Physics 2102
Lecture 19
Ch 30:
Inductors and RL Circuits

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What are we going to learn?
A road map

- Electric charge
  - Electric force on other electric charges
  - Electric field, and electric potential
- Moving electric charges: current
- Electronic circuit components: batteries, resistors, capacitors
- Electric currents ➔ Magnetic field ➔ Magnetic force on moving charges
- Time-varying magnetic field ➔ Electric Field
- More circuit components: inductors.
- Electromagnetic waves ➔ light waves
- Geometrical Optics (light rays).
- Physical optics (light waves)
Inductors are with respect to the magnetic field what capacitors are with respect to the electric field. They “pack a lot of field in a small region”. Also, the higher the current, the higher the magnetic field they produce.

**Capacitance** → how much **potential** for a given charge: \( Q = CV \)

**Inductance** → how much **magnetic flux** for a given current: \( \Phi = Li \)

Using Faraday’s law: \( EMF = -L \frac{di}{dt} \)

Units: \([L] = \frac{\text{Tesla} \cdot \text{m}^2}{\text{Ampere}} \equiv \text{H (Henry)}\)
“Self”-Inductance of a solenoid

- Solenoid of cross-sectional area \( A \), length \( l \), total number of turns \( N \), turns per unit length \( n \)
- Field inside solenoid = \( \mu_0 n i \)
- Field outside \( \sim 0 \)

\[
\Phi_B = NAB = NA\mu_0 ni = Li
\]

\[
L = \text{“inductance”} = \mu_0 NA\pi = \mu_0 \frac{N^2}{l} A
\]
Example

- The current in a 10 H inductor is decreasing at a steady rate of 5 A/s.
- If the current is as shown at some instant in time, what is the magnitude and direction of the induced EMF?

\[ EMF = -L \frac{di}{dt} \]

(a) 50 V

(b) 50 V

- Magnitude = (10 H)(5 A/s) = 50 V
- Current is decreasing
- Induced emf must be in a direction that OPPOSES this change.
- So, induced emf must be in same direction as current
The RL circuit

- Set up a single loop series circuit with a battery, a resistor, a solenoid and a switch.
- Describe what happens when the switch is closed.
- Key processes to understand:
  - What happens JUST AFTER the switch is closed?
  - What happens a LONG TIME after switch has been closed?
  - What happens in between?

Key insights:
- If a circuit is not broken, one cannot change the CURRENT in an inductor instantaneously!
- If you wait long enough, the current in an RL circuit stops changing!

At $t=0$, a capacitor acts like a wire; an inductor acts like a broken wire. After a long time, a capacitor acts like a broken wire, and inductor acts like a wire.
In an RC circuit, while charging, \( Q = CV \) and the loop rule mean:

- charge increases from 0 to \( CE \)
- current decreases from \( \frac{E}{R} \) to 0
- voltage across capacitor increases from 0 to \( E \)

In an RL circuit, while “charging” (rising current), emf = \( L\frac{di}{dt} \) and the loop rule mean:

- magnetic field increases from 0 to \( B \)
- current increases from 0 to \( \frac{E}{R} \)
- voltage across inductor decreases from \( -E \) to 0
Example

Immediately after the switch is closed, what is the potential difference across the inductor?
(a) 0 V
(b) 9 V
(c) 0.9 V

- Immediately after the switch, current in circuit = 0.
- So, potential difference across the resistor = 0!
- So, the potential difference across the inductor = \( E = 9 \text{ V} \)!
Example

- Immediately after the switch is closed, what is the current $i$ through the 10 $\Omega$ resistor?

(a) 0.375 A  
(b) 0.3 A  
(c) 0

- Long after the switch has been closed, what is the current in the 40 $\Omega$ resistor?

(a) 0.375 A  
(b) 0.3 A  
(c) 0.075 A

- Immediately after switch is closed, current through inductor $= 0$.
- Hence, current through battery and through 10 $\Omega$ resistor is $i = (3\, \text{V})/(10\, \text{Ω}) = 0.3\, \text{A}$

- Long after switch is closed, potential across inductor $= 0$.
- Hence, current through 40 $\Omega$ resistor $= (3\, \text{V})/(40\, \text{Ω}) = 0.075\, \text{A}$
“Charging” an inductor

• How does the current in the circuit change with time?

\[-iR + E - L \frac{di}{dt} = 0\]

\[i = \frac{E}{R} \left( 1 - e^{-\frac{Rt}{L}} \right)\]

“Time constant” of RL circuit = L/R
“Discharging” an inductor

The switch is in a for a long time, until the inductor is charged. Then, the switch is closed to b.

What is the current in the circuit?

Loop rule around the new circuit:

\[ iR + L \frac{di}{dt} = 0 \]

\[ i = \frac{E}{R} e^{-\frac{Rt}{L}} \]

Exponential discharge.
Inductors & Energy

- Recall that **capacitors** store energy in an **electric** field.
- **Inductors** store energy in a **magnetic** field.

\[
E = iR + L \frac{di}{dt}
\]

\[
(iE) = (i^2R) + Li \frac{di}{dt}
\]

\[
(iE) = (i^2R) + \frac{d}{dt} \left( \frac{Li^2}{2} \right)
\]

Power delivered by battery = power dissipated by R

+ \((d/dt)\) energy stored in L
The switch has been in position “a” for a long time.

It is now moved to position “b” without breaking the circuit.

What is the total energy dissipated by the resistor until the circuit reaches equilibrium?

When switch has been in position “a” for long time, current through inductor = \(\frac{9V}{10\,\Omega}\) = 0.9A.

Energy stored in inductor = \((0.5)(10H)(0.9A)^2 = 4.05\, J\)

When inductor “discharges” through the resistor, all this stored energy is dissipated as heat = 4.05 J.
$E=120V$, $R_1=10\Omega$, $R_2=20\Omega$, $R_3=30\Omega$, $L=3H$.

1. What are $i_1$ and $i_2$ immediately after closing the switch?
2. What are $i_1$ and $i_2$ a long time after closing the switch?
3. What are $i_1$ and $i_2$ 1 second after closing the switch?
4. What are $i_1$ and $i_2$ immediately after reopening the switch?
5. What are $i_1$ and $i_2$ a long time after reopening the switch?