(*We expand \( J_0[x] \) and \( J_1[x] \) in a power series about \( x = 0 \). It is easy to see that \( J_0[x] \) is an even function of \( x \) -- like \( \cos[x] \) -- and \( J_1[x] \) is an odd function of \( x \) -- like \( \sin[x] \).*

\[
\text{Series[BesselJ[0, x], \{x, 0, 5\}]}
\]
\[
\frac{x^2}{4} + \frac{x^4}{64} + O[x]^6
\]

\[
\text{Series[BesselJ[1, x], \{x, 0, 5\}]}
\]
\[
\frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384} + O[x]^6
\]

(*Also like \( \cos[x] \) and \( \sin[x] \) we have that one is the negative of the derivative of the other.*

\[D[\cos[x], x]\]
\[-\sin[x]\]

\[D[\text{BesselJ}[0, x], x]\]
\[-\text{BesselJ}[1, x]\]

(*We plot \( J_0[x] \) (blue) and \( \cos[x] \) (red) near \( x=0 \) and see \( J_0[x] \) is like a damped \( \cos[x] \).*

\[\text{Plot[\{BesselJ[0, x], \cos[x]\}, \{x, -6\pi, 6\pi\}]}\]

(*We plot \( J_1[x] \) (blue) and \( \sin[x] \) (red) near \( x=0 \) and see \( J_1[x] \) is like a damped \( \sin[x] \).*
Plot[{BesselJ[1, x], Sin[x]}, {x, -6 Pi, 6 Pi}]

(*As proved in class, for |x| >> 1 J_0 is asymptotic to Sine and Cosine.*)

Series[BesselJ[0, x], {x, Infinity, 1}]

\[
\cos\left(\frac{\pi}{4} - x\right) \left(\sqrt{\frac{2}{\pi}} + o\left(\frac{1}{x}^{3/2}\right)\right) + \left(-\frac{1}{4\sqrt{2\pi}}\frac{1}{x^{3/2}} + o\left(\frac{1}{x}^{2}\right)\right) \sin\left(\frac{\pi}{4} - x\right)
\]

(*We plot J_0 for |x| >> 1 and the leading term in the asymptotic expansion and see agreement is good.*)

Plot[{BesselJ[0, x], Cos[\(\frac{\pi}{4} - x\)] \* Sqrt[2 / (Pi \* x)]}, {x, 0, 30 Pi}]
Plot\[\text{BesselJ}[0, x], \text{Cos}\left[\frac{\pi}{4} - x\right] \ast \text{Sqrt}[2 / (\text{Pi} \ast x)]\], \{x, 0, 2 \text{Pi}\}, \text{PlotRange} \to (-1, 2)]

(*Hence we can use the asymptotic expression when |x|>1 with good results.*)

In[1]=
(*Here are plots of the first five Bessel functions of non-negative integer order.*)

In[10]= Plot[\{Table[\text{BesselJ}[n, x]], \{n, 0, 5\}\}, \{x, -2 \text{Pi}, 2 \text{Pi}\}]

Out[10]=

(*Note that all of them except J_0 vanish at the origin and alternate even and odd.*)

In[7]= (*Here we plot the first five Bessel functions of negative integer order.*)

In[11]= Plot[\{Table[\text{BesselJ}[-n, x]], \{n, 1, 5\}\}, \{x, -2 \text{Pi}, 2 \text{Pi}\}]

Out[11]=

(*Again all vanish at the origin and alternate even and odd.*)