Physics 2113
Lecture 20: WED 15 OCT

CH27: Circuits
Summary of last class:

- To solve a circuit, start at any point in it and "take a walk" around it.
- Add potential every time you traverse an EMF from - to +.
- Deduct -iR potential every time you move along a resistor "with the flow".
- Take as many walks along independent loops as needed to solve the circuit.
- Treat junctions as "water pipes".
Resistors in series:

Taking a walk:

\[ E - iR_1 - iR_2 = 0 \implies E = i(R_1 + R_2) = iR_{\text{tot}} \]

\[ R_{\text{tot}} = R_1 + R_2 \]

Behave like capacitors in parallel!

If you have \( n \) resistors in series: \( R_{\text{tot}} = \sum_{i=1}^{n} R_i \)

Resistors in parallel:

Node a: \( i_1 = i_2 + i_3 \)

Left loop: \( E - i_2R_1 = 0 \implies i_2 = \frac{E}{R_1} \)

Outer loop: \( E - i_3R_2 = 0 \implies i_3 = \frac{E}{R_2} \)

\[ i_1 = \frac{E}{R_1} + \frac{E}{R_2} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)E \]

\[ R_{\text{tot}} = \frac{1}{\left( \frac{1}{R_1} + \frac{1}{R_2} \right)} \]

Same as capacitors in series.
Example:

38E. A circuit containing five resistors connected to a battery with a 12.0 V emf is shown in Fig. 28-38. What is the potential difference across the 5.0 Ω resistor?

Bottom loop: (all else is irrelevant)

\[ i = \frac{V}{R} = \frac{12V}{8\Omega} = 1.5A \]
Example:

48P. In Fig. 28-44, $\mathcal{E}_1 = 3.00 \text{ V}$, $\mathcal{E}_2 = 1.00 \text{ V}$. $R_1 = 5.00 \Omega$, $R_2 = 2.00 \Omega$, $R_3 = 4.00 \Omega$, and both batteries are ideal. (a) What is the rate at which energy is dissipated in $R_1$? In $R_2$? In $R_3$? (b) What is the power of battery 1? Of battery 2?

1) Label currents, assume direction.

2) Top junction: $i_3 = i_1 + i_2$

3) Left loop: $E_1 - i_3R_3 - i_1R_1 = 0$

4) Right loop: $-E_2 + i_1R_1 - i_2R_2 = 0$

\[ 4) \rightarrow -1 + 5i_1 - 2i_2 = 0 \]

3) and 2) \[ 3 - 4(i_1 + i_2) - 5i_1 = 0 \Rightarrow 3 - 9i_1 - 4i_2 = 0 \]

Subst. in 4) \[ -1 + 1.31 - 2i_2 = 0 \Rightarrow i_2 = 0.16A \]

Junction: \[ i_3 = 0.42A \]

Power in $R_1$ : \( (0.26)^2 5 = 0.34W \) in $R_2$ : \( (0.16)^2 2 = 0.05W \) in $R_3$ : \( (0.42)^2 4 = 0.7W \)

Power from battery 1 : \( E_1i_3 = 1.26W \)

Power from battery 2 : \( E_2i_2 = -0.16W \)

Check : $1.26 - 0.16 = 1.10 \approx 0.34 + 0.05 + 0.7$
Comment:

Voltmeter and ammeter “in real life”.

In practice, the voltmeter will draw a (hopefully tiny) current $\Delta i$, therefore distorting the value of $V$ across $R$ we want to measure. Good voltmeters minimize this current in order to minimize the distortion.

Similarly an ideal ammeter would try to allow all of the current to pass through it, opposing the least resistance. Good ammeters minimize their internal resistance.

Good voltmeter: high internal resistance, low $\Delta i$

Good ammeter: low internal resistance, low $\Delta V$

Never plug an ammeter into a 110V outlet!

A good ammeter is essentially a “short circuit”
An application, Ohmmeters:

One is tempted to say, “I measure current $i$ going through the resistor and voltage $V$ across its ends, therefore using Ohm’s laws, $R = V/i$”.

Wrong. Voltage indeed is $V$.

Applying junction rule in a, the current across the resistor is really $i_{\text{true}} = i - \Delta i$.

Therefore, $R = \frac{V}{i - \Delta i}$

Real life Ohmmeters actually correct for this. The correction is harder than it looks, since $\Delta i$ actually depends on $R$!!
Wheatstone bridge, a simple Ohmeter

One slides the contact in Rs, varying its resistance, until the potential difference between a and b vanishes.

Solving the circuit, the current through the bottom loop is,

\[ i_{\text{bottom}} = \frac{V_{cd}}{R_s + R_x}, \]

and for the top is \( i_{\text{top}} = \frac{V_{cd}}{R_1 + R_2} \)

Now, \( V_{ad} = i_{\text{top}} R_2 \) and \( V_{bd} = i_{\text{bottom}} R_x \)

But \( V_{ad} = V_{bd} \), therefore, 

\[ \frac{R_2}{R_1 + R_2} = \frac{R_x}{R_s + R_x} \Rightarrow \frac{R_x}{R_s} = \frac{R_2}{R_1} \]

So, knowing resistances 1,2 and s, one knows resistance x.
Light bulbs in series and parallel

Manufacturer spec: Watts at 110V
More Watts = less resistance.
Less Watts = more resistance.

\[
\text{Power} = \frac{V^2}{R}
\]

Série: 
\[75\text{W} \quad 50\text{W}\]
i is given. More resistance (less spec Watts), more power!

Parallèle: 
\[75\text{W} \quad 50\text{W}\]
\(V\) is given. Less resistance (more spec Watts), more power!
Summary:

• We solve circuits involving resistors “by taking a walk”, as we discussed.
• Circuits in series share the same current, in parallel the same voltage. Beware at the time of computing powers!
• Voltmeters and Ammeters in real life behave as resistors, and therefore have non-vanishing currents and voltages across their leads respectively, that ideally would be zero.