Announcement:

- There is a class on Saturday (10/27) 1:30-3:00

Lecture Notes:
http://www.phys.lsu.edu/classes/fall2012/phys2101-6/
For 2 particles the magnitude of the attractive force between them is

\[ |F| = G \frac{m_1 m_2}{r^2} \]

\( m_1 \) and \( m_2 \) are masses and \( r \) is distance between them and...

\[ G = 6.67 \times 10^{-11} \, N \cdot m^2 / kg^2 \]

\[ = 6.67 \times 10^{-11} \, m^3 / kg \cdot s^2 \]

Gravitational Constant

(\( \neq g \), \( \neq 9.8 \, m/s^2 \))

\[ g = G \frac{m_{\text{earth}}}{r_{\text{earth}}^2} \]
Quick Review: Gravitation Notes

1) All objects -- independent of each other (Newton’s 3\textsuperscript{rd} Law)

2) Gravitational Force is a VECTOR - unit vector notation

\[ \vec{F}_{12} = G \frac{m_1 m_2}{r_{12}^2} \hat{r}_{12} \]

Force on \( m_1 \) due to \( m_2 \)

\[ \hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} \]

\[ \vec{F}_{21} = G \frac{m_1 m_2}{r_{21}^2} \hat{r}_{21} \]

Force on \( m_2 \) due to \( m_1 \)

\[ \hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|} = -\hat{r}_{12} \]

\[ \vec{F}_{21} = -\vec{F}_{12} \]

3) Principle of superposition

\[ \vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + ... + \vec{F}_{1n} = \sum_{i=1}^{n} \vec{F}_{1i} \]

VECTOR ADDITION!!

4) A uniform spherical shell of matter attracts an object on the outside as if all the shell’s mass were concentrated at its center (note: this defines the position)

\[ \text{height} = R_E + h \]
**Gravitation and the earth**

\[ F_g = \left( G \frac{M_E}{R_E^2} \right) m_{\text{apple}} = m_{\text{apple}} g \]

Net force points towards center of earth

**g differs around the earth**  (equator-9.780 & north pole-9.832 m/s\(^2\))

1) **Earth is not a perfect sphere** - height \((R_E\) is not constant):
   - On Mount Everest (8.8 km) \(g=9.77\) m/s\(^2\) (0.2% smaller)
   - At Equator earth bulges by 21 km

2) **Earth is not uniform density**: “gravity irregularities” \((10^{-6}-10^{-7})g\)
   gravimeters can measure down to \(10^{-9}g\)

2) **Earth is rotating**: centripetal force makes apparent weight change

At poles:
\[
W - mg = 0 \\
W = mg
\]

At equator:
\[
W - mg = -m\left( \frac{v^2}{R_E} \right) \\
W = m\left( g - \frac{v^2}{R_E} \right) \\
\text{Weight is less} \ (0.3\%)
Gravity and Spheres

• A uniform spherical shell of matter attracts a particle that is outside the shell as if all the shell’s mass were concentrated at its center.

• A uniform shell of matter exerts no net gravitational force on a particle located inside it.
**Gravitation Inside the Earth**

Newton proved that the net gravitational force on a particle by a shell depends on the position of the particle with respect to the shell.

If the particle is inside the shell, the net force is zero.

If the particle is outside the shell, the force is given by:

\[ F_1 = G \frac{m_1 m_2}{r^2}. \]

Consider a mass \( m \) inside the Earth at a distance \( r \) from the center of the Earth. If we divide the Earth into a series of concentric shells, only the shells with radius less than \( r \) exert a force on \( m \). The net force on \( m \) is:

\[ F = \frac{G m M_{\text{ins}}}{r^2}. \]

Here \( M_{\text{ins}} \) is the mass of the part of the Earth inside a sphere of radius \( r \):

\[ M_{\text{ins}} = \rho V_{\text{ins}} = \rho \frac{4\pi r^3}{3} \]

\[ F = \frac{4\pi G m \rho}{3} r \]

\( F \) is linear with \( r \).
Problem 13-13

With what gravitational force does the hollowed-out sphere attract a small sphere of mass m?
Gravitational Potential Energy

From Section 8.3 ⇒ \[-\Delta U = W_{\text{done by force}}\] ⇒ Conservative force-path independent

\[\Delta U_g = -W_{\text{done}} = -\int_{x_i}^{x_f} \vec{F}_g \cdot d\vec{x}\]

At Earth’s surface, \(F_g \approx \text{const.}\)

\[\Delta U_g = -W_{\text{done}} = -m(-g)\int_{y_i}^{y_f} dy = mg\Delta y\]

\[W = \int_{r_i}^{r_f} \left(-G \frac{mM}{r^2}\right) dr = -GmM \int_{r_i}^{r_f} \left(\frac{1}{r^2}\right) dr\]

If we define \(U = 0\) at \(\infty\), then the work done by taking mass \(m\) from \(R\) to \(\infty\)

\[U_\infty - U(r) = -W = -GmM\left[0 - \left(-\frac{1}{r}\right)\right]\]

\[F(r) = -\frac{dU(r)}{dr}\]

\[\frac{d}{dr}\left(-\frac{GmM}{r}\right) = -\frac{GmM}{r^2}\]

Note:

1) As before, Grav. Pot. Energy decreases as separation decreases (more negative)
2) Path independent
3) MUST HAVE AT LEAST TWO PARTICLES TO POTENTIAL ENERGY (& force)
4) Knowing potential, you can get force….
Gravitational Potential Energy

What is the gravitational Potential Energy of the three-particle system?
**Example**

Three spheres with mass $m_A$, $m_B$, and $m_C$. You move sphere B from left to right.

How much work is done by the gravitational force?

How much work do you do on sphere B?
Gravitational Potential Energy

The figure gives the potential energy $U(r)$ of a projectile, plotted outward from the surface of a planet of radius $R_s$. If the projectile is launched radically outward from the surface with a mechanical energy of $-2.0 \times 10^{-9}$ J, what are (a) its kinetic energy at radius $r = 1.25 R_s$ and (b) its turning point in terms of $R_s$?
**Escape Speed**

**Escape speed:** minimum speed \(v_{\text{escape}}\) required to send a mass \(m\), from mass \(M\) and position \(R\), to infinity, while coming to rest at infinity.

At infinity: \(E_{\text{mech}} = 0\) because \(U = 0\) and \(KE = 0\)

Thus any other place we have:

\[
E_{\text{mech}} = (KE + U_g) = 0 \quad \Rightarrow \quad E_{\text{mech}} = \left(\frac{1}{2} mv^2 - \frac{GmM}{R}\right) = 0 \quad \Rightarrow \quad v_{\text{escape}} = \sqrt{\frac{2GM}{R}}
\]

Earth = 11.2 km/s (25,000 mi/hr)

**Escape speed:**
- Moon = 2.38 km/s
- Sun = 618 km/s

<table>
<thead>
<tr>
<th>Body</th>
<th>Mass (kg)</th>
<th>Radius (m)</th>
<th>Escape Speed (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ceres</td>
<td>(1.17 \times 10^{21})</td>
<td>(3.8 \times 10^5)</td>
<td>0.64</td>
</tr>
<tr>
<td>Earth’s moon</td>
<td>(7.36 \times 10^{22})</td>
<td>(1.74 \times 10^6)</td>
<td>2.38</td>
</tr>
<tr>
<td>Earth</td>
<td>(5.98 \times 10^{24})</td>
<td>(6.37 \times 10^6)</td>
<td>11.2</td>
</tr>
<tr>
<td>Jupiter</td>
<td>(1.90 \times 10^{27})</td>
<td>(7.15 \times 10^7)</td>
<td>59.5</td>
</tr>
<tr>
<td>Sun</td>
<td>(1.99 \times 10^{30})</td>
<td>(6.96 \times 10^8)</td>
<td>618</td>
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<tr>
<td>Sirius B</td>
<td>(2 \times 10^{30})</td>
<td>(1 \times 10^7)</td>
<td>5200</td>
</tr>
<tr>
<td>Neutron star</td>
<td>(2 \times 10^{30})</td>
<td>(1 \times 10^4)</td>
<td>(2 \times 10^5)</td>
</tr>
</tbody>
</table>
Problem
A projectile is fired vertically from the Earth’s surface with an initial speed of 10 km/s (22,500 mi/hr)

Neglecting air drag, how far above the surface of Earth will it go?

\[ R_E = 6380 \cdot km \]
\[ GM_E = 4 \times 10^{14} \cdot m^3/s^2 \]
Geosynchronous Satellite: One that stays above same point on the earth (only at equator) TV, weather, communications…….

How high must it be?

Only force is gravity:

\[- \frac{G m_{sat} M_E}{r_{sat}^2} = - m_{sat} \frac{v^2}{r_{sat}}\]

for synchronous orbit, period of satellite and earth must be the same

\[v = \frac{2 \pi r}{T}\]

\[T = 1 \text{ day}\]

\[r_{sat}^3 = \frac{G M_E}{4 \pi^2} T^2\]

\[\Rightarrow r_{sat} = \frac{GM_E}{4 \pi^2} T^2\]

\[\Rightarrow r_{sat} = 42,300 \text{ km}\]

\[\Rightarrow \text{height above earth surface} = 35,000 \text{ km} \sim 6 R_E\]

Geosynchronous Satellite = 22,500 miles high

Spy Satellite (polar orbit) = 400 miles high

Space Shuttle 186 miles high
Newton’s equation: \( F = ma \)
- \( F \) is gravitational force
- \( a \) is centripetal acceleration

\[
-\frac{1}{2}U_g = \frac{GmM}{2r} = m\frac{v^2}{2} = \frac{1}{2}mv^2 = KE
\]

**Mechanical Energy**

\[
E = U + KE = U + \left(-\frac{1}{2}U\right) = \frac{1}{2}U
\]

\[
E_s = -\frac{GMm}{2r}
\]

\[
KE_s = \frac{GMm}{2r}
\]

\[
E_s = -KE_s
\]
Satellites: Energy graph

\[ E_s = -KE_s \]
\[ KE_s = -\frac{1}{2}U \]

Mechanical Energy

\[ E_{\text{mech}} = U + KE \]
\[ = -\frac{GMm}{2r} \]
\[ KE = \frac{GMm}{2r} \]
\[ U = -\frac{GMm}{r} \]
Satellites: Orbits and Energy

**Problem:** Two satellites, A and B, both of mass $m=125 \text{ kg}$, move in the same circular orbit of radius $r= 7.87 \times 10^6 \text{ m}$ around the Earth but in opposite senses of rotation and therefore on a collision course.

(a) Find the total mechanical energy $E_A + E_B$ of the two satellites + Earth before the collision.

(b) If the collision is completely inelastic so that the wreckage remains as one piece, find the total mechanical energy immediately after the collision.

© Just after the collision, is the wreckage falling directly toward the Earth’s center or orbiting around the earth?