A force acting on a system can heat it up (or cool it down) by working on it.
A change in temperature produced in a system can be used to produce mechanical work.
At any point in the process, the system (gas) will have temperature $T$, pressure $p$ and volume $V$.

$$dW = Fds = pAds = pdV$$

$$W = \int_{V_i}^{V_f} pdV$$
Changing the system’s pressure and volume from an initial to a final state can be done using different amounts of work (and heat transferred):

Going back to the original state does not mean that no work was done!

The work $W$ done by the system during a transformation from an initial state to a final state depends on the path taken. The heat $Q$ absorbed by the system during a transformation from an initial state to a final state depends on the path taken. However, the difference $Q - W$ does not depend on the path taken! We define this quantity as the change in “internal energy”:

$$\Delta E_{\text{int}} = Q - W$$

The internal energy of a system increases if energy is added as heat, and decreases if energy is lost as work done by the system.
Example

The figure shows four possible paths to take a system from an initial state to a final state. Rank the paths according to:

- the change in internal energy
- the work done by the system
- the magnitude of energy transferred as heat

\[ \Delta E_{\text{int}} = Q - W \]

**Special processes:**

1. **Adiabatic:** \( Q = 0 \)  \[ \rightarrow \Delta E_{\text{int}} = -W \]
   - thermally insulated system, or
   - very quick process
2. **Constant Volume:** \( W = 0 \)  \[ \rightarrow \Delta E_{\text{int}} = -Q \]
3. **Cyclical:** \( \Delta E_{\text{int}} = 0 \)  \[ \rightarrow Q = W \]
4. **Free expansion:** \( Q = W = 0 \)  \[ \rightarrow \Delta E_{\text{int}} = 0 \]
   - happens in isolated systems, it’s irreversible.
Example

Figure 19-39a shows a cylinder containing gas and closed by a movable piston. The cylinder is kept submerged in an ice water mixture. The piston is quickly pushed down from position 1 to position 2 and then held at position 2 until the gas is again at the temperature of the ice water mixture; it then is slowly raised back to position 1. The figure shows a p-V diagram for the process. If 140 g of ice is melted during the cycle, how much work has been done on the gas? [46600] J

Kinetic theory of gases

At the microscopic level,
- temperature is a measure of the kinetic energy in the elemental units (atoms or molecules);
- pressure exerted by a gas is produced by molecular collisions on the walls of its container;
- the fact that gases fill up the volume of the container they’re in is due to the freedom of the molecules to move around.

Useful unit for number of atoms: moles

1 mole = number of atoms in a 12g sample of carbon-12 = 6.02 \times 10^{23} \text{ (Avogadro’s number)}

number of moles = \frac{\text{total number of molecules}}{N_A} = \frac{\text{total mass}}{\text{mass of one mole}} = \frac{\text{total mass}}{(\text{molecular mass} \times N_A)}
• An Avogadro’s number of standard soft drink cans would cover the surface of the earth to a depth of over 200 miles.
• If you had Avogadro’s number of unpopped popcorn kernels, and spread them across the United States of America, the country would be covered in popcorn to a depth of over 9 miles.
• If we were able to count atoms at the rate of 10 million per second, it would take about 2 billion years to count the atoms in one mole.

Lorenzo Romano Amedeo Carlo Avogadro, conte di Quaregna e di Cerreto (1776 - 1856)

Gold has a molar mass of 197 g/mol.

a) How many moles of gold are in a 2.50g sample of pure gold?
b) How many atoms are in the sample?

If we were able to count atoms at the rate of 10 million per second, it would take about 2 billion years to count the atoms in one mole.

TRUE OR FALSE?
All gases approach a unique “ideal gas” at low densities.

An ideal gas obeys the “ideal gas law”
\[ pV = nRT = NkT \]

- \( p \) = absolute pressure (Pa)
- \( V \) = volume (m\(^3\))
- \( n \) = number of moles
- \( T \) = temperature (kelvin)
- \( R \) = gas constant = 8.31 J/(mole K) = \( kN_A \)
- \( k \) = Boltzmann’s constant = 1.38 \( \times 10^{-23} \) J/K
- \( N \) = number of molecules

---

**Work done by an ideal gas**

\[ W = \int_{V_i}^{V_f} p dV \]

- **Constant temperature:** keep temperature constant, change the volume. The pressure will change, following:
  \[ p = \frac{nRT}{V} \]
  How much work is done by the gas?
  \[ W = nRT \ln \left( \frac{V_f}{V_i} \right) \]

- **Constant pressure:** keep pressure constant, change the volume. The temperature will change, following \( T = \frac{pV}{nR} \).
  How much work is done by the gas?
  \[ W = p \Delta V \]

- **Constant volume:** pressure and temperature may change, but no work is done!
  \[ W = 0 \]
Example

A gas can be taken from the initial state \( i \) to the final state \( f \) in many different ways, usually following constant pressure curves, constant volume curves, and isotherms.

a) If the initial pressure is 1 Pa, and the initial volume is 1 \( \text{m}^3 \), how many moles are there in the gas?

b) If the final volume is 1.1 \( \text{m}^3 \), what is the final pressure?

c) What is the path from \( i \) to \( f \) where the gas does minimum work?

d) What is the temperature at intermediate points A, B?

e) If the system is taken to the final state through the 310 K isotherm, and then back to the original state through point B, what is the total heat added to the system?

---

Kinetic energy, internal energy

We learned the "ideal gas law": \( pV = nRT = nkT \).

We know that \( \Delta E_{\text{int}} = Q - W \), and we can calculate the work for a process from the area in the \( p-V \) diagram. Can we calculate \( \Delta E_{\text{int}} \) and \( Q \) from \( p, V \) and/or \( T \)?

The average translational kinetic energy of molecules in a gas is related only to temperature (not pressure, or volume). The internal energy of a gas is also related just to temperature. For a monoatomic, ideal gas,

\[
E_{\text{int}} = nN_A k \bar{v} = (3/2) n k T = (3/2) nRT
\]

(The factor 3/2 can be derived from three directions of motion for each molecule, each with \( 1/2 \) \( kT \) energy).

Thus, the internal energy of all states on an isotherm curve have the same internal energy.

The work done by a gas between two states on an isotherm must be equal to the heat absorbed by the gas.

What's the heat absorbed between states on different isotherms?

We can calculate it from \( Q = \Delta E_{\text{int}} + W \) now!
We defined heat capacity \( C \) and specific heat \( c \) (heat capacity per mass) as:
\[
Q = C \Delta T = c m \Delta T
\]
For water, \( c = 1 \text{cal/g}^{\circ}C = 1 \text{Btu/lb}^{\circ}F = 4187 \text{J/kg} K \)
The *molar specific heat* is the heat capacity per mole.

However, the amount of heat needed to raise the temperature of 1 mole by 1 kelvin depends on whether we perform the operation at constant pressure or at constant volume:
\[
Q = n C_p \Delta T \text{ or } Q = n C_v \Delta T
\]
In both cases, \( Q = \Delta E_{\text{int}} + W \): \( \Delta E_{\text{int}} \) is the same, but \( W \) is not!

Consider a process at constant volume, raising the temperature by \( \Delta T \).
\[
Q = n C_v \Delta T
\]
\[
Q = \Delta E_{\text{int}} + W
\]
\[
W = 0!
\]
\[
Q = \Delta E_{\text{int}} = (3/2) n R \Delta T = n C_v \Delta T
\]
\[
\rightarrow C_v = (3/2) R = 12.5 \text{ J/molK}
\]
\[
\rightarrow E_{\text{int}} = n C_v T
\]
Molar specific heat

Consider a process at constant pressure, raising the temperature by $\Delta T$. 

\[ Q = n \, C_P \, \Delta T \]

\[ Q = \Delta E_{\text{int}} + W \]

\[ W = p \, \Delta V = n \, R \, \Delta T \]

\[ \Delta E_{\text{int}} = (3/2) \, n \, R \, \Delta T \]

\[ Q = \Delta E_{\text{int}} + W = (5/2) \, n \, R \, \Delta T \]

$\rightarrow C_P = (5/2)R = C_V + R$

---

Ideal gases so far

For any process (1, 2, 3 or 4):

\[ pV = n \, R \, T \]

\[ \Delta E_{\text{int}} = Q - W \]

\[ E_{\text{int}} = (3/2) \, n \, R \, T = n \, C_V \, T \]

Also,

\[ C_P = C_V + R ; C_P = (3/2)R \]

Constant volume (4):

\[ W = 0, \quad Q = n \, C_P \, \Delta T \]

\[ \Delta E_{\text{int}} = n \, C_V \, \Delta T \]

Constant pressure (1):

\[ W = p \, \Delta V = n \, R \, \Delta T \]

\[ Q = n \, C_P \, \Delta T \]

\[ \Delta E_{\text{int}} = n \, C_V \, \Delta T \]

Constant temperature (2):

\[ W = n \, R \, T \ln (V_f/V_i) \]

\[ \Delta E_{\text{int}} = 0 \]

\[ Q = W = n \, R \, T \ln (V_f/V_i) \]

True for monoatomic gases, but not for others!