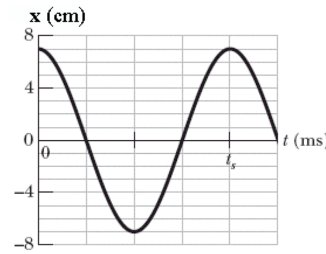
Supplementary HW #9: Ch. 15

1. The figure gives the position function of a 50g block in simple harmonic oscillation on the end of a spring. The period of the oscillation is 2 s. 1) Find the velocity and acceleration function? 2) Find the maximum kinetic energy, potential energy, and total energy. 3) Find the maximum force. 4) If you want to stop the oscillation in a quarter of period, what is minimum kinetic friction coefficient required?



(Answer: $\mu_k = 0.02$)

Solutions:

$$a)\quad x(t) = x_m\cos(\omega t) \quad \text{with } \omega = \frac{2\pi}{T} = 3.14(1/s) \quad \text{and } x_m = 0.07m$$

$$v(t) = -\omega x_m \sin(\omega t)$$

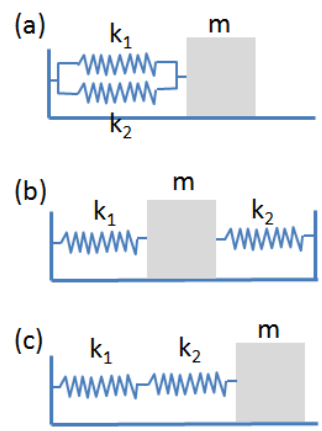
$$a(t) = -\omega^2 x_m \cos(\omega t)$$

$$(b)\quad F_{\max} = -kx_m;\quad U_{\max} = \frac{kx_m^2}{2};\quad K_m = \frac{mv_m^2}{2} = \frac{m(\omega x_m)^2}{2} = \frac{kx_m^2}{2}$$

$$(c)\quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \quad \text{so that } k = \frac{4\pi^2 m}{T^2}$$

$$\text{Since}\quad W = F\cdot x_m = \mu_k mgx_m = \frac{1}{2}kx_m^2$$

$$\mu_k = \frac{1}{2mg}kx_m = \frac{1}{2mg}\left(\frac{4\pi^2 m}{T^2}\right)x_m = \frac{2\pi^2}{T^2 g}x_m = 0.02$$



2. Two springs with the same unstretched length but different force constants k_1 and k_2 are attached to a block of mass m on a level, frictionless surface. Calculate the effect force constant k_{eff} in each of the three cases (a), (b), and (c) depicted in the figure (the effective force constant is defined as $\sum F_x = -k_{eff}x$.)

Solutions:

(a) Image the mass moves a distance of Δx, the springs move Δx_1 and Δx_2

with force $F_1 = -k_1\Delta x_1,\quad F_2 = -k_2\Delta x_2$

$\Delta x = \Delta x_1 = \Delta x_2,\quad F = F_1 + F_2 = -(k_1+k_2)\Delta x$

so $k_{eff} = k_1 + k_2$

(b) Since $\Delta x = \Delta x_1 - \Delta x_2;\quad k_{eff} = k_1 + k_2$

(c) $F = F_1 = F_2;\quad \Delta x = \Delta x_1 + \Delta x_2$

so $\Delta x = -\left(\frac{1}{k_1} + \frac{1}{k_1}\right)F$ and $k_{eff} = \frac{k_1 k_2}{k_1 + k_2}$

3. An approximation for the potential energy of a KCL molecule is $U = A[(R_0^7/8r^8) - \frac{1}{r}]$ where R_0 = 2.67 × 10^{-10} m and A = 2.31 × 10^{-28} J m. Using this approximation: (a) Show that radial component of the force on each atom is $F_r = A[(R_0^7/r^9) - \frac{1}{r^2}]$. Show that R_0 is the equilibrium separation of the K and Cl atoms. (c) Find the minimum potential energy. (d) Use $r = R_0 + x$ and the first two terms of the binomial theorem $\{(1+u)^n = 1 + nu + \frac{n(n-1)}{2!}u^2 + \frac{n(n-1)(n-2)}{3!}u^3 + \cdots$, if $u \equiv x/R_0 << 1\}$ to show that $F_r \approx -(aA/R_0^3)x$ so that the molecule's force constant is $k = 7A/R_0^3$. (e) With both the K and Cl atoms vibrating in opposite directions on opposite sides of the molecule's center of mass $\frac{m_1 m_2}{m_1 + m_2}$ = 3.06 × 10^{-26} kg is the mass to use in calculating the frequency. Calculate the frequency of small-amplitude vibration.

(Answer: (e) f = 8.39 × 10^{12} Hz)

Solutions:

(a) $F_r = -\frac{dU}{dr} = A[(\frac{R_0^7}{r^9}) - \frac{1}{r^2}]$

(b) Set $F_r = 0$ for $r = r_{eq}$ that is, $\frac{R_0^7}{r_{eq}^9} = \frac{1}{r_{eq}^2}$ or $r_{eq} = R_0$

(c) $U(R_0) = -\frac{7A}{8R_0} = -7.57 \times 10^{-19}\, J$

(d) From (a) we have

$$F_r = A[(\frac{R_0^7}{r^9}) - \frac{1}{r^2}] = \frac{A}{R_0^2}[(\frac{r}{R_0})^{-9} - (\frac{r}{R_0})^{-2}]$$

$$= \frac{A}{R_0^2}[(1+\frac{x}{R_0})^{-9} - (1+\frac{x}{R_0})^{-2}] \quad (x << R_0)$$

$$\cong \frac{A}{R_0^2}\{1 - 9(\frac{x}{R_0}) - [1 - 2(\frac{x}{R_0})]\} = -7\frac{A}{R_0^3}x$$

(e) $f = \frac{\sqrt{k/m}}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{7A}{mR_0^3}} = 8.39 \times 10^{12}\,\text{Hz}$

4. You want to construct a physical pendulum with a period of 4.0 s at a location where g = 9.8m /s^2. (a) What is the length of a *simple* pendulum having this period? (b) Suppose the physical pendulum must mounted in a case that is not more than 0.5 m high. Can you devise a physical pendulum having a period of 4.0 s that will satisfy this

requirement? (hint: consider a uniform slender rod of mass *M* and length *L* = 0.5 m and pivot the rod about an axis that is a distance *d* above the center of the rod)

(Answer: (a) *L* =3.97 M; (b) *d* = 0.0053 m)

Solutions:

(a) $T = 2\pi\sqrt{L/g}$ so $L = g(T/2\pi)^2 = 3.97m$

(b) Use a uniform slender rod of mass M and length $L = 0.5$ m

Pivot the rod about an axis that is a distance d above the center of the rod.

The rod will oscillate as a physical pendulum with period $T = 2\pi\sqrt{I/Mgd}$

Choose d so that $T = 4.0$ s

$$I = I_{cm} + Md^2 = \frac{1}{12}ML^2 + Md^2 = M(\frac{1}{12}L^2 + d^2)$$

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M(\frac{1}{12}L^2 + d^2)}{Mgd}} = 2\pi\sqrt{\frac{\frac{1}{12}L^2 + d^2}{gd}}$$

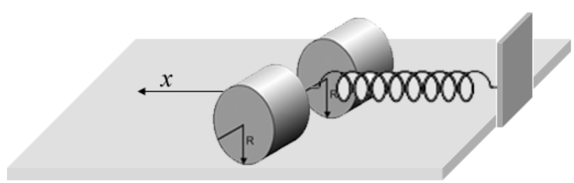
so that

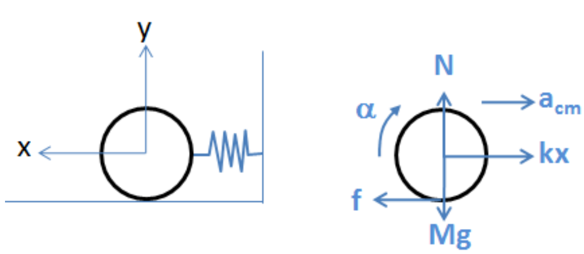
$$d^2 - (\frac{T}{2\pi})^2 gd + L^2/12 = 0 \quad \text{(set } L = 0.5 \text{ m and } T = 4.0 \text{ s)}$$

$d = 3.97$ m or $d = 0.0053$ m

since d can not bigger than L/2, the correct answer is d = 0.0053 m = 0.53 cm

5. Two solid cylinders connected along their common axis by a short, light rod have radius *R* and total mass *M* and rest on a horizontal tabletop. A spring with force constant *k* has one end attached to a clamp and the other end to a frictionless ring at the center of mass of the cylinders (see the figure). The cylinders are pulled to the left a distance *x*, which stretches the spring, and released. There is sufficient friction between the tabletop and the cylinders to roll without slipping as they move back and forth on the end of the spring. (a) Show that the motion of the center of mass of the cylinders is simple harmonic. (b) Calculate its period in terms of *M* and *k*. (c) if the amplitude of the simple harmonic motion is A, what is the maximum value of the velocity of the center of mass of the cylinders.





(Answer: (b) $T = 2\pi\sqrt{3M/2k}$)

Solutions:

(a) Apply $\sum \tau = I_{cm}\alpha$ and $\sum F_x = Ma_{cm}$

Based upon the coordination in the figure,

$\sum \tau = fR = (\frac{1}{2}MR^2)\alpha$ or $f = \frac{1}{2}MR\alpha = \frac{1}{2}Ma_{cm}$

$\sum F_x = f - kx = -Ma_{cm}$ so that

$kx = \frac{3}{2}Ma_{cm} \quad (a_x = -a_{cm})$

$a_x = -\frac{2k}{3M}x$ thus give a SHO compared with $a_x = -\omega^2 x$

(b) $\omega^2 = 2k/3M$ so that $T = 2\pi/\omega = 2\pi\sqrt{3m/2k}$

(c) $U_{\max} = \frac{1}{2}kA^2 = KE_{\max} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}(\frac{v_{cm}}{R})^2 = \frac{3}{4}Mv_{cm}^2$

$v_{cm} = \sqrt{\frac{2k}{M}}A$

Another way of solving this problem form energy point of view:

Total mechanical energy is given as

$$E_{mech} = U_s + \frac{Mv^2}{2} + \frac{I_{CM}^2\omega^2}{2}$$

$$\frac{dE_{mech}}{dt} = \frac{d}{dt}\left(\frac{3Mv_{cm}^2}{4} + \frac{kx^2}{2}\right) = \frac{3Mv_{cm}a_x}{2} + kxv_{cm}$$

However, as we know $E_{mech} = const.$ so that $\frac{dE_{mech}}{dt} = 0$

$a_x = -\left(\frac{2k}{3M}\right)x$ and $\omega = \sqrt{\frac{2k}{3M}}$