Physics 7412, Fall 2013
Problem set 4
Due date: 10/23/13

The temperature in a solid cylinder subject to heating and cooling from the outside is, for many materials, described by the equation,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{4K}\frac{\partial T}{\partial t} \tag{1}$$

where $T = T(t, r)$ is the temperature, r is the radial distance from the center of the cylinder, t is time and K is the diffusivity constant.

a) Write a Crank-Nicholson discretization for this equation.

b) Study its stability applying the Von Neumann criterion.

c) Find the temperature profile $T(r, t = 10)$, given the initial and boundary conditions,

$$\begin{aligned} T(1,t) &= 100 + 40t, \qquad 0 \le t \le 10, & (2)\\ T(0.5,t) &= t, \qquad\quad 0 \le t \le 10, & (3)\\ T(r,0) &= 200(r - 0.5), \qquad 0.5 \le r \le 1, & (4) \end{aligned}$$

and $K = 0.1$

You are welcome to use a very small grid (say, four points in the r direction, maybe twenty in t) and set up the problem "by hand" with a calculator, or write a numerical code for doing it. I got as result

$$\begin{aligned} T(0.6,10) &\approx 137 & (5)\\ T(0.7,10) &\approx 246 & (6)\\ T(0.8,10) &\approx 340 & (7)\\ T(0.9,10) &\approx 424. & (8) \end{aligned}$$