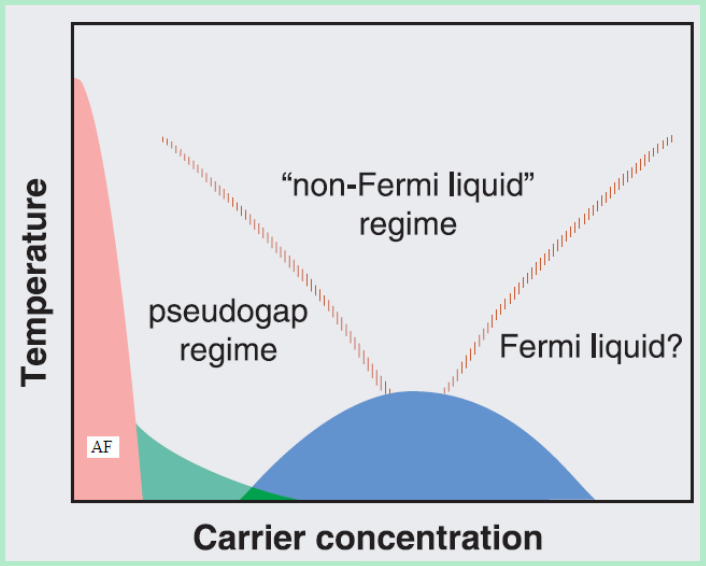
An introduction to
High Temperature Superconductors



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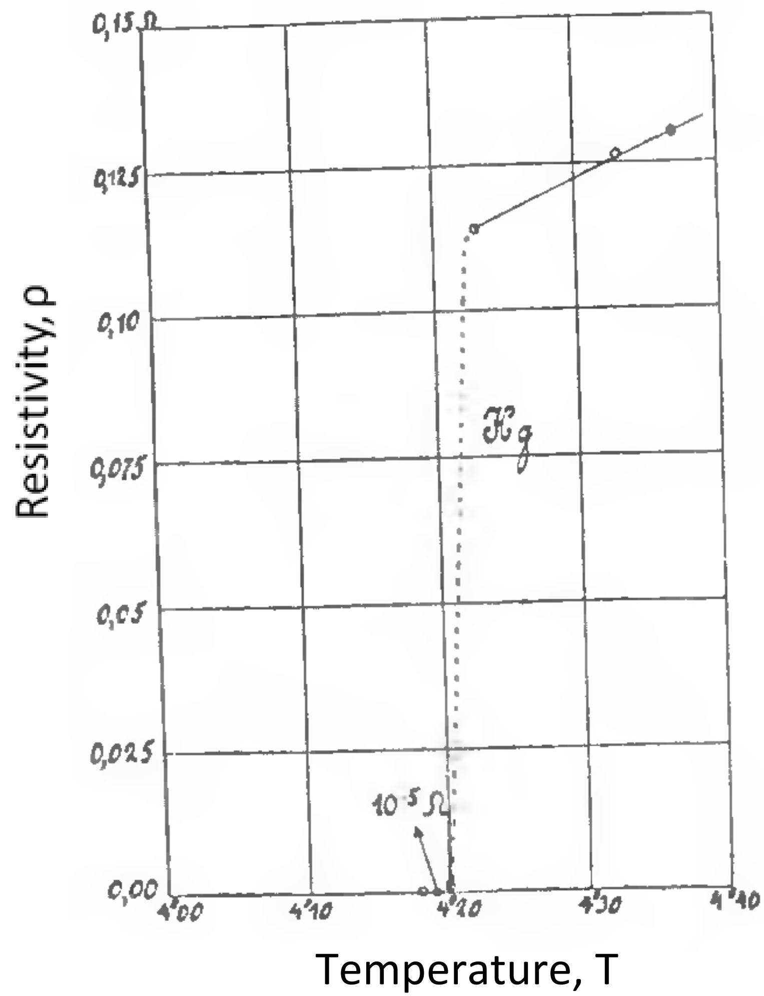
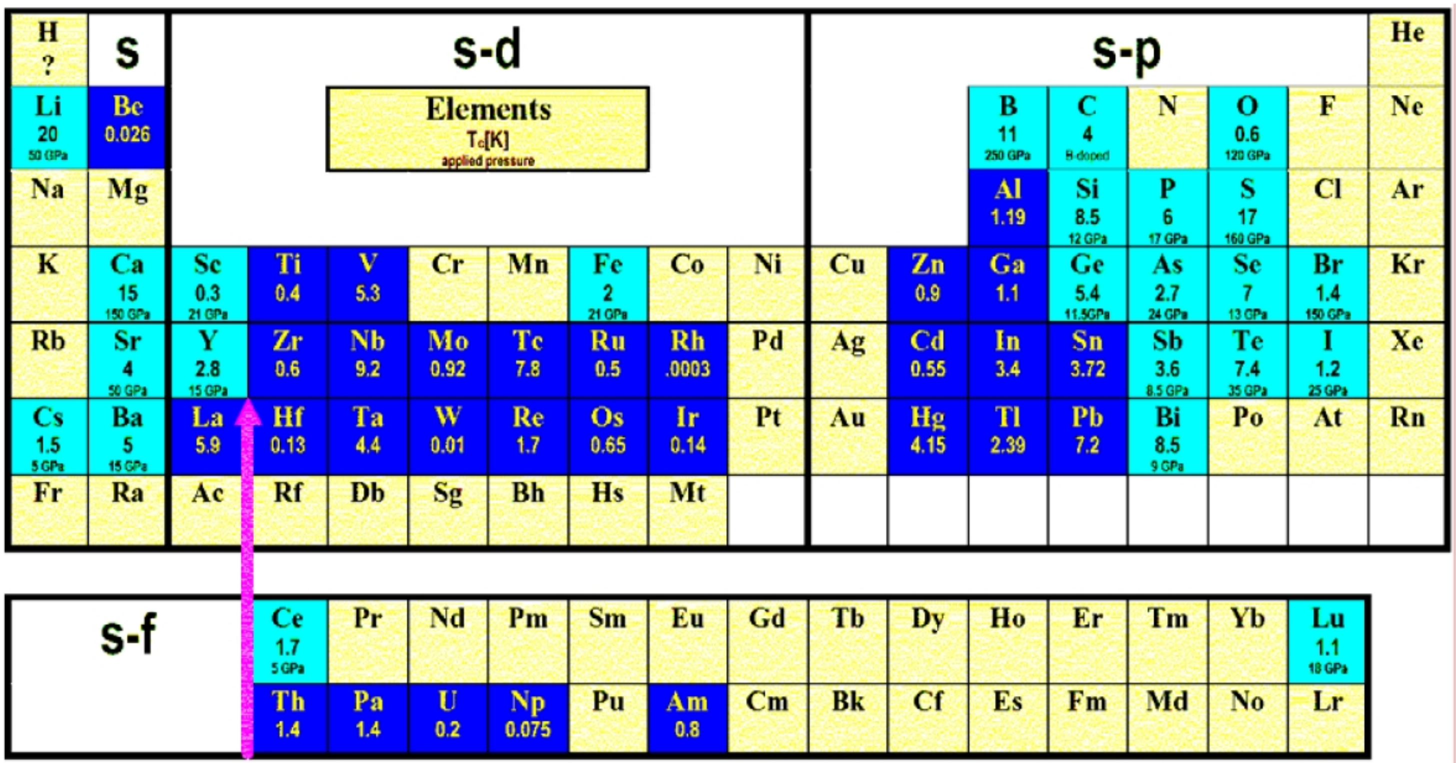
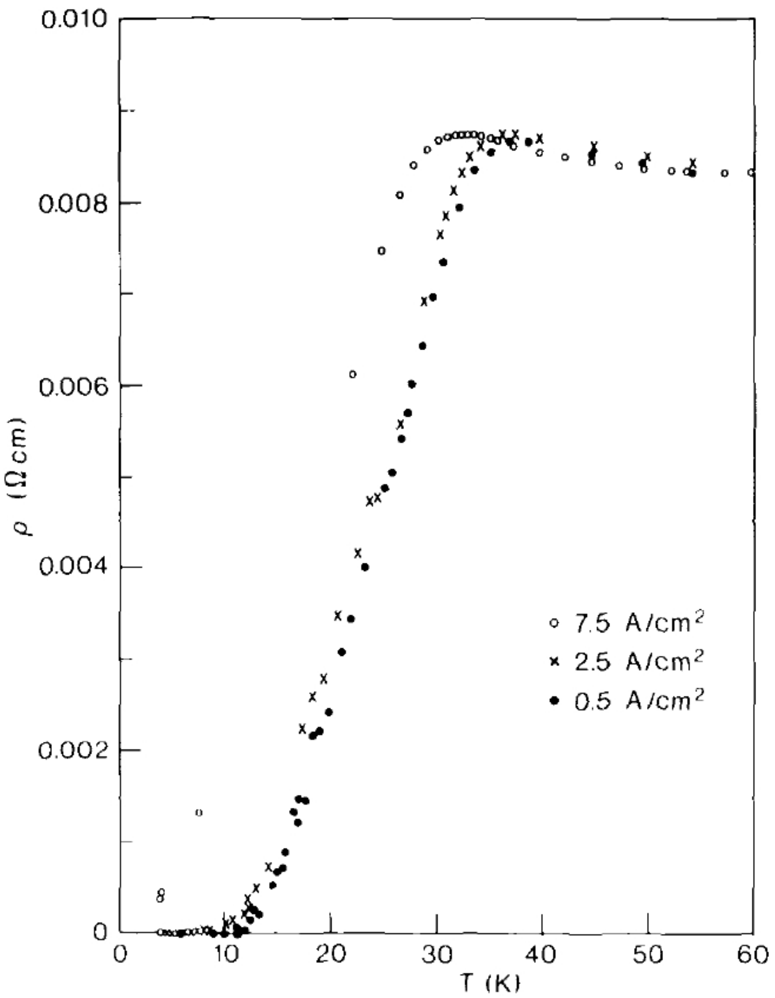
Kamerlingh Onnes - 1911



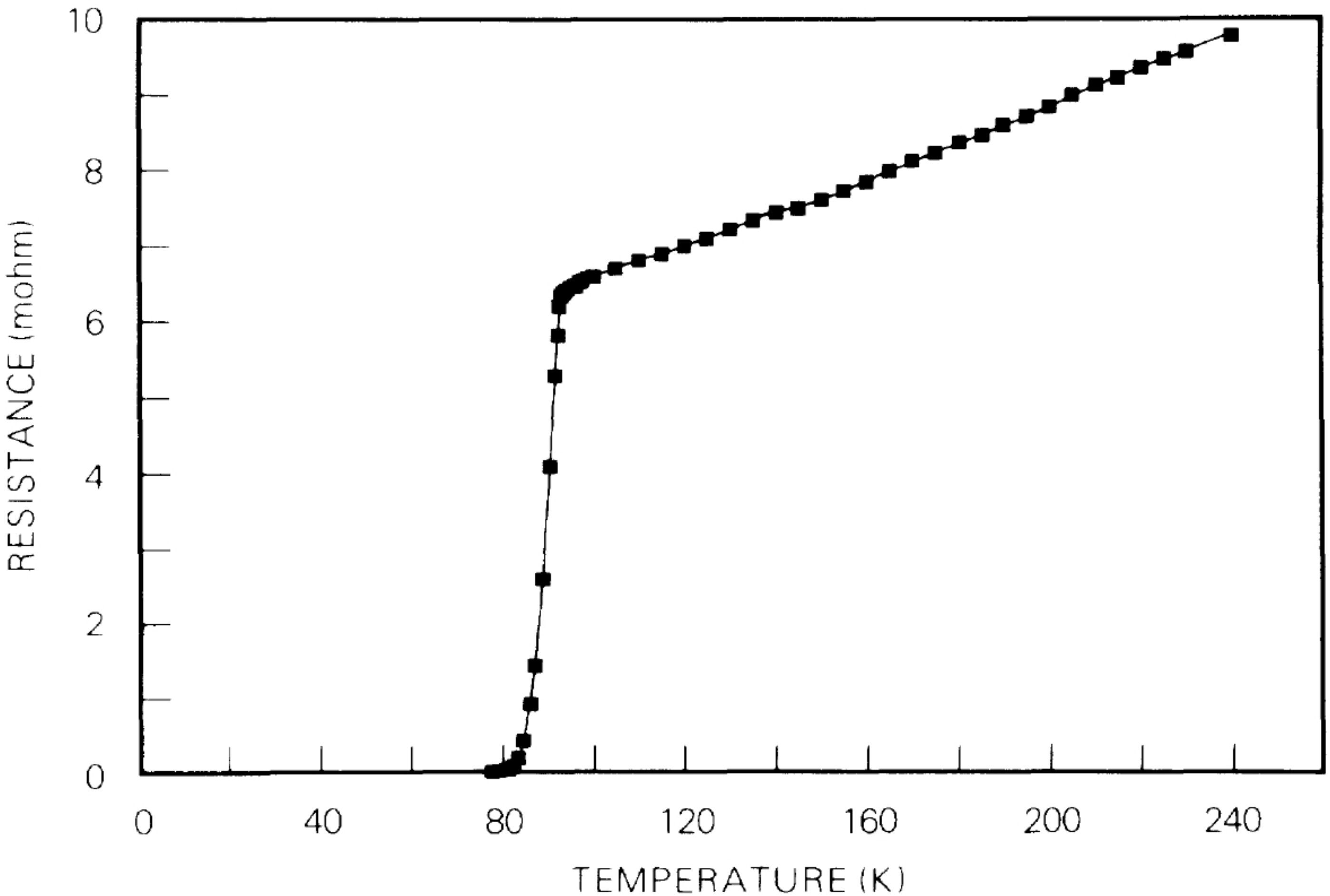
Not a rare phenomenon !!!



Bednorz & Muller - 1986



Superconductivity above Liquid Nitrogen temperature !!!

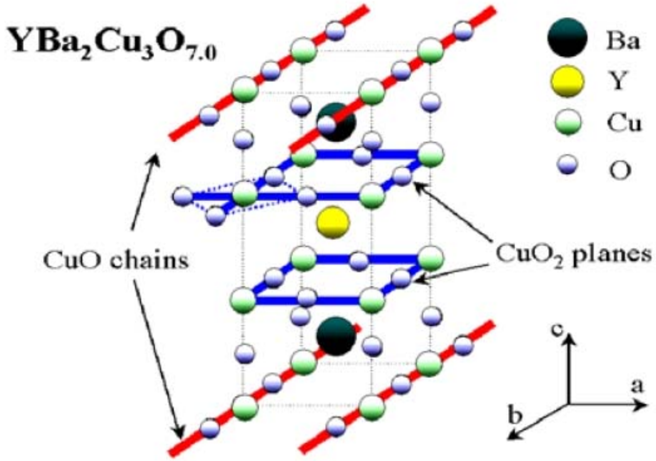


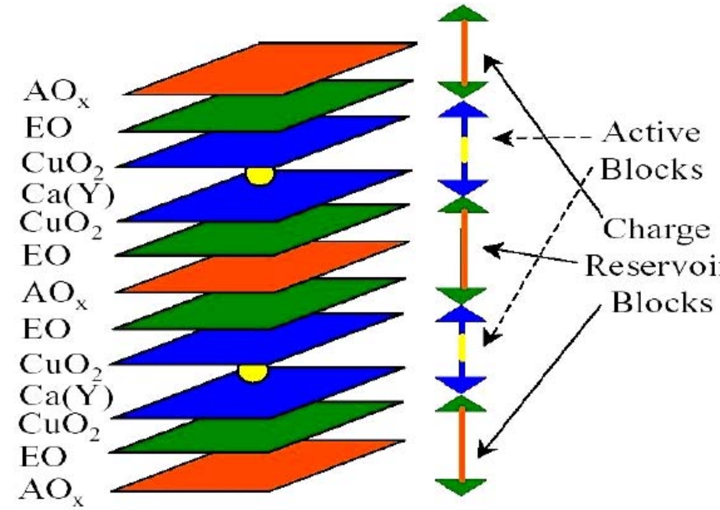
Wu et al (1987)

Plan of the talk

- High T_c Buzz - Cuprate Material
- Recap
  - Fermi Liquid Theory – Quasi-particle (QP) Idea
  - Superconductivity – Cooper Calculation – BCS Results
  - Concretizing QP Idea – Greens Function – Spectral Function – ARPES
- High T_c phases
  - AF Mott Insulator
  - Pseudo-gap regime
  - Super-conducting state
  - “Normal” metal regime
- Theoretical Models

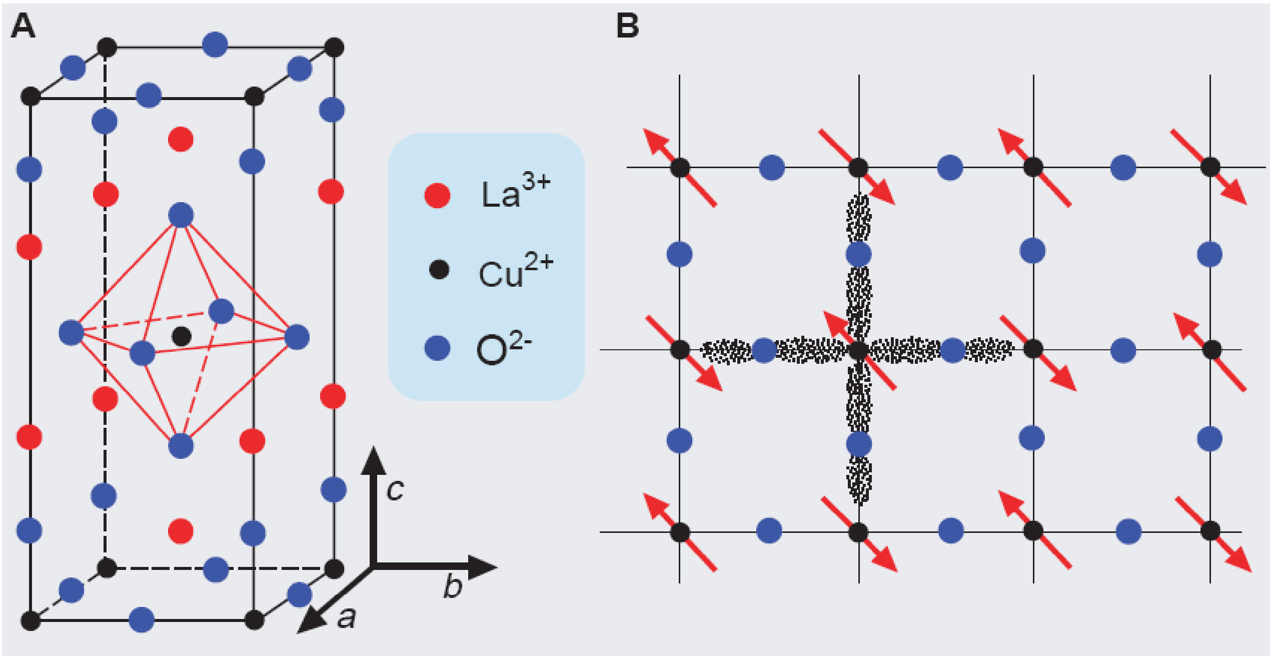
Cuprates: Material





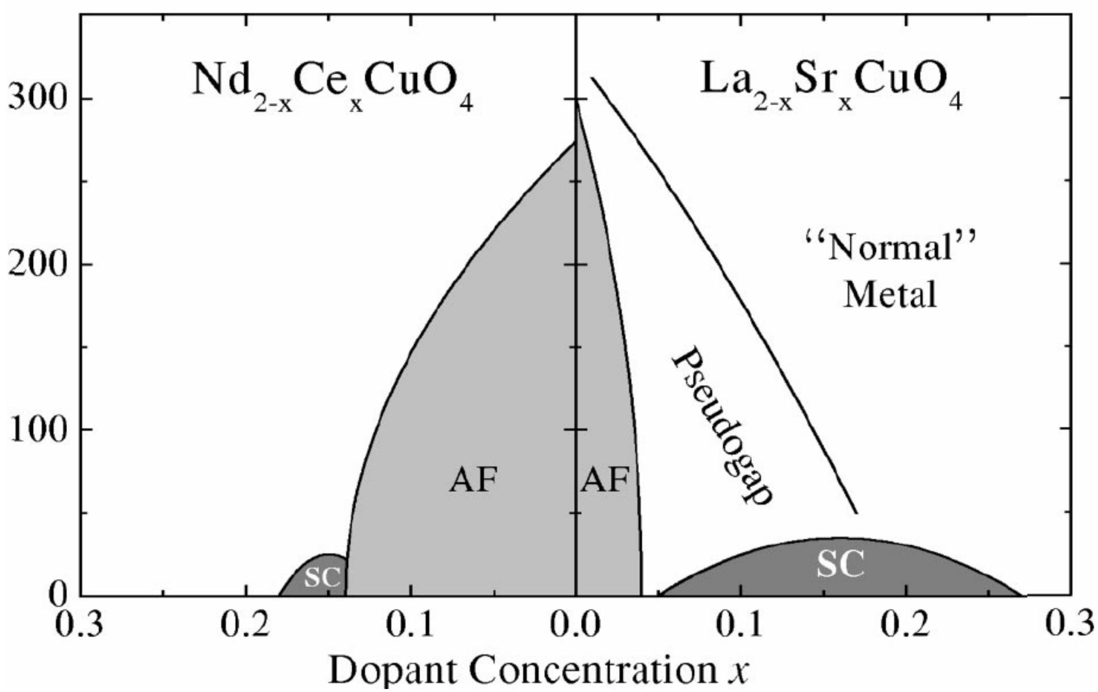
- Two sub-units – Active Blocks & Charge Reservoir Blocks
- *"Active Block"* – $(CuO_2/Ca/)_{n-1}CuO_2$ – Mobile Carriers Workspace
- *"Charge Reservoir Block"* - $EO/(AO_x)_mEO$
  - Storage for dopants
  - Provide Doping Charge
  - A = (Bi, Pb, Tl, Hg, Au, Cu, Ca, B, Al, Ga)
  - E=Alkaline Earth Metal (Sr, Ba)
- General Formula – $A_mE_2Ca_{n-1}Cu_nO_{2n+2+mx}$ – Terminology: A-m2(n-1)n
- Examples – $Bi_2Sr_2Ca_2Cu_3O_{10}$ – Bi-2223, $Y_1Ba_2Cu_3O_7$ – Y-1223

0210 compounds



- E_2CuO_4 - La_2CuO_4, Nd_2CuO_4
- Key Structural Unit – CuO_2 planes with weak inter-planar coupling
- Hole Doping: Replace x fraction of La^{3+} with Sr^{2+} - Result: x holes per Cu
- Cu^{2+} - d^9 – Spin ½ on a planar square lattice
- Spins arrange in AF fashion to gain KE through Super-exchange.
- Electron Doping: Replace x fraction of Nd^{3+} with Ce^{4+} - Result: x e^- per Cu

Cuprates Phase Diagram



- Shows very rich phase diagram as a function of Doping
- AF Mott Insulator for small doping
- On further doping – SC
- Doping at which maximal T_C occurs – optimal doping.
- Lower dopings – Under-doping and Higher dopings- Over-doping
- Transition to "Strange" or "Normal" metal for $T > T_C$
- Exotic Pseudogap phase between AF and SC phases.

Free Fermi Gas

$$H = \sum_{i=1}^{n} \frac{p_i^{\,2}}{2m}$$

- Describes a collection of particles with momenta $\mathbf{p_i}$ and energy, E_i
- Since there is translational invariance, **p** is a good quantum number, electrons occupy **p** states following Pauli – Total Energy(E_F) Contour – Fermi-surface (FS)– Ground State.
- **Excitations :** Remove an electron from $k < k_F$, thus creating a hole there and placing it arbitrarily close to FS – Particle-hole excitation – *Gapless*
- **Specific Heat :** Only a fraction of electrons α T can get thermally activated. Thus U α $T.k_BT$ → $\mathbf{C_v = \gamma T\ (\gamma\ \alpha\ g_0)}$
- **Magnetic Susceptibility :** constant α g_0 *(Pauli Paramagnet)*
- **Infinite Electrical Conductivity –** In presence of scatterers – Drude Formula

Landau Quasi-particle Idea

- What if we add Interactions between electrons?
- Example - Screening – An electron gets screened from other by those near it – For others, it is still a particle but has wore a new dress – “Quasi-particle”
- Landau’s Idea – There is one-to-one correspondence between states of non-interacting system to states of interacting system – No level crossing – Momentum still a good quantum number – An electron in a state is given a new dress by co-workers – **“Quasi-particle” or “collective excitation”**
- All the non-interacting physics remains the same in spite of high Coulomb energy per particle – mass get renormalized
- **Interactions → Fermi-Gas to Fermi-Liquid**

Superconductivity

- As metals are cooled below certain temperature, the electrical resistance goes to zero and this phase is called a superconductor (SC).
- Differences from perfect condcutor
  - A superconductor does not allow Magnetic field (H) to penetrate through it. Persistent surface currents are set in such a way that H is cancelled – ***Meissner effect (1933)*** – ***Perfect Diamagnetism*** – Magnetization, M = -H, while in metals, flux can not change on cooling through transition
  - Perfect Conductor - No persistent currents

Types of Superconductors

- ***Type I SC*** – As H is increased, M becomes zero at a critical field H_c.
- ***Type II SC*** – Two critical H_c – H_{c1} & H_{c2}. As H goes past H_{c1}, magnetic field lines penetrates in bundles (vortex). There can be surface currents on both sides of vortex. As H goes past H_{c2}, vortices come closer and kill SC.

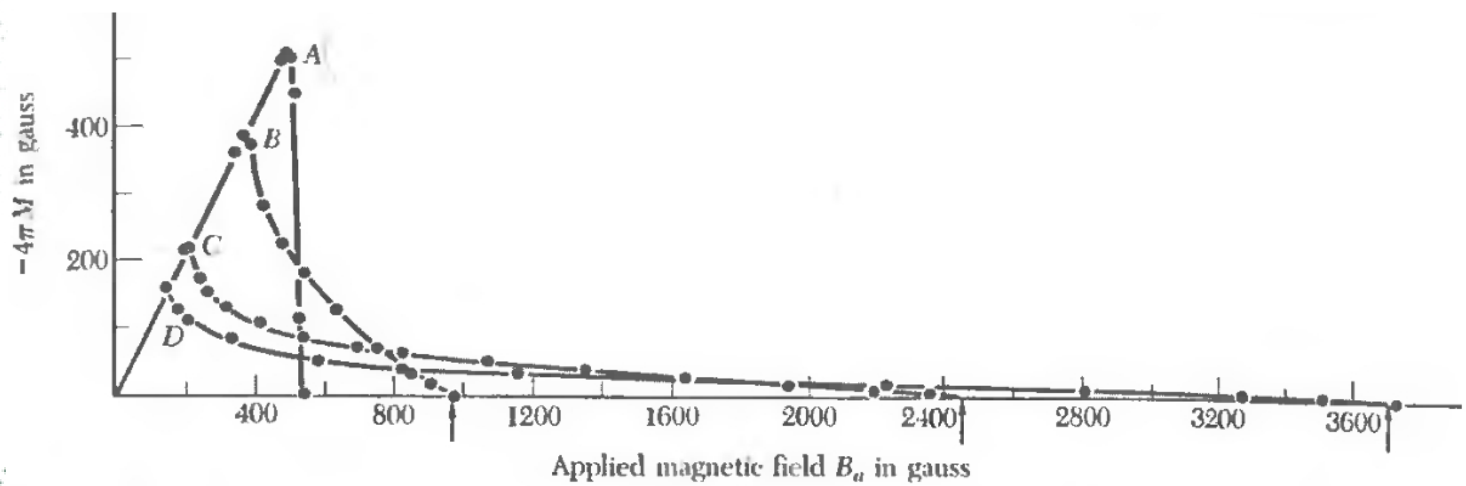
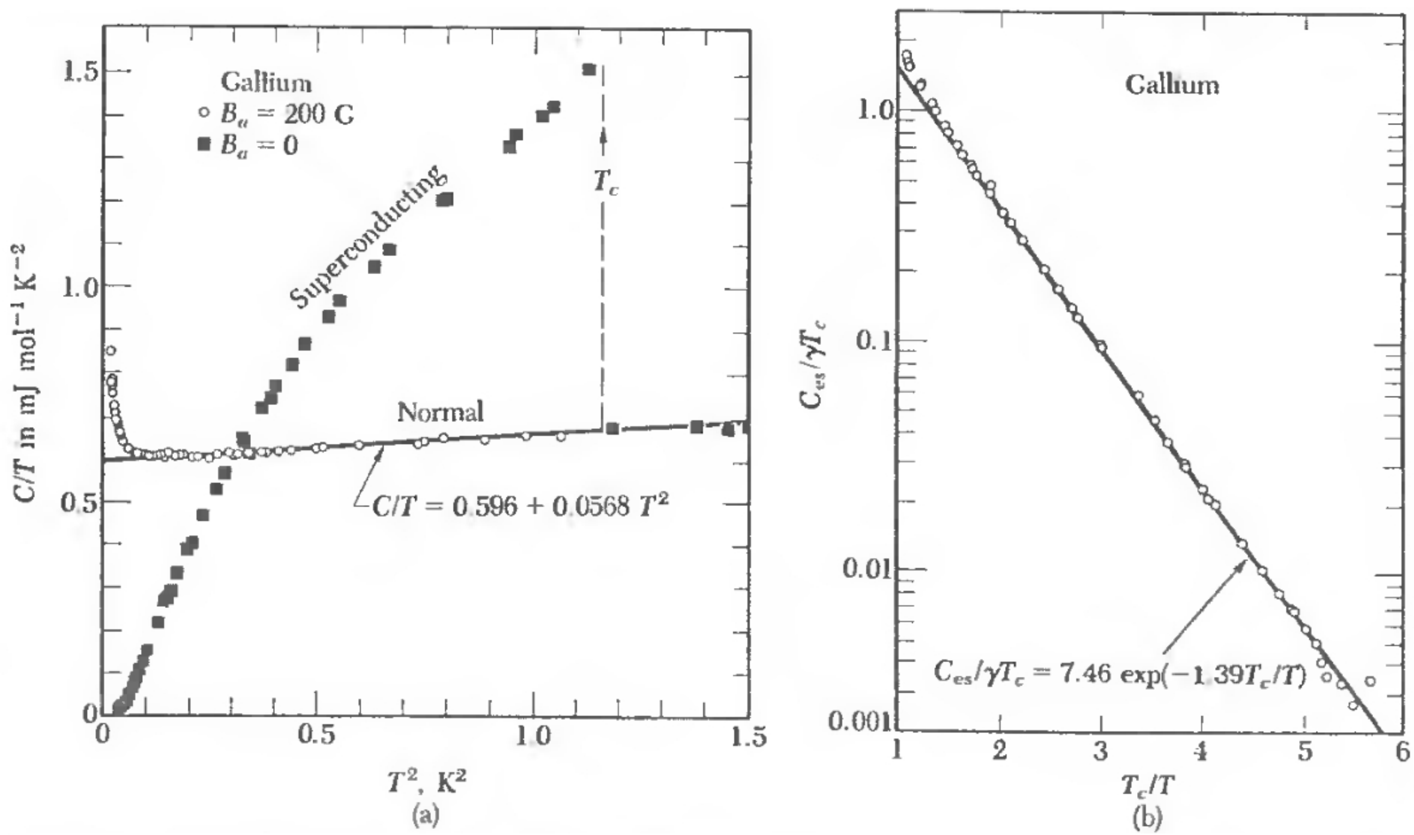


Figure 5a Superconducting magnetization curves of annealed polycrystalline lead and lead-indium alloys at 4.2 K. (*A*) lead; (*B*) lead–2.08 wt. percent indium; (*C*) lead–8.23 wt. percent indium; (*D*) lead–20.4 wt. percent indium. (After Livingston.)

Specific Heat

- Below T_c, SC state has lower entropy than normal state – more ordered. Small Difference – *Small fraction of conduction electrons participate in the transition.* Two fluid picture – *"normal" and "super"*
- Low T C_v exponential – *Excitations are gapped*



London phenomenological theory (1935)

- Postulate – Superconducting current is proportional to vector potential

$$j_s(\vec{r}) = -\frac{1}{\mu_0 \lambda^2}\vec{A}$$

$$\lambda = \left(\frac{mc}{\mu_0 q^2 n_s}\right)^{\frac{1}{2}}$$

- Explains Meissner effect

$$\nabla \times \vec{j}_s = -\frac{1}{\mu_0 \lambda^2}\vec{B}$$

$$\Rightarrow \nabla \times \vec{j}_s = -\frac{1}{\mu_0 \lambda^2}\vec{B}$$

$$\Rightarrow \nabla \times \vec{B} = \mu_0 \vec{j}_s \ : \text{Maxwell's equation}$$

$$\Rightarrow \nabla \times \nabla \times \vec{B} = \mu_0 \nabla \times \vec{j}_s = -\frac{1}{\lambda^2}\vec{B}$$

$$\Rightarrow \nabla^2 \vec{B} = \frac{1}{\lambda^2}\vec{B}$$

- Constant Magnetic field cannot reside inside a superconductor
- Since, λ diverges as transition temperature is reached, n_s was presumed to go continuously to zero at Tc – Second Order phase transition
- Standard Reference for Second Order Transitions : Landau!

G-L description of SC state (1950)

- G-L introduced a phenomenological order parameter, Ψ(**r**) s.t. $\Psi^*(\mathbf{r})\,\Psi(\mathbf{r}) = n_s$ – super-fluid density ($n_s \rightarrow 0$ as $T \rightarrow T_c^+$)
- In the vicinity of SC-Metal transition, Free energy density of SC state, F_s can be expanded in terms of Ψ(**r**)

$$F_s = F_N + a(T)|\psi|^2 + \frac{1}{2}b(T)|\psi|^4 + \frac{1}{2m}|(-i\hbar\nabla - qA/c)\psi|^2 + \frac{B^2}{2\mu_0}$$

- Uniform space solution is given by

$$|\psi_0|^2 = -\frac{a(T)}{b(T)} = n_s$$

or

$$\psi = 0$$

- This implies a(T) should be less than 0 for $T<T_c$ and vice-versa.

$$a(T) = a_1(T - T_c),\ a_1 > 0$$

- It is easy to see that

$$H_c(T) \sim (T - T_c)$$

G-L Theory: London equation

- By minimizing w.r.t. **A** and requiring that $\Psi \approx \Psi_0$

$$j_s(\vec{r}) = -(\imath q\hbar / 2m)(\psi^* \nabla \psi - \psi \nabla \psi^*) - (q^2 / mc)\psi^* \psi \vec{A}$$

$$j_s(\vec{r}) = -(q^2 / mc)\psi^* \psi \vec{A} \text{ : London equation}$$

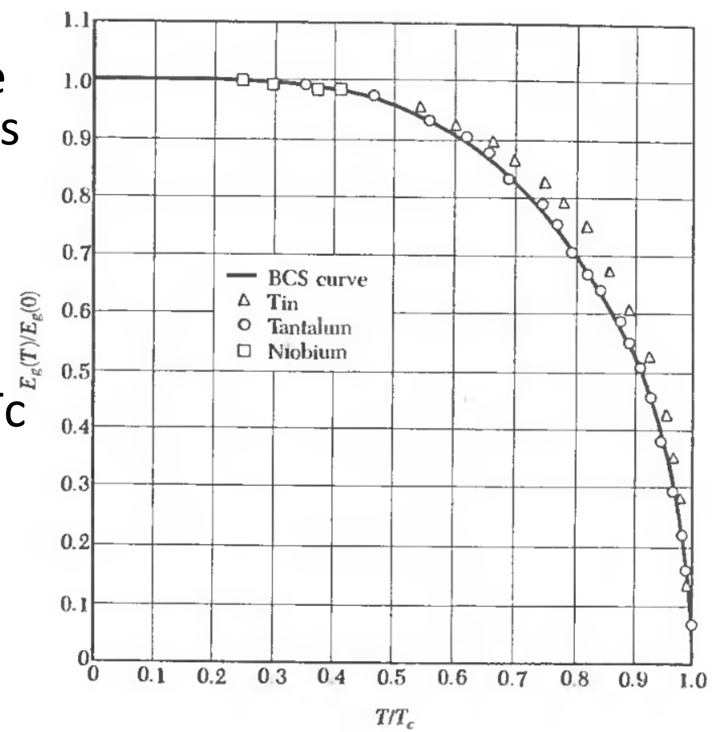
- Length Scale describing super-fluid stiffness

$$\xi^2 = \frac{\hbar^2}{2ma}$$

- Larger the magnitude of ξ, harder for |ψ| to fluctuate – measure of the distance within which the SC electron concentration cannot change drastically in a spatially varying field.
- Criterion for Type I or Type II Superconductors
- Type I behavior supported when $\xi > \lambda$ and Type II for $\xi < \lambda$
- Define Landau-Ginzburg constant, $\kappa = \lambda / \xi$
- For usual SC, κ does not exceed 3 usually, but, for High Tc Cuprates , κ = 100 !!!

BCS Theory (1957)

- T_c depends on isotope mass of A cation – Phonons at play !
- ***BCS Theory*** based on phonon mediated electron-electron attraction has been extremely successful
- Cooper – Any attractive interaction between electrons outside Fermi surface will produce a bound state – Cooper pair
- There is a pairing wave-function which has an amplitude and an phase. The wave function has zero total angular momentum and thus, s-wave symmetry.
- In super-conducting state, the phase of all the pairs gets fixed and electrons move as a coherent cloud – zero resistance
- Gap arises as finite energy is required to break electron pairs.
- Gap goes smoothly to zero as T $\longrightarrow$ Tc – Second Order transition



Standard Results Summary

Property	Fermi Liquid	BCS Superconductors
Excitations	Gapless	Gapped
Specific Heat	~T	$\sim Exp(-\Delta/K_B T)$
Electrical Conductivity	$\sim T^2$	∞
Magnetism	Pauli Paramagnet	Diamagnet
	(non-zero Constant)	

Greens Function

- Suppose we have a Hamiltonian $H = H_0 + V$
- We ask the question, given state |Ψ0> at time t=0, what is time evolution of the state? $\left(i\frac{\partial}{\partial t} - H_0\right)|\psi> = V(t)|\psi>$
- Define Greens function to be solution of

$$\left(i\frac{\partial}{\partial t} - H_0\right)G(t-t') = \delta(t-t')$$

- In terms of Greens Function,

$$|\psi(t)> = |\psi(t)>_0 + \int_0^t dt' G(t-t')V(t')|\psi(t')>$$

$$|\psi(t)>_0 = U(t)|\psi_0>, \; U(t) = e^{-itH_0}$$

- Consider V=0 (non-interacting) case,

$$H_0 = \sum_\alpha \varepsilon_\alpha |\alpha><\alpha|$$

$$U(t) = \sum_\alpha e^{-i\varepsilon_\alpha t}|\alpha><\alpha| \;\Rightarrow\; U(\omega) = \sum_\alpha \delta(\omega - \varepsilon_\alpha)|\alpha><\alpha|$$

$$U_{\alpha\alpha}(\omega) = \delta(\omega - \varepsilon_\alpha)$$

Thus, $trU(\omega) =$ DOS

$U_{\alpha\alpha}(\omega)$ - Probability that state α has energy ω - Spectral function

Properties of Greens Function

- Greens function is *causal* → Kramer's Kronig relations – Real and Imaginary parts of the Green's function are not independent.

- Spectral function completely determines Greens function

$$\left(i\frac{\partial}{\partial t} - H_0\right)G(t) = \delta(t)$$

$$\Rightarrow (\omega I - H_0)G(\omega) = I \Rightarrow G(\omega) = (\omega I - H_0)^{-1}$$

$$\Rightarrow G(\omega) = \sum_\alpha \frac{|\alpha><\alpha|}{\omega - \varepsilon_\alpha}$$

$$G_{\alpha\alpha}(\omega^+) = P\left(\frac{1}{\omega - \varepsilon_\alpha}\right) - i\pi\delta(\omega - \varepsilon_\alpha)$$

$$\text{Thus, } \boxed{-\frac{1}{\pi}\text{Im}(G_{\alpha\alpha}(\omega^+)) = \text{Spectral function, } A_\alpha(\omega)}$$

$$\text{and, } \boxed{-\frac{1}{\pi}\text{Im}(trG(\omega^+)) = DOS}$$

- Take Fourier transform – $G_{\alpha\alpha}(t) = -i\theta(t)e^{-i\varepsilon_\alpha t}$

$$G(\omega) = \int d\omega' \frac{\sum_\alpha \delta(\omega' - \varepsilon_\alpha)|\alpha><\alpha|}{\omega - \omega'}$$

$$\boxed{G(\omega) = \int d\omega' \frac{\rho(\omega')}{\omega - \omega'}}$$

Importance of Spectral Function

- If Greens function has pole at $\omega = \Omega - \Gamma$, i.e.

$$G_{\alpha\alpha}(\omega) = \frac{Z_\alpha}{\omega - (\Omega - i\Gamma)}$$

$$\text{Then, } G_{\alpha\alpha}(\omega) = \frac{Z_\alpha(\omega - \Omega)}{\left((\omega-\Omega)^2 + \Gamma^2\right)} - i\frac{Z_\alpha \Gamma}{\left((\omega-\Omega)^2 + \Gamma^2\right)}$$

$$-\frac{1}{\pi}\mathrm{Im}(G_{\alpha\alpha}(\omega^+)) = \frac{Z_\alpha \Gamma / \pi}{\left((\omega-\Omega)^2 + \Gamma^2\right)} = A_\alpha(\omega)$$

- Then, $G_{\alpha\alpha}(t) = -i\theta(t)e^{-i\Omega t - \Gamma t}$
- This means that whenever Greens function has pole on negative imaginary axis or Spectral function has a Lorentzian part, We have a state which has life-time, $1/\Gamma$ and there are Quasi-particles excitations. These are particles which have well-defined energy dispersion and a finite lifetime (so that they can be observed).

Many Body Spectral Function, A(k,ω)

- Define $A^{+}(\vec{k},t-t') = < N | c_{\vec{k}}(t) c_{\vec{k}}^{\dagger}(t') | N >$

  $A^{-}(\vec{k},t-t') = < N | c_{\vec{k}}^{\dagger}(t') c_{\vec{k}}(t) | N >$

  $$A(\vec{k},\omega) = A^{+}(\vec{k},\omega) + A^{-}(\vec{k},\omega)$$

- If a particle (A^+) or hole (A^-) is added to a N-particle system at time,t', does it still look like a particle or hole at time, t ?

- Since particle is to be added at an energy, ω > μ and hole at energy, ω < μ

  $$A^{+}(\vec{k},\omega) = \theta(\omega-\mu) A(\vec{k},\omega)$$

  $$A^{-}(\vec{k},\omega) = \theta(\mu-\omega) A(\vec{k},\omega) = n_F^{-}(\omega) A(\vec{k},\omega)$$

- After simple algebra, it turns out that

  $$A^{+}(\vec{k},\omega) = \sum_n |< n, N+1 | c_{\vec{k}}^{\dagger} | N >|^2 \delta(\omega - (\omega_n + \mu))$$

  $$A^{-}(\vec{k},\omega) = \sum_n |< n, N-1 | c_{\vec{k}} | N >|^2 \delta(\omega - (-\omega_n + \mu))$$

- Delta function counts if the n^{th} excited state has energy ,ω measured from chemical potential and if it does, what is the weight of the added excitation in that state. *Thus, A(**k,** ω) measures the probability that an excitation (particle or hole) added to the **k**-state has energy ω – generalization* of non-interacting DOS

A(k,ω) – Properties & Sum Rules

- General Structure – $A(\vec{k},\omega) = \dfrac{1}{\pi} \dfrac{Z_{\vec{k}} \left(\dfrac{1}{\Gamma_{\vec{k}}} \right)}{(\omega - E_{\vec{k}})^2 + \left(\dfrac{1}{\Gamma_{\vec{k}}} \right)^2} + A^{inc}(\vec{k},\omega)$

- Sum Rules

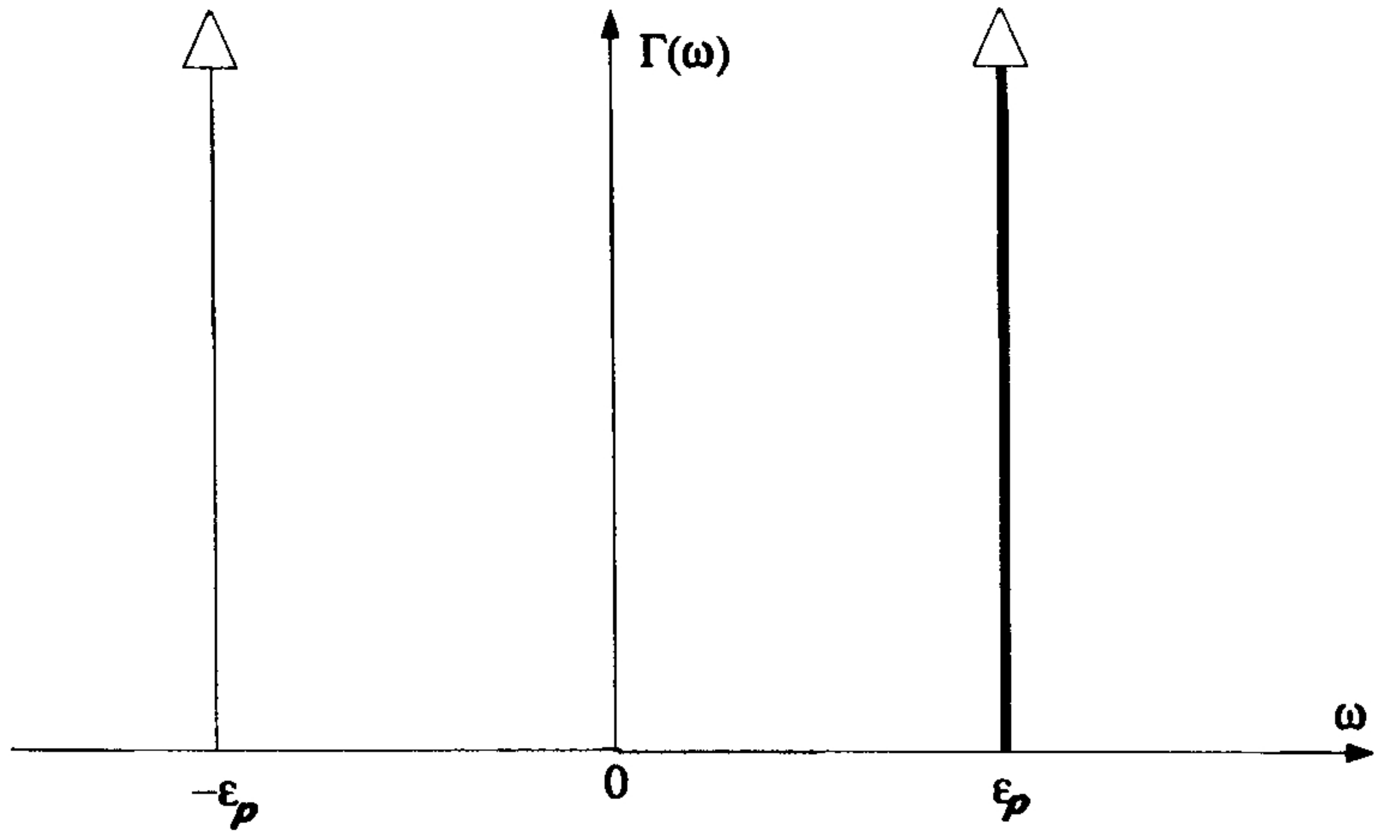
$$\int_{-\infty}^{\infty} d\omega A(\vec{k},\omega) = 1$$

$$\int_{-\infty}^{\infty} d\omega A^{+}(\vec{k},\omega) = < c_{\vec{k}} c_{\vec{k}}^{\dagger} >$$

$$\int_{-\infty}^{\infty} d\omega A^{-}(\vec{k},\omega) = < n_{\vec{k}} >$$

- Spectral function for
  - *Fermi System – Delta function peaked at* $\xi_{\mathbf{k}}$
  - *BCS Superconductor* **–** *Two Delta functions peaked at +,- E_k – One for particle addition and other for hole addition (pair breaking)*

A(**k**,ω) for superconductors



Spectral density of the retarded Green's function in the superconductor.

- Note that QP are infinitely lived even inspite of interactions.

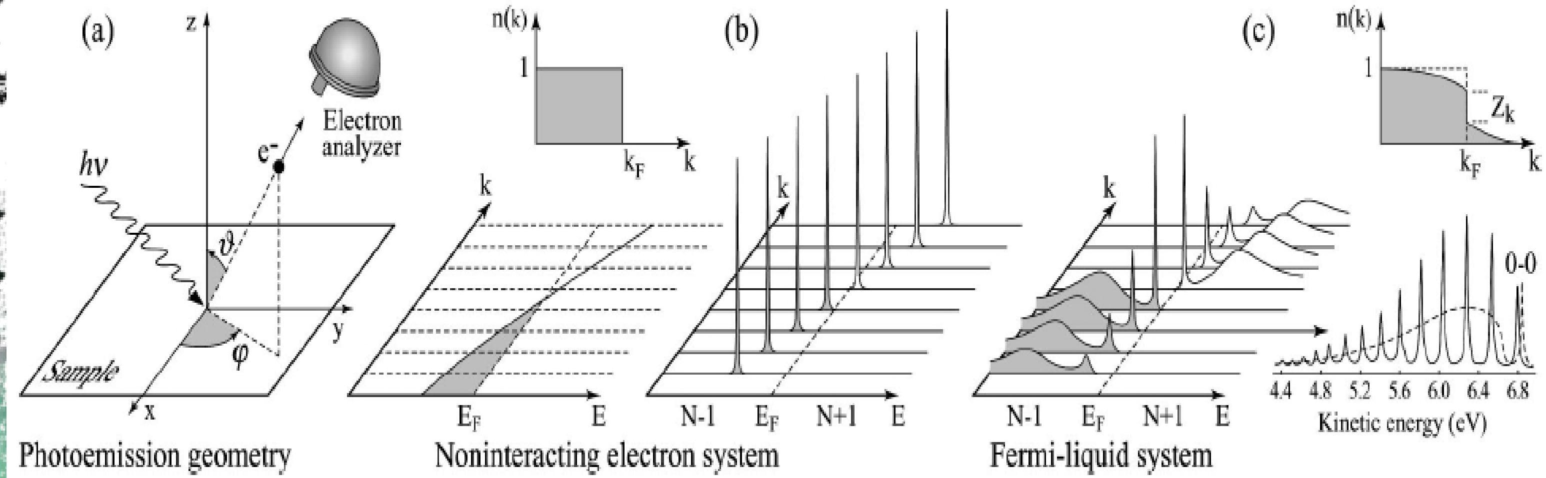
Many Body Greens Function

- Taking analogy from non-interacting case, define

$$G(\vec{k},\omega)=\int d\omega'\frac{A(\vec{k},\omega')}{\omega-\omega'}$$

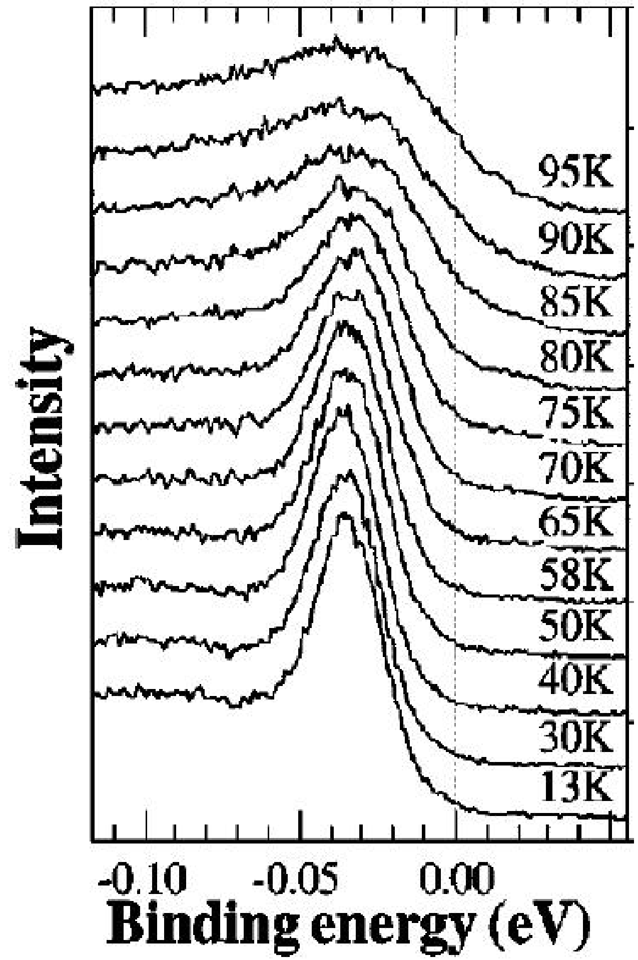
$$\Rightarrow G(\vec{k},t)=-i\theta(t)<G|\{c_{\vec{k}}(t),c_{\vec{k}}^{\dagger}(t')\}|G>$$

Angle Resolved Photoemission Spectroscopy (ARPES)



- Technique to measure $A^{-}(\mathbf{k},\omega)$
- A monochromatic radiation is incident on properly aligned single crystal sample – Emission of Photoelectrons – collected by Electron Analyzer – E_{kin} & **p** observed.
- Energy-momentum conservation : $$E_{kin} = \hbar\nu - \phi - |E_B|$$ $$p_{\parallel} = \hbar k_{\parallel} = \sqrt{2mE_{kin}}\sin\vartheta$$
- Photon momentum & k_{perp} neglected – photon energies kept small to obtain good resolution - 2D Geometry
- Intensity is plotted as a f^n of E_B for fixed **k**

ARPES: SC – Normal Transition



- $Bi_2Sr_2CaCu_2O_{8+\delta}$ (T_c = 87 K) Randeria et al, 1995

Surprises awaiting to appear !

- Undoped Cuprate – Mott Insulator – Expected to be metal by band theory
- Non-BCS d-wave superconductor below Tc
- Non-Fermi Liquid metal for T>Tc
- For very high doping, fermi metal