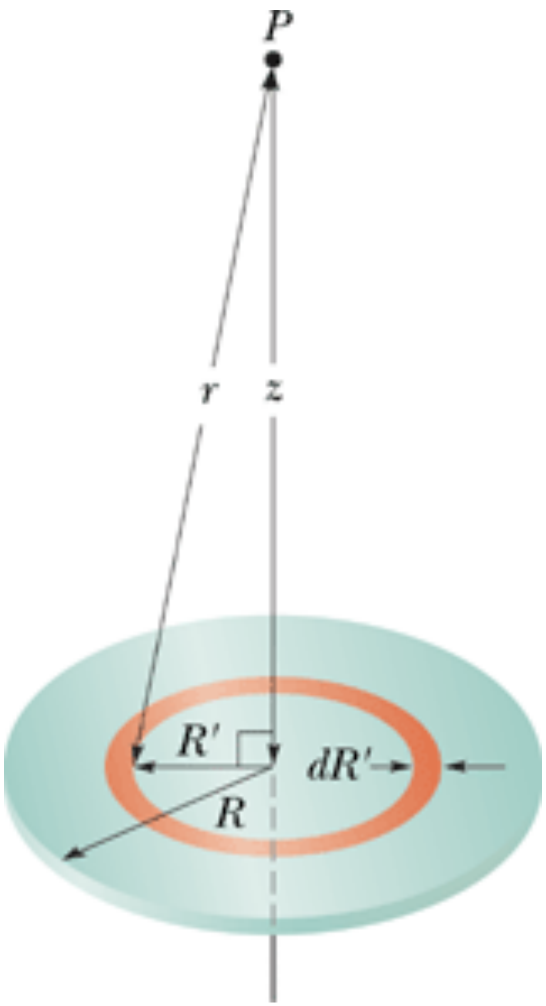
$$dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda dx}{4\pi\varepsilon_0 \sqrt{a^2 + x^2}} \qquad V = \int_0^L dV$$

$$V = \frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{L + \sqrt{L^2 + a^2}}{a}\right]$$

$$\text{Check: if } a \to \infty, \text{ then } [\,] \to 1, \quad V \to 0$$

Since the argument of log is greater than unity, V is always positive

Potential due to a charged disk



Consider a charged disk of radius R with a uniform charge density. We wish to compute the potential at point P lying on the central axis of the disk.

We consider a differential element of radius R' and width dR', enclosing a surface area $2\pi R' dR'$

Enclosed charge: $dq = \sigma(2\pi R' dR')$

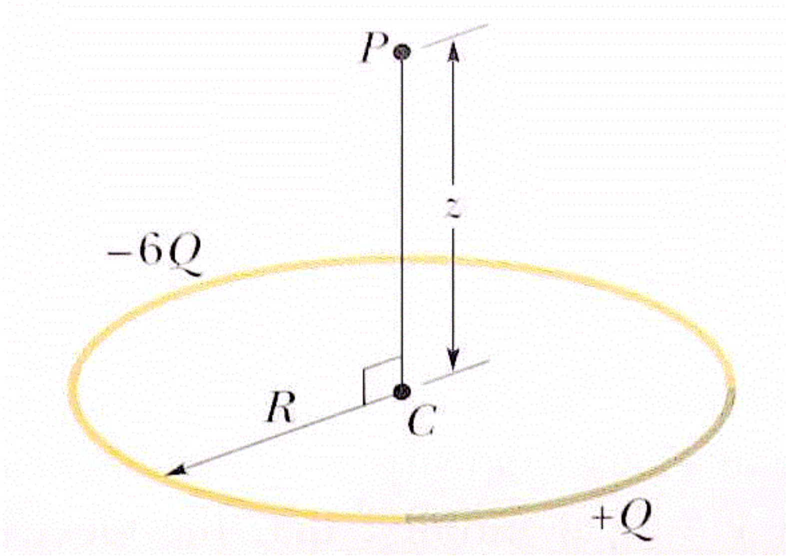
This enclosed charge leads to the potential:

$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

We can then integrate this potential from 0 to R to get the net potential due to the disk:

$$V = \int_0^R \frac{dq}{4\pi\varepsilon_0 r} = \frac{\sigma}{2\varepsilon_0}\left(\left(z^2 + R^2\right)^{1/2} - z\right)$$

Example



All the charge is at the same distance R from C, so the potential at C is,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \frac{6Q}{R}\right) = -\frac{5Q}{4\pi\varepsilon_0 R}$$

All the charge is at the same distance from P, so the potential at P is,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\sqrt{R^2+z^2}} - \frac{6Q}{\sqrt{R^2+z^2}}\right) = -\frac{5Q}{4\pi\varepsilon_0\sqrt{R^2+z^2}}$$

Summary

- Like electric fields, potentials for configurations involving many charges or continuous charge distributions are obtained by superposing.
- But the superposition is a scalar one, so it is usually easier to do than superposing fields.
- Next class we will learn that one can obtain the fields from the potentials, so this simplifies calculations quite a bit.