Physics 2113
Lecture 12: WED 24 SEP

CH24: Electric Potential

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Definition of electric potential:

Potential energy of a system per unit charge \( V = \frac{U}{q} \)

Units… Units…

\[ V_f - V_i = \frac{U_f - U_i}{q} = \int_{i}^{f} \vec{E} \cdot d\vec{s} \]

Units: \([V] = \frac{\text{Joule}}{\text{Coulomb}} \equiv \text{Volt}\]

Unit most commonly used for electric fields

\[ \Delta V = \frac{\Delta U}{q} \Rightarrow \Delta U = q \Delta V \]

\( eV = \text{electron-volt}, \) the energy that an electron acquires when placed in an electric potential of 1V

\[ 1 \ eV = (1.6 \times 10^{-19} \text{ C})V = 1.6 \times 10^{-19} \text{ J} \]
Potential due to a point charge:

Change in potential in bringing q0 from infinity to a point P.

\[ \Delta V = -\int_A^B E \cdot d\vec{s} = -\int_{\infty}^{r} E \, dr' = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0 r'^2} \, dr' = -\frac{q}{4\pi\varepsilon_0} \int_{\infty}^{r} \frac{1}{r'^2} \, dr' = -\frac{q}{4\pi\varepsilon_0} \left[ -\frac{1}{r'} \right]_{\infty}^{r} = \frac{q}{4\pi\varepsilon_0 r} \]

- If charge is negative, then potential is negative.
- At infinity, potential is zero, as expected for isolated sources.
- For several charges, potentials are simply superposed:

\[ V = \sum_i V_i = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i} \]

As was the case with electric fields, the potential outside a charged sphere or charged shell coincides with the potential of a point charge at the origin.
(a) In Fig. 24-9a, 12 electrons (of charge \(-e\)) are equally spaced and fixed around a circle of radius \(R\). Relative to \(V = 0\) at infinity, what are the electric potential and electric field at the center \(C\) of the circle due to these electrons?

**KEY IDEAS**

1. The electric potential \(V\) at \(C\) is the algebraic sum of the electric potentials contributed by all the electrons. (Because electric potential is a scalar, the orientations of the electrons do not matter.)
2. The electric field at \(C\) is a vector quantity and thus the orientation of the electrons is important.

**Calculations:** Because the electrons all have the same negative charge \(-e\) and are all the same distance \(R\) from \(C\), Eq. 24-27 gives us

\[
V = -12 \frac{1}{4\pi \varepsilon_0} \frac{e}{R} \quad \text{(Answer) (24-28)}
\]

Because of the symmetry of the arrangement in Fig. 24-9a, the electric field vector at \(C\) due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at \(C\),

\[
\vec{E} = 0. \quad \text{(Answer)}
\]

**Reasoning:** The potential is still given by Eq. 24-28, because the distance between \(C\) and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.
Potential due to a Dipole

At point P, the total potential is due to that of +q and -q

\[
V = \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{r_+} - \frac{q}{r_-} \right) = \frac{q}{4\pi\varepsilon_0} \left( \frac{r_- - r_+}{r_+ r_-} \right)
\]

If point P is at “infinity” or \( r >> d \), then in this approximation we can consider fig (b):

\[
r_- - r_+ = d \cos \theta \quad \text{and} \quad r_- r_+ \approx r^2
\]

Then,

\[
V = \frac{q}{4\pi\varepsilon_0} \left( \frac{d \cos \theta}{r^2} \right)
\]

Electric dipole: defined as \( p = d \ q \)

\[
V = \frac{1}{4\pi\varepsilon_0} \left( \frac{p \cos \theta}{r^2} \right)
\]
CHECKPOINT 5

Suppose that three points are set at equal (large) distances $r$ from the center of the dipole in Fig. 24-10: Point $a$ is on the dipole axis above the positive charge, point $b$ is on the axis below the negative charge, and point $c$ is on a perpendicular bisector through the line connecting the two charges. Rank the points according to the electric potential of the dipole there, greatest (most positive) first.

\[ V = \frac{1}{4\pi\varepsilon_0} \left( \frac{p \cos \theta}{r^2} \right) \]
Induced dipole

As we discussed, some molecules (H2O) have a permanent dipolar nature. Others do not, the distribution of electrons is spherical and its center coincides with the center of the nucleus.

But when a field is applied, a dipole moment is induced
Like for electric fields, you break it up into small pieces, treat each little piece like a point charge, and add up the resulting potentials. Unlike electric fields, you superpose the potentials as scalars, not vectors.

So it is messy, but a bit simpler.
Potential due to continuous distributions of charge

Strategy: same as for field calculations, break up into infinitesimal pieces, integrate. It is easier than for the field, since the potential is a scalar.

Example: charged rod

\[ \lambda = \frac{q}{L} \quad dq = \lambda \, dx \]

\[ dV = \frac{dq}{4\pi\varepsilon_0 r} = \frac{\lambda \, dx}{4\pi\varepsilon_0 \sqrt{a^2 + x^2}} \quad V = \int_0^L dV \]

\[ V = \frac{\lambda}{4\pi\varepsilon_0} \ln \left[ \frac{L + \sqrt{L^2 + a^2}}{a} \right] \]

Check: if \( a \to \infty \), then \([\ln] \to 1\), \( V \to 0 \)

Since the argument of log is greater than unity, \( V \) is always positive
Potential due to a charged disk

Consider a charged disk of radius $R$ with a uniform charge density. We wish to compute the potential at point $P$ lying on the central axis of the disk.

We consider a differential element of radius $R'$ and width $dR'$, enclosing a surface area $2\pi R' dR'$

Enclosed charge: $dq = \sigma (2\pi R' dR')$

This enclosed charge leads to the potential:

$$dV = \frac{dq}{4\pi \varepsilon_0 r}$$

We can then integrate this potential from $0$ to $R$ to get the net potential due to the disk:

$$V = \int_0^R \frac{dq}{4\pi \varepsilon_0 r} = \frac{\sigma}{2\varepsilon_0} \left( \left( z^2 + R^2 \right)^{1/2} - z \right)$$
Example

All the charge is at the same distance \( R \) from \( C \), so the potential at \( C \) is,

\[
V = \frac{1}{4\pi\varepsilon_0} \left( \frac{Q}{R} - \frac{6Q}{R} \right) = -\frac{5Q}{4\pi\varepsilon_0 R}
\]

All the charge is at the same distance from \( P \), so the potential at \( P \) is,

\[
V = \frac{1}{4\pi\varepsilon_0} \left( \frac{Q}{\sqrt{R^2 + z^2}} - \frac{6Q}{\sqrt{R^2 + z^2}} \right) = -\frac{5Q}{4\pi\varepsilon_0 \sqrt{R^2 + z^2}}
\]
Summary

• Like electric fields, potentials for configurations involving many charges or continuous charge distributions are obtained by superposing.
• But the superposition is a scalar one, so it is usually easier to do than superposing fields.
• Next class we will learn that one can obtain the fields from the potentials, so this simplifies calculations quite a bit.