Homework review:

Prob. 30.34: Platinum has a work function of 6.35 eV , and iron has a work function of 4.50 eV. Light of frequency 1.86×10^{15} Hz ejects electrons from both of these surfaces.

a) From which surface will the ejected electrons have a greater maximum kinetic energy?

b) Explain.

c) Calculate the maximum kinetic energy of ejected electrons for each surface.

Solution: **Einstein theory of Photoelectric Effect:** **Photon energy =** $hf = \frac{hc}{\lambda}$

$$hf = K_{\max} + W_0 \Rightarrow K = hf - W_0$$

$$h = 6.63\times10^{-34}\, J\cdot s = 4.14\times10^{-15}\, eV\cdot s$$

$$f = 1.86\times10^{15}\, Hz$$

$$K_{Pt} = hf - W_{0,Pt} = 4.14\times1.86 eV - 6.35 eV = 1.35 eV$$

$$K_{Fe} = hf - W_{0,Fe} = 4.14\times1.86 eV - 4.50 eV = 3.20 eV$$

Homework review:

Prob. 30.57: A photon has an energy *E* and wavelength λ before scattering from a free electron. After scattering through a 140° angle, the photon's wavelength has increased by 11.5%.

a)Find the initial wavelength of the photon.

b)Find the initial energy of the photon.

c)Find the momentum of the initial photon.

Solution **Compton scattering:** $\lambda - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$

$$\lambda = 1.115\lambda_0 \qquad 0.115\cdot\lambda_0 = \frac{h}{m_e c}(1-\cos 140^\circ)$$

a): $$\lambda_0 = \frac{h}{0.115\cdot m_e c}(1-\cos 140^\circ)$$

b): $$\text{Energy} = hf = \frac{hc}{\lambda_0} = \frac{0.115\cdot m_e c^2}{1-\cos 140^\circ}$$

c): $$\text{Momentum} = \frac{\mathrm{h}}{\lambda} = \frac{\text{Energy}}{\mathrm{c}} = \frac{0.115\cdot m_e c}{1-\cos 140^\circ}$$

Homework review:

Prob. 30.71: The uncertainty in position of a proton confined to the nucleus of an atom is roughly the diameter of the nucleus. If this diameter is d = 7.8×10^{-15} m, what is the uncertainty in the proton's momentum?

Solution **Heisenberg Uncertainty Principle:** $\Delta x \cdot \Delta p \geq \frac{\hbar}{2} = \frac{h}{4\pi}$

$$\Delta p \geq \frac{\hbar}{2\cdot\Delta x} = \frac{\hbar}{2\cdot d} = \frac{h}{4\pi\cdot d} = \frac{6.63\times10^{-34}\, J\cdot s}{4\pi\cdot 7.8\times10^{-15}\, m}$$

Prob. 30.75: An excited state of a particular atom has a mean lifetime of 5.9×10^{-10} s, which we may take as the uncertainty Δt, What is the minimum uncertainty in any measurement of the energy of this state?

Solution **Heisenberg Uncertainty Principle:** $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

$$\Delta E \geq \frac{\hbar}{2\cdot\Delta t} == \frac{6.63\times10^{-34}\, J\cdot s}{4\pi\cdot 5.9\times10^{-10}\, s}$$

Homework review:

Prob. 31.44: The electron in a hydrogen atom with an energy of -0.544 eV is in a subshell with 18 states.

a) What is the principal quantum number, *n*, for this atom?
b) What is the maximum possible orbital angular momentum this atom can have?
c) Is the number of states in the subshell with the next lowest value of equal to 16, 14, or 12?
d) Explain.

Solution **Bohr model:**

$$E_n = -(\frac{h^2}{4\pi^2 m_e k e^2})\frac{Z^2}{n^2} = -(2.18\times10^{-18}J)\frac{Z^2}{n^2} = -(13.6eV)\frac{Z^2}{n^2}$$

a): $$n^2 == -(13.6eV)\frac{Z^2}{E_n} = -13.6eV\frac{1}{-0.544eV} = 25 \quad \Rightarrow \quad n = 5$$

b): In a subshell of 18 states: $2(2l+1) = 18 \Rightarrow l = 4$

c) & d): Next subshell $l = 3 \quad \Rightarrow \quad 2(2l+1) = 14$

Homework review:

How to construct the structure of a atom? Example: Fe

1s					
2s	2p				
3s	3p	3d			
4s	4p	4d	4f		
5s	5p	5d	5f	5g	
6s	6p	6d	6f	6g	etc.

Subshell ordering: 1s 2s 2p 3s 3p 4s 3d 4p 5s 4d 5p 6s 4f …

$$n \cdot l^{2(2l+1)}$$

$$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 6s^2\ 4f^{14}....$$

Fe: $Z = 26$

$$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^6$$

Homework review:

Prob. 31.65: Consider an X-ray tube that uses platinum (Z= 78-1=77 see page 957 example 11) as its target.

a) Use the Bohr model to estimate the minimum kinetic energy electrons must have in order for K_α X-rays to just appear in the X-ray spectrum of the tube;

b) Assuming the electrons are accelerated from rest through a voltage V, estimate the minimum voltage necessary to produce the K_α X-rays.

Solution **Bohr model:**

a): $$K_{\min} = hf_{K_\alpha} = E_{n=2} - E_{n=1} = (13.6\text{ eV})Z^2(1-\frac{1}{4}) = 60.5\times 10^3 eV$$

b): $$\Delta V = 60.5\times 10^3 V$$

In general:

$$hf_{K_\alpha} = E_{n=2} - E_{n=1} = (13.6\text{ eV})Z^2(1-\frac{1}{4})$$

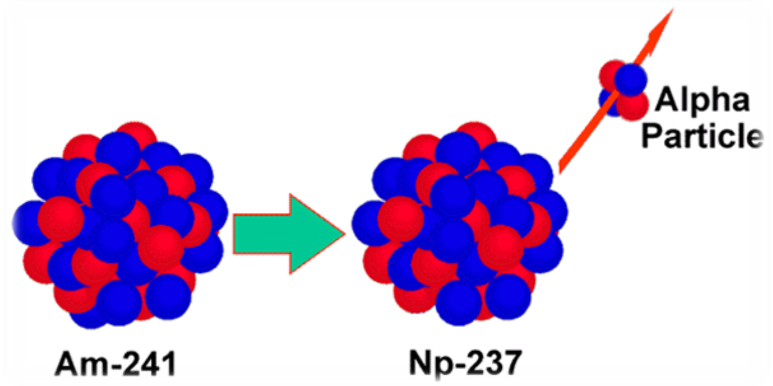
$$hf_{K_\beta} = E_{n=3} - E_{n=1} = (13.6\text{ eV})Z^2(1-\frac{1}{9})$$

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right), \quad n_i = \begin{cases} 2 & \text{for } K_\alpha \\ 3 & \text{for } K_\beta \end{cases} \quad \text{and} \quad n_f = 1$$

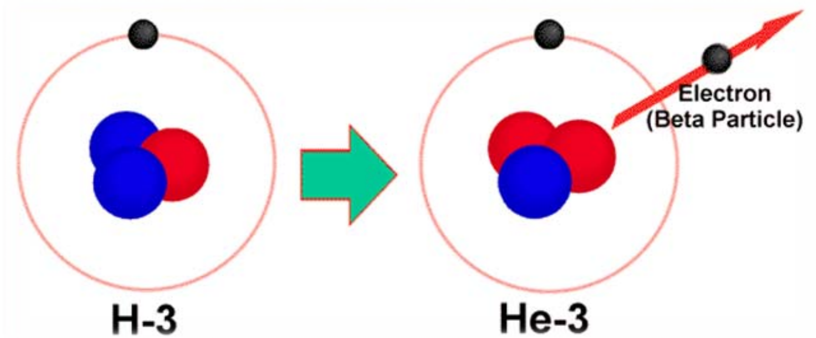
$$\text{where} \quad R = 1.097\times 10^7 \text{ m}^{-1}$$

Nuclear decay:

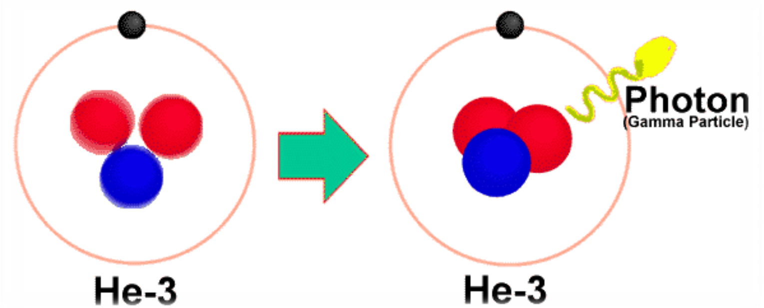
$$\boxed{{}^{A}_{Z}\mathrm{P} \rightarrow {}^{A\text{-}4}_{Z\text{-}2}\mathrm{D} + {}^{4}_{2}\mathrm{He}}$$



$$\boxed{{}^{A}_{Z}\mathrm{P} \rightarrow {}^{A}_{Z+1}\mathrm{D} + {}^{0}_{-1}\mathrm{e}}$$

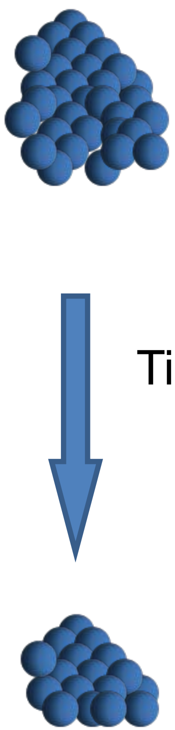


$$\boxed{{}^{A}_{Z}\mathrm{P}^{*} \rightarrow {}^{A}_{Z}\mathrm{P} + {}^{0}_{0}\gamma}$$



31.6 Radioactivity

Assume I have a chunk of radioactive material that contains N radioactive nuclei, or parent nuclei.



N_0

Time elapse

$T_{1/2}$

$(1/2)N_0$

Since radioactivity is a quantum-mechanical process, we can't know with certainty. We can only predict when a particular nucleus will decay. Thus, radioactivity is a statistical process.

As time passes, some of the nuclei will decay, and N will decrease.

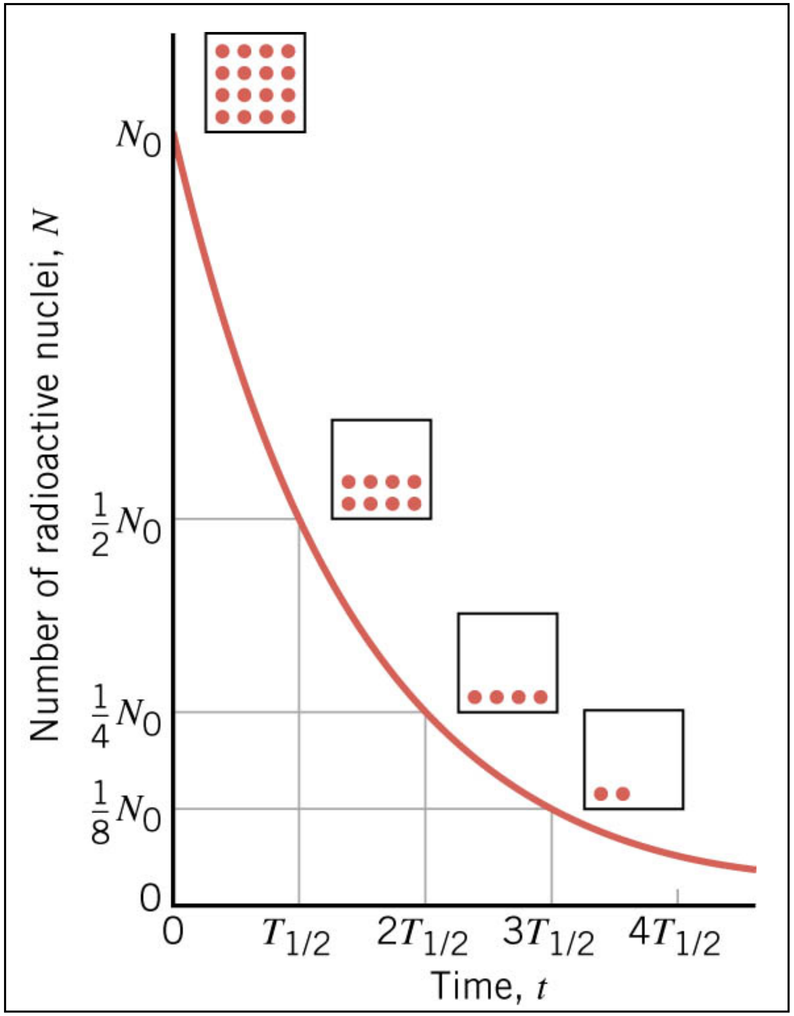
$$N=N(t)$$

One characteristic time: *Half-life*

Time	# of X atoms left	
0	100	
1 year (1 half-life)	50	(1/2 left)
2 years (2 half-lives)	25	(1/4 left)
3 years (3 half-lives)	12.5	(1/8 left)
4 years (4 half-lives)	6.25	(1/16 left)
.		.
n years (n half-lives)	$\frac{100}{2^n} = \frac{N}{2^n}$	($1/2^n$ left)

$$\boxed{\begin{gathered} N = N_0\left(\frac{1}{2}\right)^n \\ t = nT_{1/2} \end{gathered}}$$

Let's make a plot of the atoms left versus time:



Example: Assume I have a box of 5 × 10^8 Kr atoms sitting on the table. How many will be left in the box after 45 minutes?

Solution: **From the table we just looked at, we see that the half-life of Kr is 3.16 min.**

Each ½ life reduces the number of Kr nuclei by ½. Thus, after *n* half-lives, I have:

$$\frac{N}{2^n}$$ **nuclei left.** **So how many half-lives do we have in 45 min?**

$$n = \frac{45\,\text{min}}{3.16\,\text{min}} = 14.24$$ **or, 14 complete half-lives.**

$$\text{So, } N_f = \frac{N}{2^n} = \frac{5\times10^8}{2^{14}} = \boxed{30{,}517}$$

We can talk about the rate at which any particular radioactive sample decays.

This is called the Activity, and it equals the # of disintegrations per second.

As each nuclei disintegrates, N decreases. The more nuclei I start with, the more that will disintegrate in a given time period:

$$\frac{\Delta N}{\Delta t} \propto N \quad \Rightarrow \boxed{\frac{\Delta N}{\Delta t} = -\lambda N}$$

The Activity Equation

λ is the proportionality constant, and it's called the decay constant. It has units of inverse time.

The minus sign indicates that N decreases as time passes.

The units on activity: $$\frac{\Delta N}{\Delta t} = \left[\frac{\text{Disintegrations}}{\text{s}}\right] = [\text{Bq}]$$ (Becquerel) [decays/s]

Activity can also be measured in Curies: $$\boxed{1\,\text{Ci} = 3.70 \times 10^{10}\ \text{Bq}}$$

So the radioactive decay of atoms vs. time is an exponential process:

$$\boxed{N = N_o e^{-\lambda t}}$$

N is the # of nuclei left after time t.

N_o is the # of nuclei initially.

Notice, that at t = 0, $N = N_o$.

Remember, e is the base for natural log ($\ln$). It is a constant: e = 2.178…

We can relate the decay constant λ to the half-life $T_{1/2}$ since we know that when:

$$t = T_{1/2}, \qquad N = \frac{N_o}{2}. \qquad \textbf{Thus,} \quad \frac{N_o}{2} = N_o e^{-\lambda T_{1/2}} \quad \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

Take the $\ln$ of both sides: $\ln(\tfrac{1}{2}) = \ln\left(e^{-\lambda T_{1/2}}\right) \Rightarrow \cancelto{0}{\ln(1)} - \ln(2) = -\lambda T_{1/2}$

$$\Rightarrow \ln(2) = \lambda T_{1/2} \quad \Rightarrow \boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

Example: Let's redo the Kr problem: Assume I have a box of 5×10^8 Kr atoms sitting on the table. How many will be left in the box after 45 minutes?

First, we calculate the decay constant:

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.16\ \mathrm{min}} = 0.219\ \mathrm{min}^{-1}$$

Recall that $T_{1/2}$ for Kr is 3.16 min.

Now that we have λ, we can calculate the # of radioactive nuclei left:

Same units

$$N = N_o e^{-\lambda t} = (5\times 10^8)e^{-(0.219)(45)} = \boxed{26{,}242}$$

Notice, this # is different than we calculated before (30,517), since we didn't round to the nearest number of complete half-lives.

So here is your toolbox of equations for radioactivity:

$$\boxed{N = N_o e^{-\lambda t}}$$

$$\boxed{\lambda = \frac{0.693}{T_{1/2}}}$$

$$\boxed{\frac{\Delta N}{\Delta t} = -\lambda N}$$

Here, λ must be in s^{-1}.

Given the half-life, $T_{1/2}$, you can get λ and then find N or the activity, etc.

Homework review:

Prob. 32.3: a) What is the nuclear radius of ${}^{197}_{79}Au$?
b) What is the nuclear radius of ${}^{60}_{27}Co$?

Solution **Empirical nuclear radius:**

$$\boxed{r \approx (1.2\times10^{-15}\ \text{m})A^{1/3}}$$

a): ${}^{197}_{79}Au$ A =197 $r \approx (1.2\times10^{-15}\ \text{m})(197)^{1/3}$

b): ${}^{60}_{27}Co$ A =60 $r \approx (1.2\times10^{-15}\ \text{m})(60)^{1/3}$

Homework review:

Prob. 32.31: The number of radioactive nuclei in a particular sample decreases over a period of t =16 days to one-sixteenth the original number. A) What is the half-life of these nuclei? B) What is the decay constant? C) What is the activity at the end of 16 days if the initial # of nuclei is $5x10^{15}$?

Solution :

a):

$$\boxed{\begin{gathered} N = N_0 (\frac{1}{2})^n \\ t = nT_{1/2} \end{gathered}}$$

$$T_{1/2} = \frac{t}{n} = \frac{16 \times 24 \times 60 \times 60 s}{4}$$

b):

$$\boxed{\lambda = \frac{0.693}{T_{1/2}}}$$

c): $$\text{activity} = \left| \frac{\Delta \mathrm{N}}{\Delta \mathrm{t}} \right| = \lambda N = \lambda N_0 e^{-\lambda t} = \lambda N_0 (\frac{1}{2})^{n=4}$$