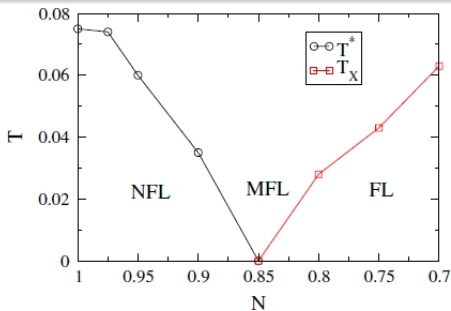


# The Role of the Van Hove Singularity in the Quantum Criticality of the Hubbard Model?

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# Overview: Quantum Critical Point in Hubbard Model



Vidhyadhiraja *et. al* PRL 102, 206407 (2009)

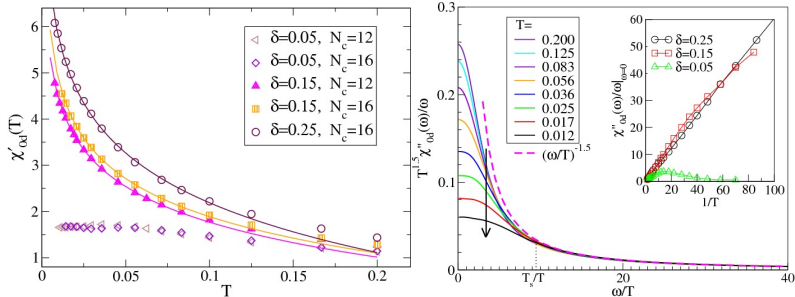
- Hubbard model on square lattice

$$\mathcal{H} = - \sum_{ij} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

- A quantum critical point exist in the Hubbard model
- Characterized by quasi-particle weight going to zero
- At finite temperatures, MFL separates a NFL phase from FL phase

LSU

# Scaling behavior of $d$ -wave susceptibility



Yang et. al PRL, 106, 047004 (2011)

- At QCP, pairing polarization decays algebraically  $\chi'_{0d}(T) \propto \frac{1}{\sqrt{T}}$
- *Scaling* -  $T^{1.5}\chi''_{0d}(\omega)/\omega$  curves for different  $T$  fall on top of each other

$$T^{1.5}\chi''_{0d}(\omega)/\omega = H(\omega/T) = (\omega/T)^{-1.5}$$

Kramers-Krönig -

$$\chi'_{0d}(T) = \frac{1}{\pi} \int d\omega \chi''_{0d}(\omega)/\omega = \frac{1}{\pi} \int dx H(x) \times \frac{1}{\sqrt{T}} \propto \frac{1}{\sqrt{T}}$$

- Bethe-Salpeter equation for  $\chi$

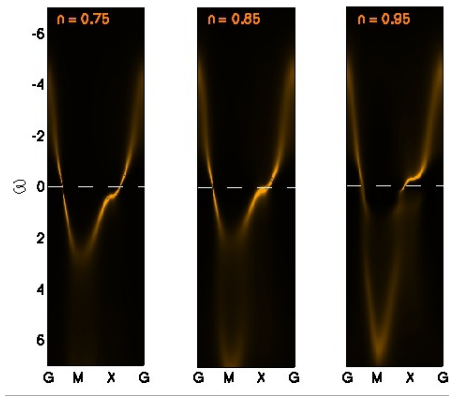
The diagram illustrates the Bethe-Salpeter equation for the susceptibility  $\chi$ . On the left, a circle labeled  $\chi$  contains three vertical lines representing fermion propagators. On the right, the equation is shown as a fraction. The numerator is a circle labeled  $\chi_0$ . The denominator consists of a vertical line, a minus sign, a square labeled  $U$ , and a circle labeled  $\chi_0$ .

- BCS equation for transition temperature  $T_c$

The diagram illustrates the BCS equation for the transition temperature  $T_c$ . It shows a vertical line equal to a square labeled  $U$  multiplied by a circle labeled  $\chi_0$ .

- $\chi'_{0d}(T) \propto \frac{1}{\sqrt{T}} \Rightarrow T_c \sim U^2$

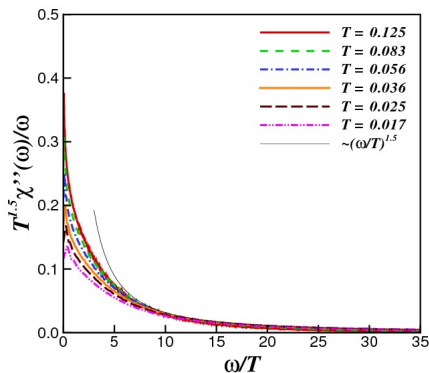
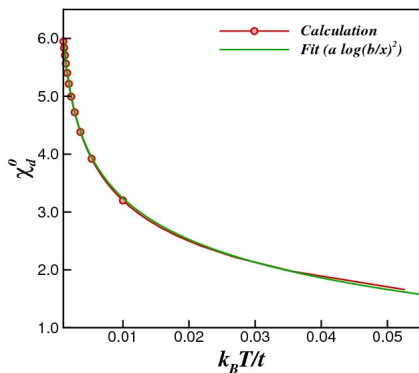
# Energy Dispersion



- vHS is pinned to the Fermi level near anti-nodal point X ( $\pi, 0$ ) at the QCP!
- New result - pinning at *finite* filling

*Role of vHS in the quantum criticality of Hubbard model?*

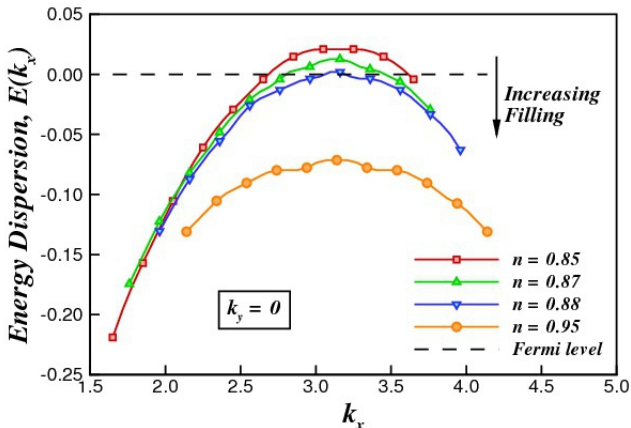
# Bare $d$ -wave susceptibility does not show good scaling!



$$\chi'_{0d}(T) = \frac{1}{\beta} \sum_{\mathbf{k}, i\omega_n} g_d^2(\mathbf{k}) G(\mathbf{k}, i\omega_n) G(-\mathbf{k}, -i\omega_n) = \sum_{\mathbf{k}} g_d^2(\mathbf{k}) \left( \frac{1 - 2n_F(\epsilon_{\mathbf{k}})}{2\epsilon_{\mathbf{k}}} \right)$$

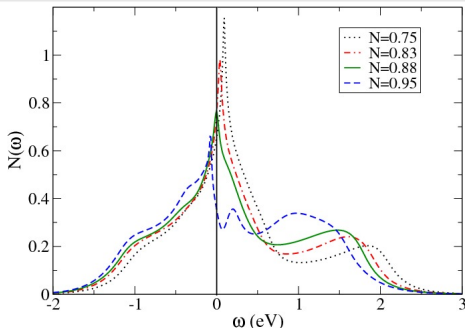
- For standard dispersion  $\epsilon(\mathbf{k}) = -2t(\cos k_x + \cos k_y)$ , DOS  $N(\omega) \sim \log |\omega|$
- By converting sum to integral cutoff by  $T$ , we get  $\chi'_{0d} \sim -(\log T)^2$

# Closer look at dispersion shows that vHS is flatter than quadratic!



- Dispersion around the Fermi vector along the anti-nodal direction

# Critical DOS is ALGEBRAICALLY singular, not logarithmic!



- Peaks in DOS are algebraic near  $\omega = 0$  (Using analytic continuation of self energy, Wang et. al PRB 80, 045101 (2009))

$$N(\omega) \sim \frac{1}{(\omega - \omega_p)^\alpha},$$

where,  $0 < \alpha < 0.5$  and  $\omega_p$  is location of maximum of peak

- Peaks moves through  $\omega = 0$  at quantum critical filling
- Quantum critical  $N(\omega)$  shows low  $\omega$  particle-hole symmetry

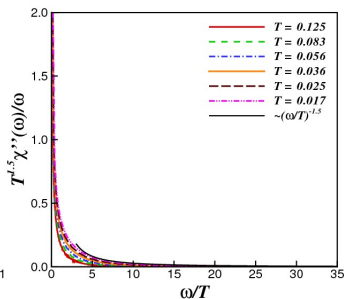
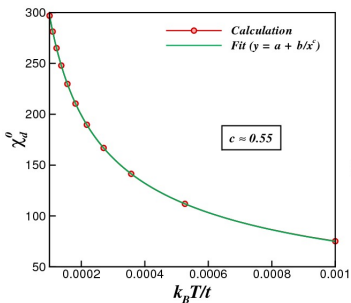


# Quartic dispersion show algebraic decay!

- Toy Model with Quartic Dispersion

$$\epsilon(\mathbf{k}) = -\frac{4}{\pi^4} ((|k_x| - \pi)^4 - k_y^4)$$

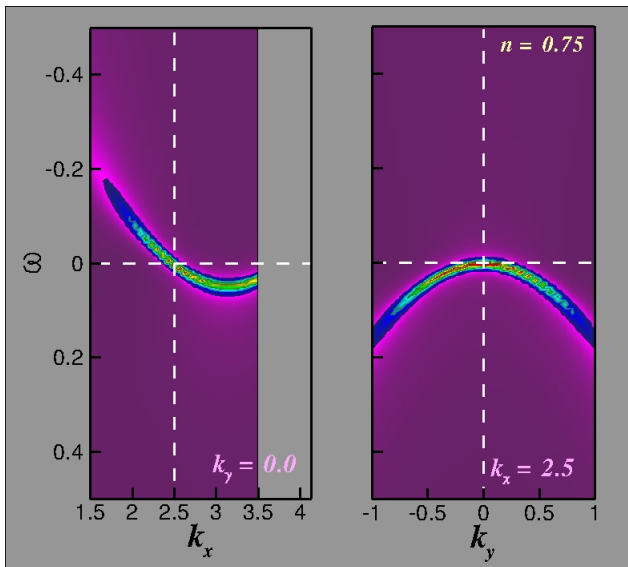
- Similar argument shows that for  $N(\omega) \sim \frac{1}{\sqrt{\omega}}$ ,  $\chi_d^0 \sim \frac{1}{\sqrt{T}}$



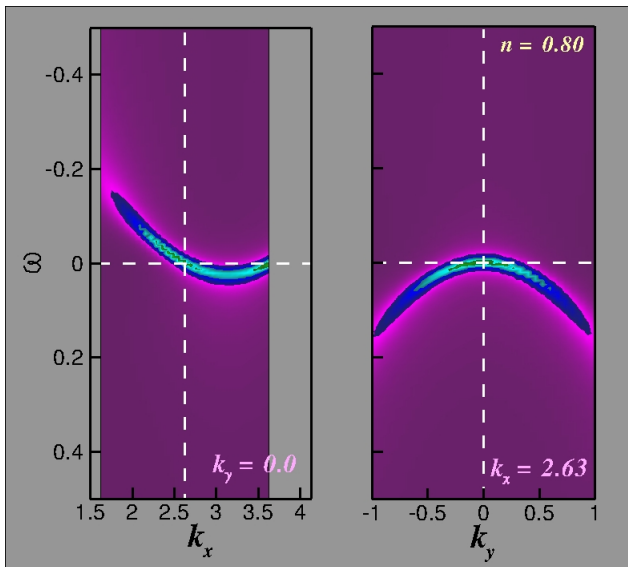
# Effect of $t'$ ( $t' = -0.1$ )

- vHS pinned to Fermi level for larger range of doping
- Affects transport properties like linear  $T$  resistivity, etc. for larger doping range

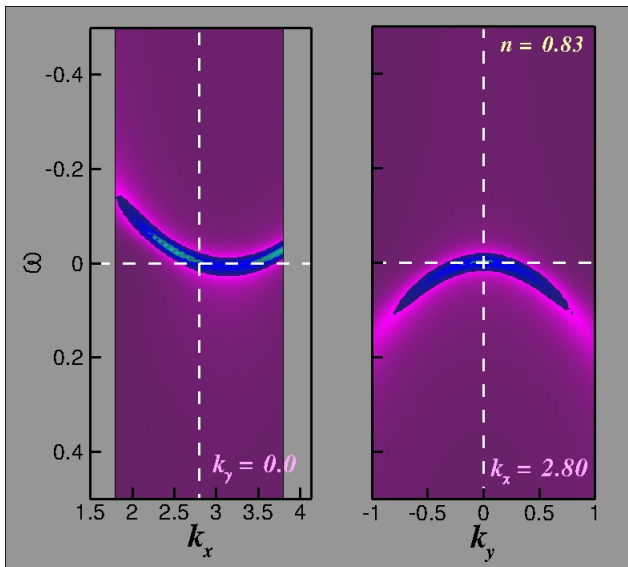
$$n = 0.75$$



$$n = 0.80$$

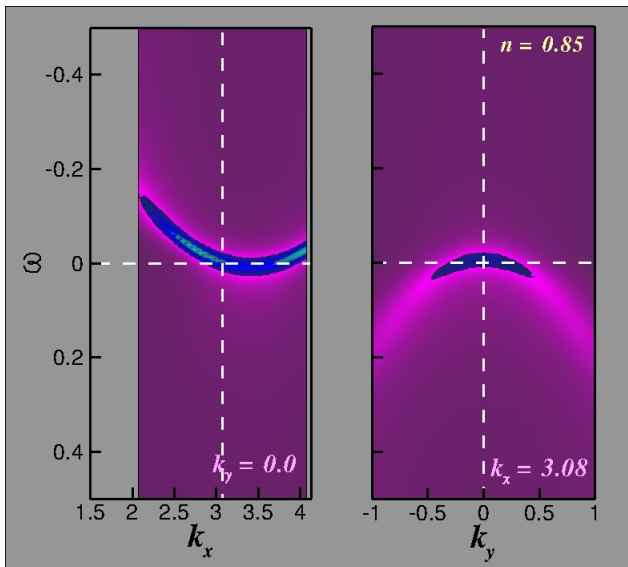


$$n = 0.83$$

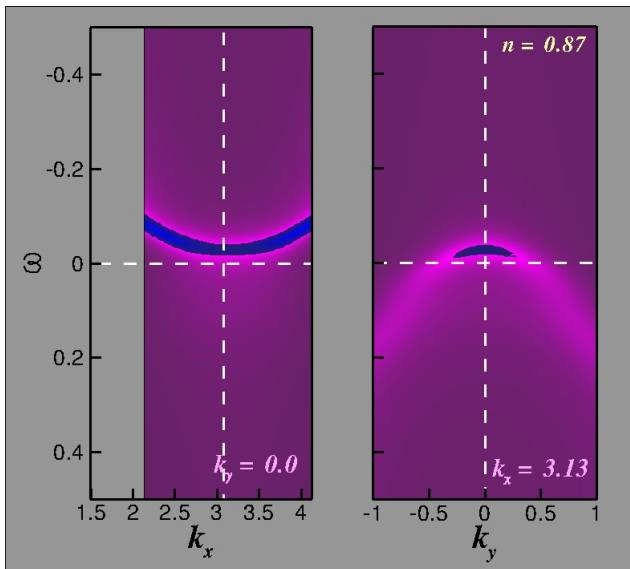


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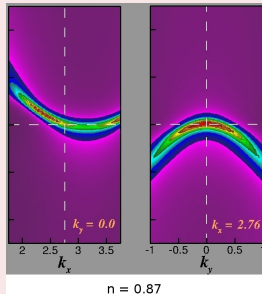
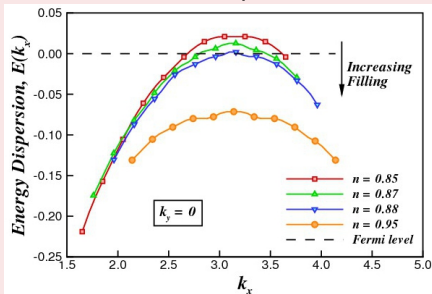
$$n = 0.85$$



$$n = 0.87$$



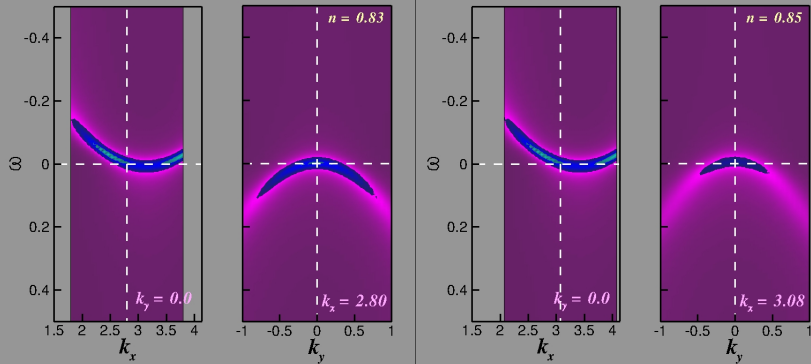
- The algebraic decay of  $\chi'_{0d}$  cannot be understood simply from vHS at Fermi level picture



- Combined effect of vHS at Fermi level, extendedness of the singularity and thermal broadening of the quasi-particle peaks



For negative  $t'$ , vHS pinned to Fermi level for larger range of doping



- leads to larger range of doping where MFL behavior is observed

# Phase Diagram

