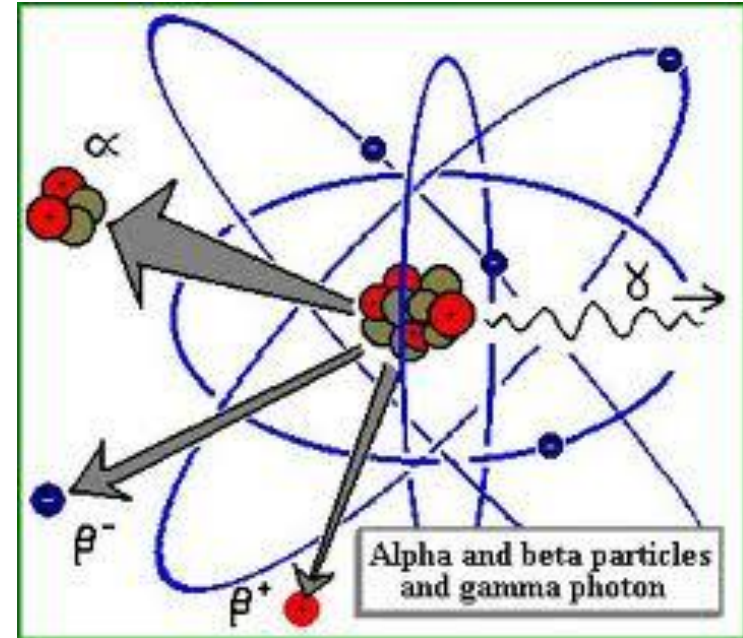# Algebra-based Physics II

Dec. 3rd: **Chap 31 Nuclear Physics and Radioactivity**

• **How to find nuclear binding energy?**

• **How to calculate radius of a nucleus?**

• **How to identify different radioactive decay?**

• **How to determine radioactive activity?**

Alpha and beta particles and gamma photon

Announcements:

• Final exam: Mon-Wed.

• Extra Review session: Sunday 5-7:00 PM, Nicholson #130

Chapter 31 – Nuclear Physics and Radioactivity

31.1 Nuclear Structure

We've been studying atomic structure – the Rutherford model of the atom, where electrons orbit around a nucleus.

And now we know that it takes Quantum Mechanics to correctly describe the nature of these orbits.

But what about the **nucleus**?

Well, so far we just know about **protons.**

But, in 1932 scattering experiments by English physicist James Chadwick discovered another particle in the nucleus which had no charge → **neutron**.

The nucleus is composed of positive protons and neutral neutrons.

These are called <u>nucleons.</u>

What distinguishes different elements in the periodic table is the # of protons they have in their nucleus. This is called the <u>Atomic Number (Z).</u>

Each element also has an <u>Atomic Weight</u> or <u>Atomic Mass Number (A).</u>

Let N be the # of neutrons in the nucleus, then: $\boxed{A = Z + N}$

Take oxygen, for example:

Atomic number

Atomic mass number

15.9994

8

-2

MP → 90.18

BP → 50.15

1.429

$1s^2 2s^2 2p^4$

Symbol

O

Density

Name

Electronic configuration

Oxygen

So oxygen has 8 protons and $N = A - Z = 16 - 8 = 8$

8 Neutrons!

Shorthand notation for element representation:

$$^A_Z X$$

So oxygen would be: $^{16}_{8} O$
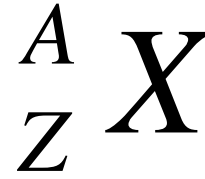
The mass of the elements is usually given in <u>atomic mass units (u).</u>

$$1u = 1.6605 \times 10^{-27} \text{ kg}$$

Nuclei that contain the same number of protons but a different number of neutrons are called <u>isotopes.</u>

For example, boron exists in nature as two stable isotopes:

$$^{11}_{5} B \qquad ^{10}_{5} B$$
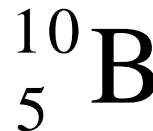
Most boron atoms have 6 neutrons (81.1%), but some (18.9%) have only 5 neutrons.

The atomic mass number (A) listed on the periodic table is the average atomic mass of the isotopes, for boron this is 10.811.

What does the nucleus look like?

Why Coulomb repulsion does tear apart nucleus?

The protons and neutrons are clustered together in a blob that is approximately spherical. The radius of the nucleus is ~ 1 fm = 1 $\times$ 10⁻¹⁵ m.
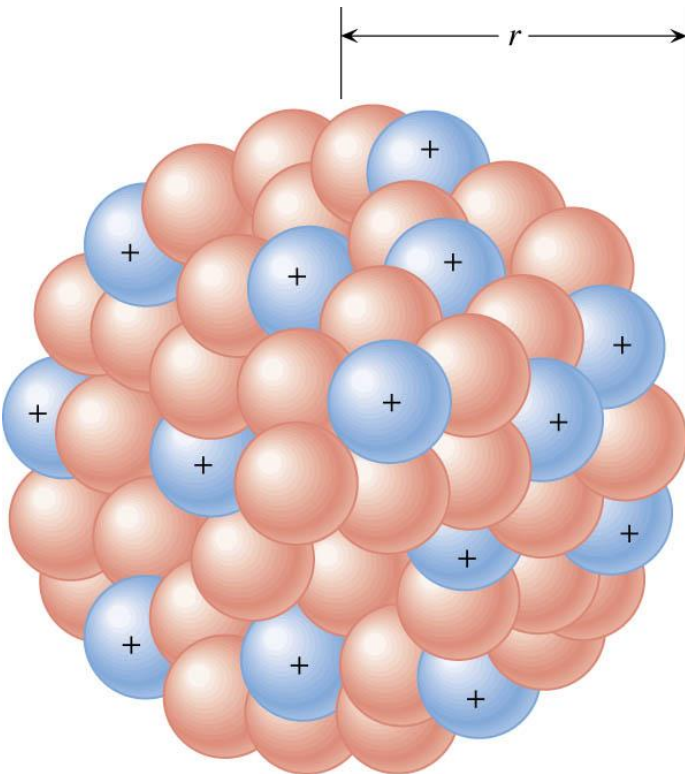
Experiments show that:

$$r \approx (1.2 \times 10^{-15} \text{ m}) A^{1/3}$$

The Nucleon Density, which is the # of protons and neutrons per unit volume, is the same for all atoms!

So we have this spherical blob of positive protons and neutral neutrons.

How does it stay together???

Example: He nucleus: 2 protons and 2 neutrons
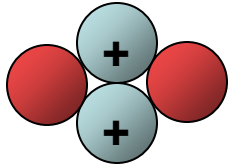
He nucleus, 2 protons and 2 neutrons.

Now I have 2 protons very close together.

We know that like charges repel, and we can calculate the repulsive force between the two protons using Coulomb's Law.

Assuming the charges are separated by 1 fm, we get a force of 231 N. This results in an acceleration of the protons = 1.4×10^{29} m/s^2. HUGE!!!

There must be an attractive force holding them together, and it's not gravity, because the gravitational attraction between to subatomic particles is very small: ~1.9×10^{-34} N!

There is a new type of force. It's the <u>Strong Nuclear Force</u>.

<u>Characteristics of the strong nuclear force:</u>

1. It is independent of charge. Thus, the attractive force between 2 protons is the same as that between 2 neutrons or between a neutron and a proton.

2. The range of the force is extremely short. It is very strong for distances ~1 fm and essentially zero at farther distances.

Most nuclei listed in the periodic table are stable, but some are not.

> **The stability of a nucleus depends on the balancing between the electrostatic repulsion between protons and the strong nuclear attractive force between all nucleons.**
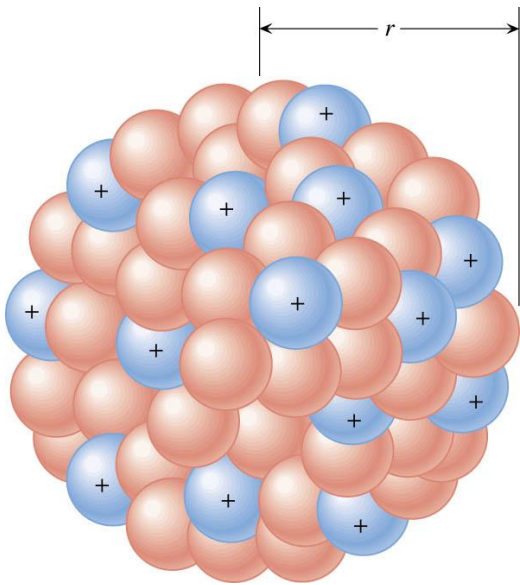
Every proton in the nucleus feels Coulombic repulsion from every other proton, since the electrostatic force is long-ranged.

But each proton and neutron only feels the strong nuclear force from its closest neighbors.

To compensate for this, the more protons I add to the nucleus, an even greater number of neutrons must be added to try and balance the electrostatic force.

As I create larger and larger nuclei, the neutron number keeps getting bigger and bigger.

Notice how the neutron # deviates from the $N = Z$ line for large nuclei.

Eventually, as more and more protons are added, no # of extra neutrons can compensate for the large electrostatic repulsion, and the nucleus breaks apart.

This occurs at Z = 83, which is bismuth (Bi).

Any element with an atomic number Z > 83 will be unstable and break apart over time. It will rearrange itself into a stable nuclei. This process is called Radioactivity.
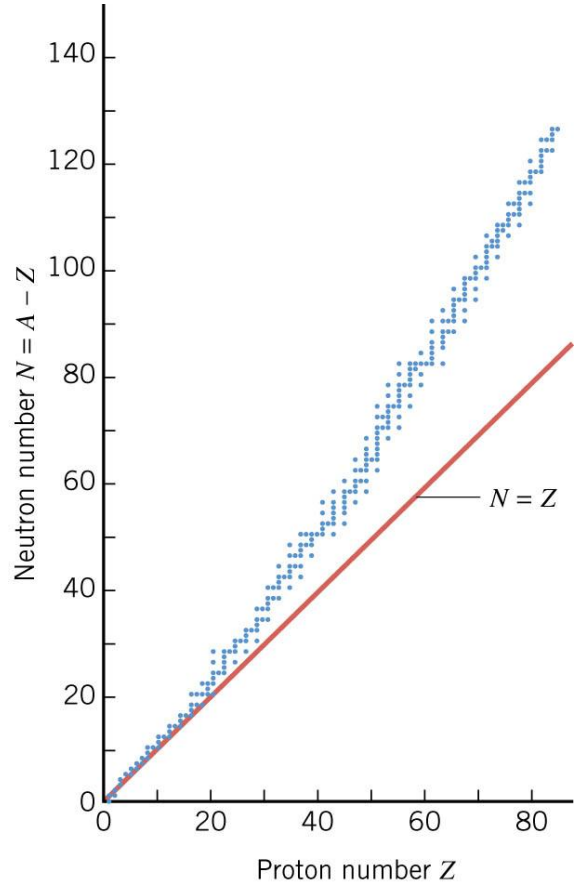
Two different nuclei have different numbers of protons and a different number of neutrons. Which of the following could be true?

1. They are different isotopes of the same element.

2. They have the same electric charge.

3. Their nucleon density is the same.

54%

33%

13%

They are different iso...

They have the same e...

Their nucleon density ..
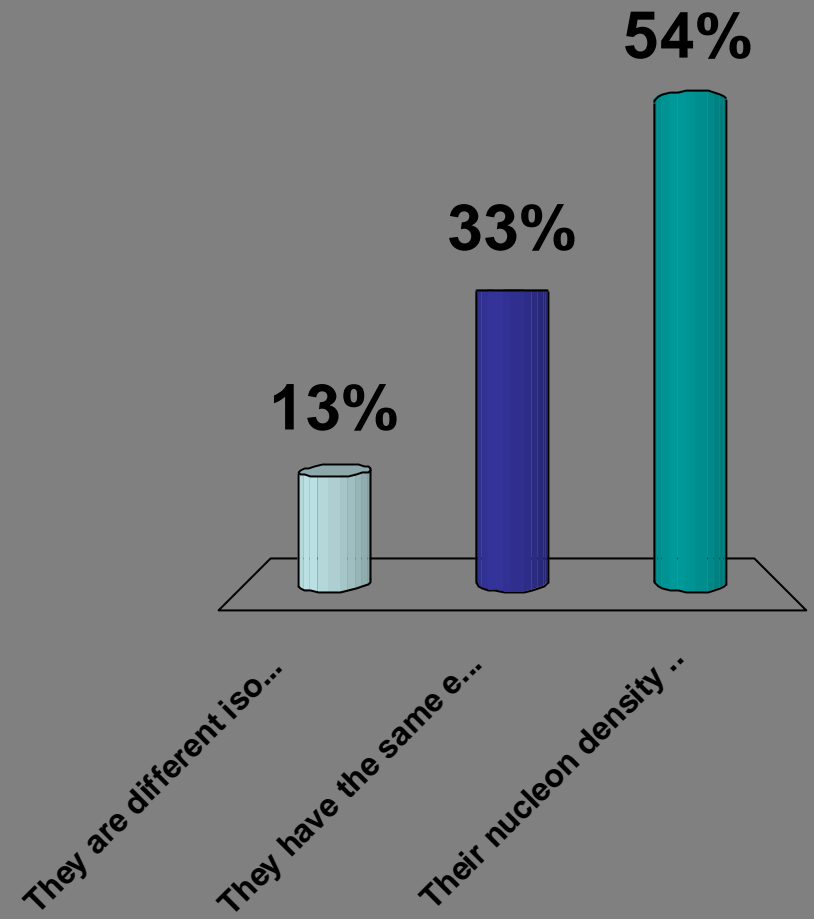
31.3 Nuclear Binding Energy

In order to separate two nucleons, we have to overcome the **strong nuclear force.**

We refer to this as the **Nuclear Binding Energy.**
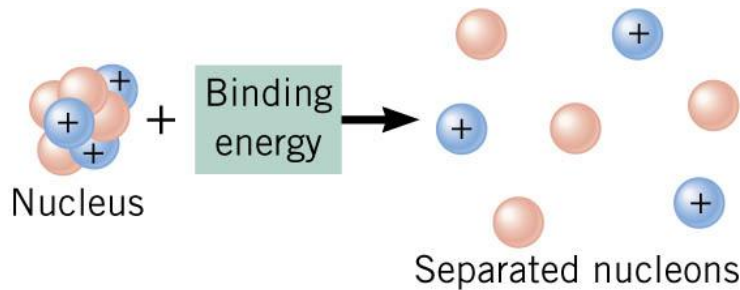
To analyze this problem quantitatively, we have to use Einstein's famous $E = mc^2$.

This equation gives us the **rest mass energy.**

Thus, a change in mass of a system equals a change in rest mass energy: $\Delta E = \Delta mc^2$.

The binding energy of the nucleus appears as **extra mass** in the separated nucleons.

In other words, the sum of the masses of the individual nucleons is greater than the mass of the stable nucleus by an amount Δm, where Δm = **the mass defect of the nucleus.**

The nuclear binding energy then is just this mass converted to energy:

$$\text{Binding Energy} = (\text{mass defect})c^2 = \Delta mc^2$$

Example: **The binding energy of a helium nucleus:**

The most abundant isotope of helium has a ^4_2He nucleus whose mass is 6.6447×10^{-27} kg. For this nucleus, calculate (a) its mass defect and (b) its binding energy.

+ **Binding Energy** →

Nucleus

2 separated protons and 2 separated neutrons
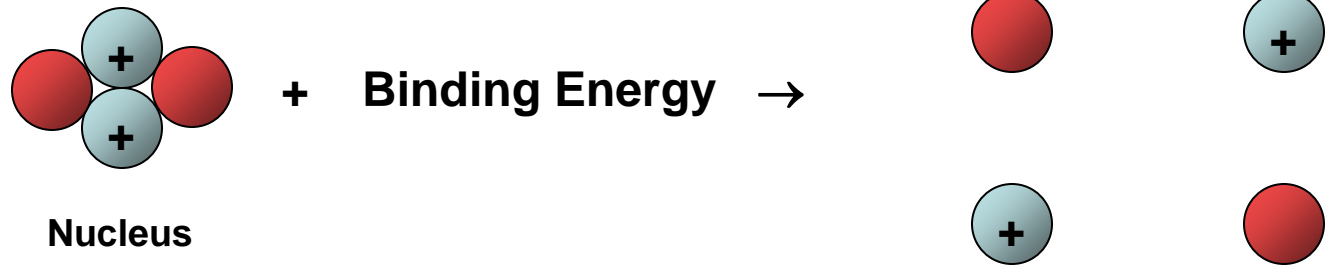
Solution:

(a) The mass defect = The mass of the separated nucleons - The mass of the stable nucleus

$$\Delta m = 2m_p + 2m_n - m_{\text{He nucleus}}$$

$$\Rightarrow \Delta m = 2(1.6726 \times 10^{-27}) + 2(1.6749 \times 10^{-27}) - 6.6447 \times 10^{-27}$$

$$= 5.03 \times 10^{-29} \text{ kg}$$

(b) Binding Energy, $E = (\Delta m)c^2 \Rightarrow E = (5.03 \times 10^{-29})(3 \times 10^8)^2$

$$= 4.53 \times 10^{-12} \text{ J} = 2.83 \times 10^7 \text{ eV} = \boxed{28.3 \text{ MeV}}$$

This is 2 million times greater energy than it takes to remove an electron from an atom!

In the previous example we used kg for the mass units, but it is customary in these types of problems to use atomic mass units, u.

Example: **let's say we wanted to calculate the binding energy of a He nucleus given its atomic mass: 4.0026 u. Now, this mass includes the mass of two electrons, since it's the atomic mass instead of the nuclear mass.**

$^4_2$He

$^1_1$H $^1_0$n

Binding energy

$^1_1$H $^1_0$n

Mass = 4.0026 u

Mass = 4.0330 u
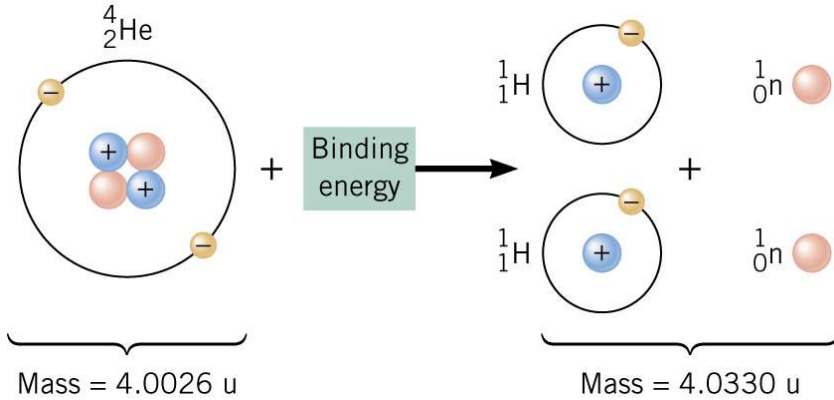
Mass defect:

$$\Delta m = 2m_{\text{Hydrogen atoms}} + 2m_n - m_{\text{He atom}}$$

$$= 2(1.0078\,\text{u}) + 2(1.0087\,\text{u}) - 4.0026\,\text{u}$$

H mass   **Neutron mass**   **He atom mass**

$$\Rightarrow \Delta m = \boxed{0.0304\,\text{u}}$$

$$\boxed{1\,\text{u} \equiv 931.5\,\text{MeV}}$$

$$\boxed{E = (\Delta m\,[\text{u}]) \times 931.5\,\text{MeV}}$$
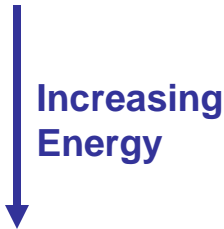
Binding Energy: $$E = (0.0304) \times 931.5\,\text{MeV} = \boxed{28.3\,\text{MeV}}$$

31.4 Radioactivity

So any nucleus with more than 83 protons is unstable, or <u>radioactive.</u>

These nuclei disintegrate over time, and during this process, 3 types of radiation may be produced:

The 3 types of radiation were discovered by Becquerel in 1896, and then later labeled by Rutherford:

1. α **Rays**

2. β **Rays**   **Increasing Energy**

3. γ **Rays**

1. α Rays are actually α particles (helium nuclei). They consist of 2 protons and 2 neutrons.

2. β Rays are electrons.

3. γ Rays are high-energy photons.

A radioactive nucleus is emitting all 3 types of radiation. The radiation travels through a uniform magnetic field that is directed everywhere into the page and follows the paths shown. What kind of radiation follows path 3?

1. Alpha rays
2. Beta rays ✓
3. Gamma rays
4. No way to tell.

Magnetic field (into paper)

Lead cylinder

Evacuated chamber

Radioactive material

Photographic plate

× × ×
× 1 × ×
  2
× × ×
  3
× × ×

68%

17%

12%

3%

Alpha rays   Beta rays   Gamma rays   No way to tell...
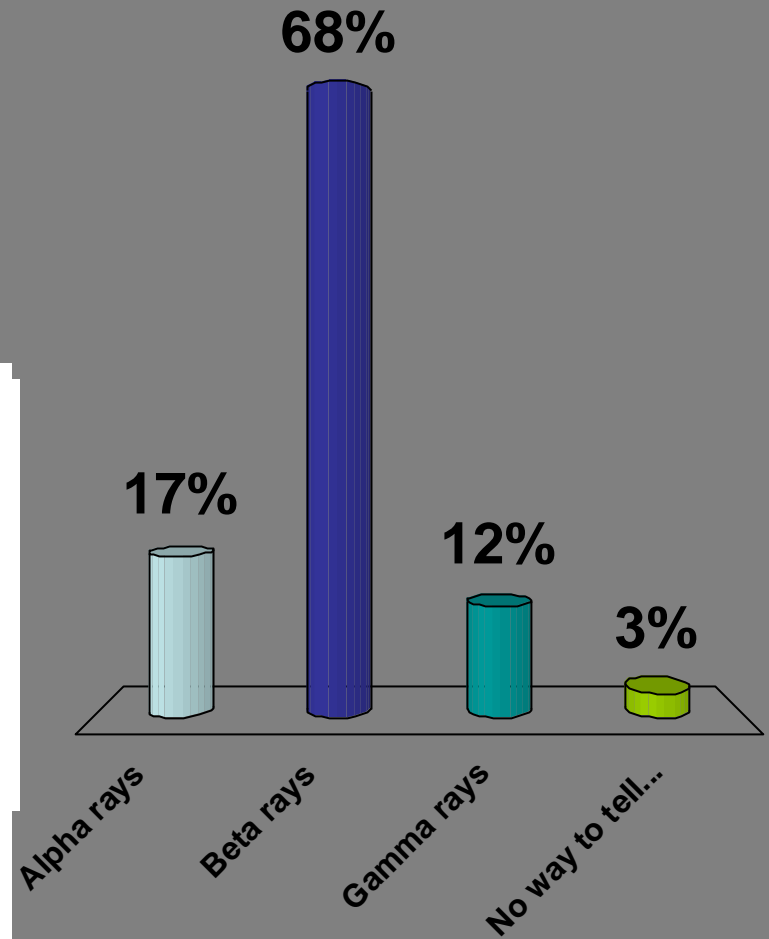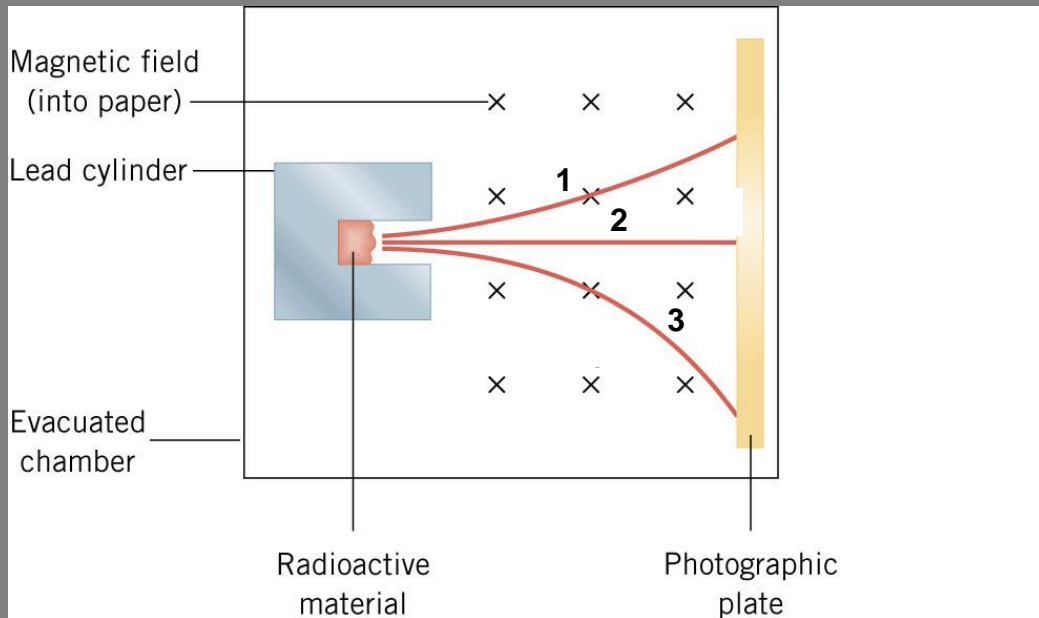
1. α Radiation: A nucleus that is emitting α particles is undergoing <u>α decay</u>.

Each α particle is a He nucleus, i.e.: ^4_2He

Let's take for example, ^{238}U which α decays to ^{234}Th: $\qquad U \rightarrow Th + \alpha$

Explicit expression: $\qquad ^{238}_{92}\text{U} \rightarrow\ ^{234}_{90}\text{Th} + ^4_2\text{He}$

Parent Nucleus   **Daughter** Nucleus   α particle
(P)   (D)
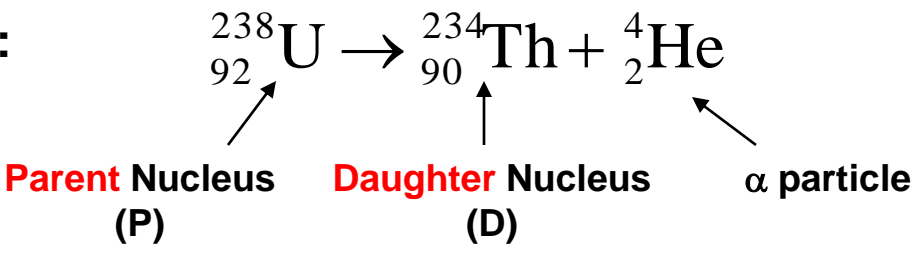
When a parent nucleus is converted into another element, it is called a <u>transmutation.</u>

*Thus, for a transmutation to occur, the proton # (Z) of the parent <u>must</u> change.

So, in general then, for <u>α decay</u>, we have:

$$^A_Z\text{P} \rightarrow\ ^{A\text{-}4}_{Z\text{-}2}\text{D} + ^4_2\text{He}$$
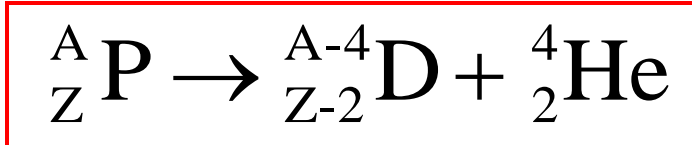
*Notice that the number of nucleons is conserved!

2. β Radiation:

Beta rays are negatively charged particles, and experiments show them to be electrons.

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$
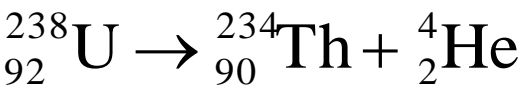
After the initial alpha decay, the daughter nucleus (^{234}Th) is still radioactive. It then undergoes beta decay:

$$Th \rightarrow Pa + \beta^{-}$$

Explicitly, we write this as:
$$^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e$$
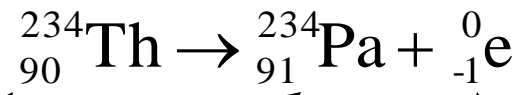
Parent Nucleus (P)   **Daughter Nucleus (D)**   β particle (electron)

So, in general then, for β decay, we have:

$$\boxed{^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + {}^{0}_{-1}e}$$
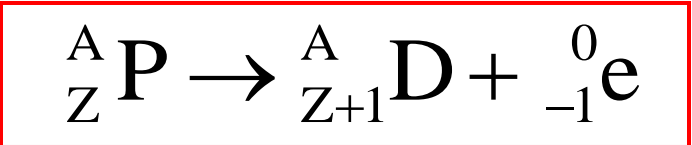
***Notice again, that the charge is conserved!**

So where does the electron come from???

It turns out that neutrons themselves are unstable, and they will decay into a proton.

When the neutron decays into a proton, it releases an electron (β⁻) and an antineutrino ($\bar{v}_{e}$)

There is a 2nd type of β decay called β+ decay.

Here, the nucleus emits a positron.

The positron is the electron's antiparticle: it's antimatter!

The positron has the same mass and spin of the electron, but opposite electric charge.

The positron is created when a nuclear proton is transformed into a neutron:

$$\ _{Z}^{A}P \rightarrow\ _{Z-1}^{A}D +\ _{+1}^{0}e$$

Parent Nucleus   **Daughter** Nucleus   β+ particle
(P)   (D)   (electron)
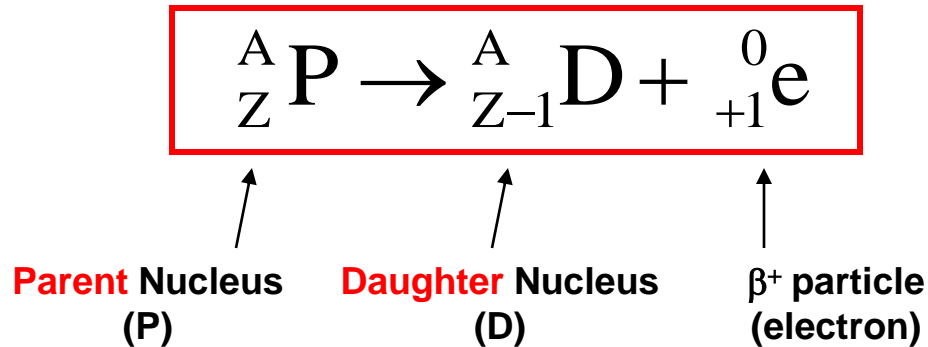
There is a 3rd type of β decay called electron capture:

Here, the nucleus pulls in an orbital electron from the K-shell.

3. γ Radiation:

In gamma decay, a nucleus that is in an **excited state** decays to a lower energy state and emits a photon.

The nucleon energy, like the energy of the orbital electrons, exist only in discreet (quantized) levels.

This decay is the nuclear analogue of the emission of light by atoms, i.e. electrons making transitions between two atomic energy levels.

$\Big|$ Orbital electron energy levels ~eV → Visible spectrum

$\Big|$ Nuclear energy levels ~MeV → High energy photons (γ rays)

γ decay usually follows α- and β-decay. For example:
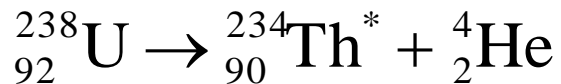
$$^{238}_{92}\text{U} \rightarrow \,^{234}_{90}\text{Th}^{*} + \,^{4}_{2}\text{He}$$ **The * means the Th nucleus is in an excited state:**

$$^{234}_{90}\text{Th}^{*} \rightarrow \,^{234}_{90}\text{Th} + \gamma$$ **So, in general then, for <u>γ decay</u>, we have:**

Excited state   **Lower energy state**   **γ ray**

$$\boxed{^{A}_{Z}\text{P}^{*} \rightarrow \,^{A}_{Z}\text{P} + \,^{0}_{0}\gamma}$$
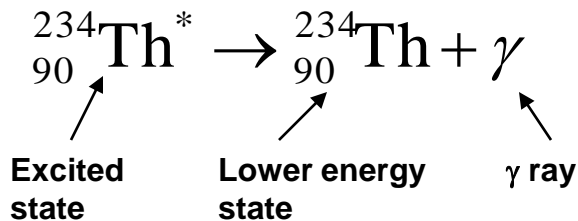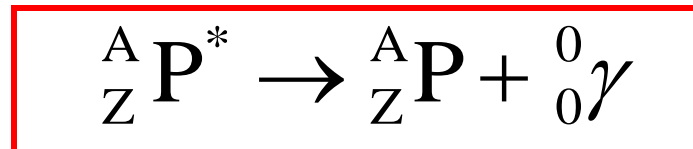
Assume I have a chunk of radioactive material that contains N radioactive nuclei, or parent nuclei.

Which one of these nuclei is going to decay, and when?

Since radioactivity is a **quantum-mechanical process**, we can't know with certainty. We can only predict when a particular nucleus will decay. Thus, radioactivity is a **statistical process**.

As time passes, some of the nuclei will decay, and N will decrease.

To help describe this process, it's useful to define something called the **half-life**:

It is represented by $T_{1/2}$, and it is the time it takes for ½ of the nuclei present to decay.

So $T_{1/2}$ will have units of time [s]. If the half-life is long, it can also be represented in years.

As an example, let's say that I have some radioactive element X which has a half-life of 1 year, and I originally start with 100 atoms of it.

How many atoms will be left after 1 year, 2 years, 3 years, and so on???

Time	# of X atoms left	
0	100	
1 year (1 half-life)	50	(1/2 left)
2 years (2 half-lives)	25	(1/4 left)
3 years (3 half-lives)	12.5	(1/8 left)
4 years (4 half-lives)	6.25	(1/16 left)
.	.	
n years (n half-lives)	$\dfrac{100}{2^n} = \dfrac{N}{2^n}$	(1/2^n left)

Let's make a plot of the atoms left versus time:
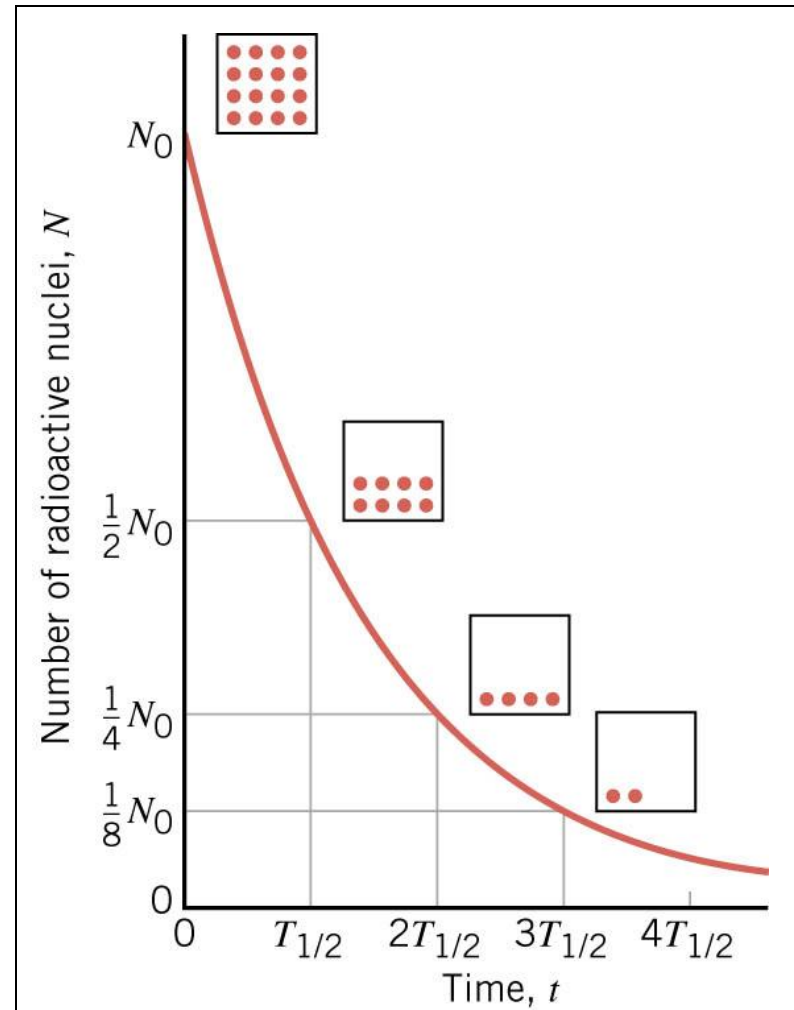
Isotope		Half-life
Polonium	$^{214}_{84}Po$	1.64×10^{-4} s
Krypton	$^{89}_{36}Kr$	3.16 min
Radon	$^{222}_{86}Rn$	3.83 da
Strontium	$^{90}_{38}Sr$	28.8 yr
Radium	$^{226}_{88}Ra$	1.6×10^3 yr
Carbon	$^{14}_{6}C$	5.73×10^3 yr
Uranium	$^{238}_{92}U$	4.47×10^9 yr
Indium	$^{115}_{49}In$	4.41×10^{14} yr

Example: Assume I have a box of 5×10^8 Kr atoms sitting on the table. How many will be left in the box after 45 minutes?

<u>Solution:</u> **From the table we just looked at, we see that the half-life of Kr is 3.16 min.**

Each ½ life reduces the number of Kr nuclei by ½. Thus, after *n* half-lives, I have:

$\dfrac{N}{2^n}$ **nuclei left.** **So how many half-lives do we have in 45 min?**

$$n = \frac{45 \, \text{min}}{3.16 \, \text{min}} = 14.24 \qquad \text{or, 14 complete half-lives.}$$

$$\text{So, } N_f = \frac{N}{2^n} = \frac{5 \times 10^8}{2^{14}} = \boxed{30{,}517}$$

We can talk about the rate at which any particular radioactive sample decays.

This is called the Activity, and it equals the # of disintegrations per second.

As each nuclei disintegrates, N decreases. The more nuclei I start with, the more that will disintegrate in a given time period:

$$\frac{\Delta N}{\Delta t} \propto N \implies \boxed{\frac{\Delta N}{\Delta t} = -\lambda N}$$

The Activity Equation

λ is the proportionality constant, and it's called the decay constant. It has units of inverse time.

The minus sign indicates that N decreases as time passes.

The units on activity: $\quad \dfrac{\Delta N}{\Delta t} = \left[\dfrac{\text{Disintegrations}}{\text{s}} \right] = [\text{Bq}]$ **(Becquerel) [decays/s]**

Activity can also be measured in Curies: $\quad \boxed{1\,\text{Ci} = 3.70 \times 10^{10}\,\text{Bq}}$

So the radioactive decay of atoms vs. time is an exponential process:

$$N = N_o e^{-\lambda t}$$

N is the # of nuclei left after time t.

N$_o$ is the # of nuclei initially.

Notice, that at t = 0, $N = N_o$.

Remember, e is the base for natural log (ln). It is a constant: e = 2.178…

We can relate the decay constant λ to the half-life $T_{1/2}$ since we know that when:

$$t = T_{1/2}, \quad N = \frac{N_o}{2}. \quad \textbf{Thus,} \quad \frac{N_o}{2} = N_o e^{-\lambda T_{1/2}} \quad \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

Take the ln of both sides: $\ln(\tfrac{1}{2}) = \ln\left(e^{-\lambda T_{1/2}}\right) \Rightarrow \ln(1) - \ln(2) = -\lambda T_{1/2}$

0

$$\Rightarrow \ln(2) = \lambda T_{1/2} \Rightarrow \boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

Example: Let's redo the Kr problem: Assume I have a box of 5×10^8 Kr atoms sitting on the table. How many will be left in the box after 45 minutes?

First, we calculate the decay constant: $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{3.16 \text{ min}} = 0.219 \text{ min}^{-1}$

Recall that $T_{1/2}$ for Kr is 3.16 min.

Now that we have λ, we can calculate the # of radioactive nuclei left:

Same units

$$N = N_o e^{-\lambda t} = (5 \times 10^8) e^{-(0.219)(45)} = \boxed{26,242}$$

Notice, this # is different than we calculated before (30,517), since we didn't round to the nearest number of complete half-lives.

So here is your toolbox of equations for radioactivity:

$$N = N_o e^{-\lambda t} \qquad \lambda = \dfrac{0.693}{T_{1/2}} \qquad \dfrac{\Delta N}{\Delta t} = -\lambda N$$

Here, λ must be in s^{-1}.

Given the half-life, $T_{1/2}$, you can get λ and then find N or the activity, etc.