Physics 2203, Fall 2012
Modern Physics

- Friday, Aug. 31st, 2012:
  Finish Ch. 2
- Announcements:
  • Tutorial session 4:30 pm Tuesday in Nicholson 102
  • Quiz today
  • Fall break canceled!
  • Monday is Labor Day.
Relativistic E, $E_K$, and Momentum

\[ E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma m_0 c^2 \equiv E_K + m_0 c^2 \]

\[ E_K = m_0 c^2 (\gamma - 1) = \gamma m_0 c^2 - m_0 c^2 \]

\[ E^2 = (pc)^2 + (mc^2)^2 \]

\[ E_K = \sqrt{p^2 c^2 + (mc^2)^2} - mc^2 \]

\[ \beta = \frac{\nu}{c} \equiv \frac{pc}{E} \]

\[ E = \gamma m_0 c^2 = Mc^2 \]

\[ M = \gamma m_0 \]
**Units**

Energy is measured in Joules. Convert J to eV

An electron volt (eV) is the energy to move an electron though one volt.

\[
1.0 \text{ eV} = e(1.0 \text{ V}) = \left(1.602 \times 10^{-19} \text{ C}\right)(1.0 \text{ V}) = 1.602 \times 10^{-19} \text{ J}
\]

\[
J = \frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ C}}
\]

**Example:** Rest energy of an electron

\[
m_0c^2(\text{electron}) = \frac{8.19 \times 10^{-15} \text{ J}}{1.602 \times 10^{-19} \text{ J}} = 5.11 \times 10^5 \text{ eV} = 0.511 \text{ MeV}
\]

\[
m_0c^2(\text{electron}) = 0.511 \text{ MeV}
\]

\[
m_0c^2(\text{neutron}) = 939.57 \text{ MeV}
\]

\[
m_0c^2(\text{proton}) = 938.3 \text{ MeV}
\]

Units for mass

\[
m_0 \rightarrow \frac{\text{MeV}}{c^2}
\]

\[
m_0(\text{electron}) = \frac{0.511 \text{ MeV}}{c^2}
\]
Units

Units for momentum: \( E^2 = (pc)^2 + (m_0c)^2 \)

\( pc \rightarrow \text{Energy} \rightarrow eV, KeV, MeV \)

\[ pc = \frac{m_0vc}{\sqrt{1 - v^2 / c^2}} \]

**Periodic Table notation:** \( _Z^AP \)

\( P \) The chemical element --H, He, Ar, etc.

\( A=Z+N \rightarrow Z \# \text{protons}, N \# \text{neutrons} \)

Atomic Mass unit \( u \), defined as \( \frac{1}{12} \) mass of \(^{12}C\)

\( 1u = 931.494013 Mev / c^2 \)

**Uranium** \( ^{238}U \rightarrow 238.0507u \)
A simple experiment: two blocks of wood with equal mass $m$ and kinetic energy $K$, are moving toward each other with velocity $v$. A spring placed between them is compressed and locks in place as they collide. Let's look at the conservation of mass-energy.

**Mass – Energy before:** \( E = 2mc^2 + 2K \)

**Mass – Energy after:** \( E = Mc^2 \)

Since Energy is conserved we have \( E = 2mc^2 + 2K = Mc^2 \)

\( M \) is greater than \( 2m \) because \( K \) went into mass.

\[ \Delta M = M - 2m = \frac{2K}{c^2} \]

*In Relativistic Mechanics*

Momentum and Total Energy are conserved!
Equivalence of Mass and Energy

Examples of Energy to Mass Exchange

• Ionization $H \rightarrow p + e : 13.6 \text{ eV: } \Delta M = \frac{13.6 \text{ eV}}{c^2}$

• Chemical Binding Energy $H_2O \rightarrow 2H + O : \sim 3 \text{ eV: } \Delta M = \frac{3 \text{ eV}}{c^2}$

• Fusion $^1H + ^1H \rightarrow ^2He + \text{ energy: } \Delta M = \frac{23.9 \text{ MeV}}{c^2}$

• Fission $^{232}Th \rightarrow ^{228}Ra + ^4He: \Delta M = \frac{4 \text{ MeV}}{c^2}$

• Nuclear reactions: $p + p \rightarrow p + p + p + p : \text{ antiproton}$
The antiproton $\bar{p}$ was discovered in 1956 through the following reaction

$$p + p \rightarrow p + p + p + \bar{p}$$

Find the minimum kinetic energy of the accelerated proton in the figure. This is called the **threshold** kinetic energy, for which the **final particles move together** as if they were a single unit.

**Conservation of Energy**

$$E_p + m_p c^2 = 4E'_p$$

**Conservation of Momentum**

$$p_p = 4p'_p$$

*Here $E'_p$ and $p'_p$ are for each of the 4 particles*

$$E'_p = \frac{E_p + m_p c^2}{4}$$

$$\sqrt{E_p^2 - (m_p c^2)} = 4\sqrt{E_{p'}^2 - (m_p c^2)}$$

$$E_p^2 - (m_p c^2) = \left[\left(E_p + m_p c^2\right)^2 - 16(m_p c^2)\right]$$

$$E_p = 7m_p c^2$$

$$K = E_p - m_p c^2 = 6m_p c^2 = 6(938 \text{ MeV}) = 5628 \text{ MeV} = 5.628 \text{ GeV}$$
A neutral K meson (mass 497.7 MeV/c²) is moving with a kinetic energy of 77.0 Mev. It decays into a π meson (mass 139.6 MeV/c²) and another particle (A) of unknown mass. The π meson is moving in the direction of the original K meson with a momentum 381.6 MeV/c. (a) Find the momentum and total energy of the unknown particle. (b) Find the mass of the unknown particle.

**Total Energy** and **Momentum** of K meson are

\[ E_K = K_K + m_K c^2 = 77.0 \text{MeV} + 497.7 \text{ MeV} = 574.7 \text{ Mev} \]

\[ p_K = \frac{1}{c} \sqrt{E_K^2 + (m_K c^2)^2} = 287.4 \text{ Mev} / c \]

**Total Energy** of π meson is

\[ E_\pi = \sqrt{(c p_\pi)^2 + (m_\pi c^2)^2} = \sqrt{(381.6 \text{MeV})^2 + (139.6 \text{MeV})^2} = 406.3 \text{ MeV} \]

**Conservation of momentum** requires \( p_K = p_\pi + p_A \)

\[ p_A = p_K - p_\pi = (287.4 - 381.6) \text{ MeV} / c = -94.2 \text{ MeV} / c \]

**Conservation of Energy** requires \( E_K = E_\pi + E_A \)

\[ E_A = E_K - E_\pi = (574.7 - 406.3) \text{ MeV} = 168.4 \text{ MeV} \]

(b) find mass: \( m_A c^2 = \sqrt{E_A^2 + (c p_A)^2} = \sqrt{(168.4)^2 + (94.2)^2 \text{ MeV}} = 139.6 \text{ MeV} \)
Equivalence of Mass and Energy: H atom

Binding Energy of the Hydrogen Atom: The binding energies of electrons to the nuclei of atoms are much smaller than nuclear binding energies. The binding energy for an electron to a proton (Bohr model) is 13.6 eV. How much mass is lost when an electron and proton from a hydrogen atom?

This is again a before and after problem: Before you have an isolated electron and proton. After you have a H atom whose binding energy is 13.6 eV.

\[
\Delta mc^2 = E(binding) = 13.6eV
\]

\[
m_Hc^2 \approx m_p c^2 = 938.3MeV
\]

\[
\frac{\Delta m}{m_H} \approx 1.4 \times 10^{-8}
\]
Equivalence of Mass and Energy: Dissociation

(a) How much lighter is a molecule of water than two hydrogen atoms and an oxygen atom? The binding energy of water is $\sim 3eV$. 

(b) Find the fractional loss of mass per gram of water formed.

(c) Find the total energy released (mainly as heat and light) when 1 gram of water is formed?

\[ \Delta M = (m_H + m_H + m_O) - M_{H_2O} = \frac{E(binding)}{c^2} \]

\[ \Delta M = \left(3.0eV\right) \left(1.6 \times 10^{-19} J / eV\right) = 5.3 \times 10^{-36} \text{ kg} \quad \text{Really small!} \]

\[ \frac{\Delta M}{M_{H_2O}} = \frac{E(binding)}{M_{H_2O}c^2} \]

\[ M_{H_2O} = 18u : \quad u \text{ is } 1/12 \text{ of the mass of Carbon (6 protons, 6 neutrons)} \]

\[ M_{H_2O} = 18 \left(1.66 \times 10^{-27} \text{ kg}\right) \]

\[ \frac{\Delta M}{M_{H_2O}} = \frac{5.3 \times 10^{-36} \text{ kg}}{18 \left(1.66 \times 10^{-27} \text{ kg}\right)^2} = 1.8 \times 10^{-10} \quad \text{Still small!} \]

\[ E = \Delta mc^2 = \left(1.8 \times 10^{-10}\right) \left(10^{-3} \text{ kg}\right) \left(3 \times 10^8 \text{ m/s}\right)^2 = 16kJ \quad \text{Big!} \]
Equivalence of Mass and Energy: Fusion

**Fusion:** Energy is gained by taking two light atoms and combining them into an atom with a heavier nuclei-later this semester. For example:

\[
\frac{1}{2}H + \frac{1}{2}H = \frac{4}{2}He + \text{Energy}
\]

\[
\frac{E}{c^2} = \text{mass} \left( \frac{1}{2}H + \frac{1}{2}H \right) - \text{mass} \left( \frac{4}{2}He \right)
\]

\[
E = \left( 3751.226 - 3727.379 \right) \text{MeV}
\]

\[
E = 23.9 \text{MeV}
\]
**Fission Reactions:** The decay of a heavy radioactive nucleus at rest into several lighter particles emitted with large kinetic energies is a great example of mass-energy conversion. A nucleus of mass \( M \) undergoes \textit{fission} into particles with masses \( M_1, M_2, \) and \( M_3 \), with speeds of \( u_1, u_2, \) and \( u_3 \).

The conservation or Relativistic Energy Requires that

\[
M c^2 = \frac{M_1 c^2}{\sqrt{1 - \frac{u_1^2}{c^2}}} + \frac{M_2 c^2}{\sqrt{1 - \frac{u_2^2}{c^2}}} + \frac{M_3 c^2}{\sqrt{1 - \frac{u_3^2}{c^2}}}
\]

\textit{This is a very important equation to remember}

The Equation above is true if

\( M > (M_1 + M_2 + M_3) \)

\textit{Disintegration Energy} \( Q \) defined

\[
Q = [M - (M_1 + M_2 + M_3)] c^2
\]

\textit{Example} in our text (pg 61)

\( ^{232}\text{Th} \rightarrow ^{228}\text{Ra} + ^4\text{He} \)

The offspring have 4 Mev Kinetic Energy

\begin{align*}
\text{Atom} & & \text{Mass (in u)} \\
^{232}\text{Th} & & 232.038 \quad \text{Initial} \\
^{228}\text{Ra} & & 228.031 \\
^4\text{He} & & + 4.003 \\
\{ & & \{ \frac{232.034}{0.004} \}
\end{align*}

\text{Initial Total} \quad \text{Difference}

\text{Appendix D:} \ 1 \ u = 1.66 \times 10^{-27} \text{kg:}

\[
\Delta M = (0.004 u) \left(1.7 \times 10^{-27} \text{kg}\right) u
\]

\[
\Delta M = 7 \times 10^{-30} \text{ kg}
\]

\[
\Delta Mc^2 = 6.3 \times 10^{-13} J = 4 \text{ MeV}
\]
Change in the Solar Mass: Compute the rate at which the Sun is losing mass, given that the mean radius $R$ of the Earth’s orbit is $1.50 \times 10^8$ km and the intensity of the solar radiation on the Earth is $1.36 \times 10^3$ W/m$^2$ (called solar constant).

Assume that the sun radiates uniformly as a sphere of radius $R$

$$P = (\text{area of sphere})(\text{solar constant})$$

$$P = (4\pi R^2)(1.36 \times 10^3 \text{ W} / \text{m}^2)$$

$$P = 4\pi (1.50 \times 10^{11} m)^2 (1.36 \times 10^3 \text{ W} / \text{m}^2)$$

$$P = 3.85 \times 10^{26} \text{ J} / \text{s}$$

This is about 4 million metric tons

$$m = \frac{E(\text{lost})}{c^2}$$

$$m = \frac{3.85 \times 10^{26} \text{ J} / \text{s}}{(3 \times 10^8 \text{ m} / \text{s})^2} = 4.3 \times 10^9 \text{ kg} / \text{s}$$

If this rate of mass loss remains constant and with a fusion mass-to-energy conversion efficiency of about 1 percent, the Sun’s present mass of $\sim 2.0 \times 10^{30}$ kg will “only” last for about $10^{11}$ more years!
In Classical Mechanics we can’t have a particle with m=0, because E and p are zero.

In Relativistic Mechanics we can have a particle with m=0

\[ E = \gamma m_0 c^2 \]
\[ \vec{p} = \gamma m_0 \vec{u} \]

\[ E^2 = (pc)^2 + (m_0 c^2)^2 \]
\[ \beta = \frac{v}{c} = \frac{pc}{E} \]

Now look at what happens to the equations in the right box if m=0.

\[ E = pc \]
\[ \beta = 1 \]

Now look at what happens to the equations in the left box if m=0.

\[ E = \gamma m_0 c^2 \approx \infty \cdot 0 \]
\[ \vec{p} = \gamma m_0 \vec{u} \approx \infty \cdot 0 \]
In Relativistic Mechanics we can have a particle with m=0. It is a photon. Chapter 4

A photon is a consequence of Quantum Mechanics (Einstein). It is the particle nature or light waves. The energy comes in terms of photons each with a discrete energy and momentum depending upon the wave length of the light.

You must have done the experiment with a supported plate with one side polished and the other side black. When light shines on this the plate rotates because of the momentum change from the photons.

*Our* example of ionization of H

\[ \gamma + H \rightarrow e + p \]

Photons are massless particles v=c: E=pc

**Neutrinos** were believed to be massless, but recent experiments show m~10^{-5}m_e.

**Gravitons** are related to gravity the way photons are to light should have m=0. No experimental evidence--LIGO
Quiz
Quiz #1

Prove that by using the new definition of momentum that momentum is conserved in the $S'$ coordinate system.

Consider the inelastic collision of two equal mass objects shown in the figure.

Now consider the $S'$ system moving with object #1 at a velocity of $v$.

\[
\vec{p} = \frac{m\vec{u}}{\sqrt{1 - \frac{u^2}{c^2}}}
\]

\[
u' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}}
\]

we need to find $v_2'$ and $V'$ using the transform

\[
v_2' = \frac{v_2 - v}{1 - \frac{v_2 v}{c^2}} = \frac{-v - v}{1 - \frac{c^2}{c^2}} = -2v
\]

\[
V' = \frac{V - v}{1 - \frac{V v}{c^2}} = \frac{0 - v}{1 - (0)v} = -v
\]