STABILIZING A COLD DISK WITH A $1/r$ FORCE LAW

Joel E. Tohline
Department of Physics and Astronomy
Louisiana State University

Massive "dark" halos are currently believed to surround individual galaxies and systems of galaxies primarily because: (a) individual galaxies exhibit "flat" rather than Keplerian rotation curves (see the review by Faber and Gallagher 1979); (b) the measured ratio of mass to light in systems of galaxies continues to increase roughly linearly with radius as one observes larger and larger systems (Rood 1982); and (c) rapidly spinning "cold" stellar disks are not dynamically stable if they are fully self-gravitating (Ostriker and Peebles 1973). It can be shown, however, that properties (a) and (b) of galaxies and systems of galaxies can be explained without invoking dark halos if one assumes that the force of gravity has the form

$$F = - \frac{G m n}{r^2} \left[ 1 + \frac{r}{a} \right]$$

(1)

where $r$ is the distance between the attracting bodies of masses $M$ and $m$, and the scale $a$ must assume a value $a \sim 1$ kpc. (See Milgrom 1982 and Bekenstein, these proceedings, for a discussion along similar lines.) Newton's Law is retrieved on scales $\ll a$ but the force is proportional to $1/r$ on scales $\gg a$. Equation (1) should not be adopted in place of Newtonian gravity to explain the dynamical properties of galaxies unless it also explains property (c) without invoking the presence of dark halos.

Here a dimensionless form of Equation (1)

$$f = - \frac{1}{\eta^2} \left[ 1 + \eta \right]$$

(2)

where $\eta = r/a$, has been used to describe the gravitational attraction between individual particles in an "n-body" computer code and the stability of cold stellar disks has been tested on different scales. A softening length $\delta \eta = 0.03 \eta_{\text{max}}$ ($\eta_{\text{max}}$ is defined below) has also been added to each $\eta$ in Equation (2) in order to soften direct 2-body encounters.
The following initial disk model has been chosen for each evolution:
1.) 512 particles were placed in a disk of zero initial thickness, in eleven concentric rings so that the disk of radius $\eta_{\text{max}}$ had a uniform surface density; 2.) Each particle was given a circular velocity consistent with centrifugal balance in the self-gravitating disk; 3.) Each particle was given an initial $z$-velocity $v_z$ randomly chosen in the range $-v_z \text{ max} < v_z < +v_z \text{ max}$ where $v_z$ was, for a given evolution, chosen to be a constant fraction of the local initial circular velocity throughout the disk. A larger $v_z \text{ max}$ produced a "hotter" initial disk.

A number of models, differing only in their initial values of $\eta_{\text{max}}$ and $v_z \text{ max}$, were run for 2-3 rotation periods (measured in terms of the disk's initial central rotation period) and the stability of each disk was examined. The most obvious result was evident in plots of particle positions in the equatorial plane of the rotating system: Models having $\eta_{\text{max}} = 0.01$ developed strong nonaxisymmetric structure in much less than one rotation period (this is exactly as was observed by Ostriker and Peebles 1973) while models having $\eta_{\text{max}} = 100.0$ remained axisymmetric throughout each extended evolution. In addition to this rather subjective measure of relative disk stability, the quantity $\gamma = (\text{Random kinetic energy})/(\text{Rotational kinetic energy})$ was evaluated throughout each evolution. (In terms of the familiar criterion proposed by Ostriker and Peebles [1973], a disk in virial equilibrium should remain stable only if $\gamma \leq 2.6$.) Disks having $\eta_{\text{max}} = 0.01$ heated up monotonically with time during evolutions from initial states of $\gamma = 0.3$ and $\gamma = 0.05$. In contrast to this, disks having $\eta_{\text{max}} = 100.0$ did not heat up at all from initial states of $\gamma = 0.3$ and $\gamma = 0.05$.

It appears, then that cold stellar disks are dynamically stable if the force governing long range interactions between stars has a $1/r$ dependence rather than a $1/r^2$ dependence. It is extremely curious that the same force (Equation [1]) that has been empirically formulated to explain properties (a) and (b) of galaxies as listed above will also stabilize a cold axisymmetric disk if the disk's radius is greater than, or on the order of, 100 times the scale "a". Perhaps properties (a), (b), and (c) are not indicating the existence of dark halos but are instead pieces of empirical data that are trying to tell us: "Gravity has a $1/r$ dependence on the scale of galaxies."

REFERENCES