Algebra-based Physics II

Nov. 15th: Chap 29. 1-3

- Wave-particle duality
- Blackbody radiation
- Photons & the photoelectric effect

Ch. 29 – Particles and Waves

Here we go! Modern physics!

29.1 The wave-particle duality

Newton developed classical physics (kinematics and dynamics) back in the 1600's.

It was Faraday's experiments and Maxwell's mathematics that shaped the field of electromagnetism

From here we discovered that EM waves move at the speed of light, and therefore light itself is an EM wave.

This led to our study of light and optics:

- **Geometric Optics**
  - **Reflection (mirrors)**
  - **Refraction (lenses)**
- **Wave Optics**
  - **Interference**
  - **Diffraction**

Is light behaves just like wave?

In1879 a very important experiment was performed by Josef Stefan.

His experiment deals with radiation that is emitted and absorbed by blackbodies.

29.2 Blackbody radiation

So what is a blackbody????

FACT: All objects are continuously absorbing and emitting radiation.

When light (or any EM radiation) falls on an opaque body, part of it is reflected, and part of it is absorbed.

Light-colored bodies reflect most of the radiation incident on them.

Dark-colored bodies absorb most of the radiation incident on them.

→ Choose wisely the color of your tee-shirt.

If an opaque body is in thermal equilibrium with its surroundings, then it must be absorbing and emitting radiation at the same rate (equally).

It has to, or otherwise it would either heat up or cool off, and then no longer be in thermal equilibrium.

This radiation is known as thermal (heat) radiation.

For objects whose temperatures are $<\sim 600^{o}$ C, this radiation is not in the visible part of the EM spectrum, but in the infrared.

The human body is $\sim 37^{o}$ C, so we emit radiation in the infrared. That's why you can't see us in the dark!

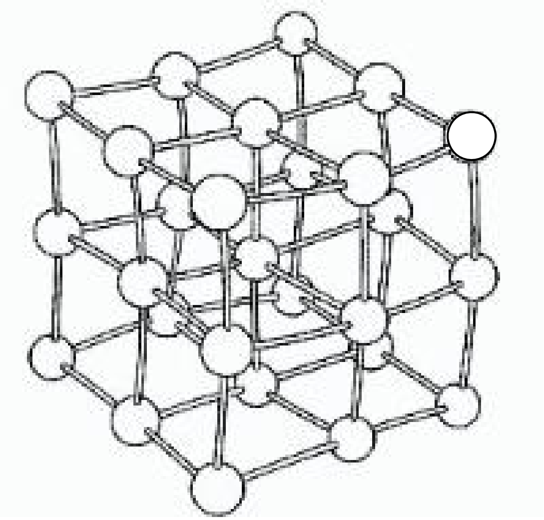
At temperatures between 600 – 700° C, there is enough energy in the visible spectrum that a body begins to glow dull red.

At much higher temperatures it will grow bright red, or even white-hot.

A body that absorbs and emits *all* of the radiation incident on it is called an ideal blackbody.

A blackbody is a piece of matter, and like all matter, it is composed of atoms.

We can treat the atoms in the solid as being connected by invisible springs:



Each atom will vibrate, or oscillate, in 3-dimensions.

This is called the simple harmonic approximation.

It is strictly classical physics.

The vibrating atoms absorb and emit radiation, and classical physics tells us that the intensity of the radiation emitted by the oscillators is proportional to the temperature of the solid:

$$I \propto T$$

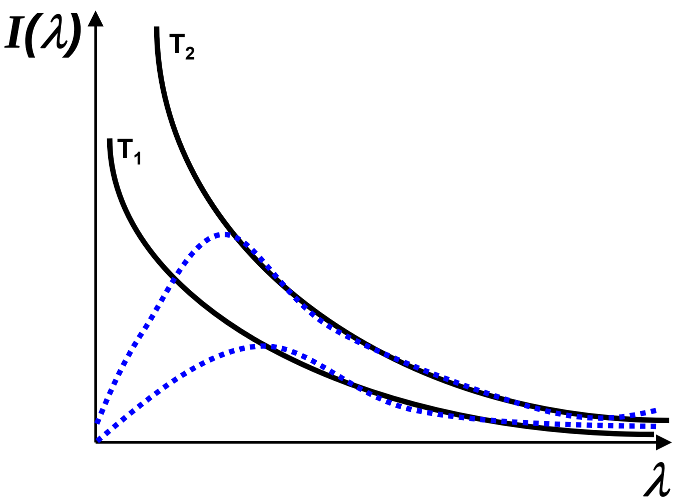
The equality relationship is: $$I(\lambda) = \frac{4kT}{\lambda^2}$$

Yet again, here is another example of an inverse-square law in physics.

This is called the Rayleigh-Jeans Law. It is a classical result.

k in the above equation is the Boltzmann constant: $k = 1.38 \times 10^{-23}$ J/K

So, if we make a plot of the radiation intensity emitted by the atomic oscillators versus wavelength, we would get:

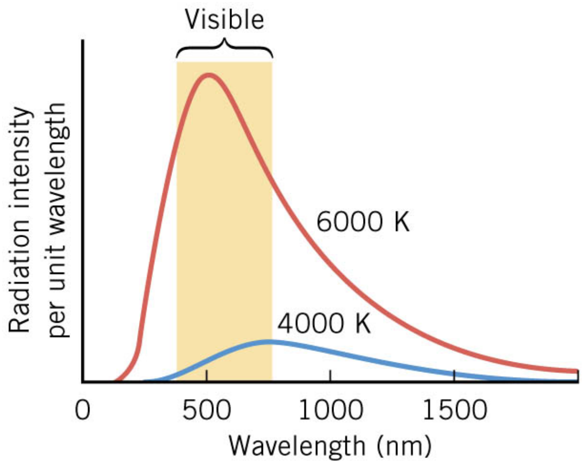


The plots are shown here for two temperatures T_1 and T_2, where $T_2 > T_1$.

Classical theory predicts that $I \to \infty$ as $\lambda \to 0$.

Experimentally, however, we find the following result: in blue.

The classical theory only gets it right at large wavelengths, but fails miserably at low wavelengths. This was known as the Ultraviolet Catastrophe.



Thus, we see that the radiation intensity from a perfect blackbody varies from wavelength to wavelength.

At higher temperatures, the intensity per unit wavelength is greater, and the maximum occurs at smaller wavelengths.

Now it's December, 1900 and a German physicist Max Planck is trying to interpret the blackbody data.

He worked to come up with a theoretical expression that agreed with the experimental data.

He discovered that he could get good agreement between theory and data if he assumed that the energy of the atomic oscillators was a discreet variable.

In other words, the energy could only have certain discreet values, i.e.

$$E = 0, \varepsilon, 2\varepsilon, 3\varepsilon, \ldots n\varepsilon$$

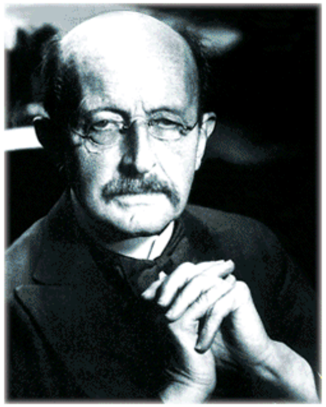
Furthermore, this discreet value, ε, had to be proportional to frequency: $\varepsilon \propto f$

Make this an equality: $\varepsilon = hf$

The proportionality constant, h, is Planck's constant: $h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$

$$E = nhf \, , n = 0,1,2,3,...$$

This was Planck's assumption:



Energy is allowed to only have certain values: It is quantized.

Quantizing the energy has some radical effects.
Take conservation of energy for example:

EM waves carry energy, so a vibrating atom emitting radiation must be losing energy.

Conservation of energy tells us that the energy carried away by the EM wave must be equal to the energy lost by the vibrating atom.

Let's say a vibrating atom has energy, $E = 3hf$.

Since it's emitting radiation, it must be losing energy, and according to Planck, the next lowest energy it could have would be $E = 2hf$.

That means the EM wave must have an energy, $E = hf$.

So energy comes in discreet packets of (hf) called quanta, or quantum of energy.

Planck's work paved the way for the development of the New Physics, or Quantum Mechanics.

Planck wasn't satisfied with his results and conclusion. He tried for several years to reconcile his results with classical physics, but never could.

The significance of his work was never realized until 1905.

<u>1905 – The Miracle Year in Physics</u>

A young former patent clerk named Albert Einstein published 4 papers between March 17 and Sept. 27, 1905.

1. On a Heuristic Viewpoint of the Generation and Conversion of Light

2. A New Measurement of Molecular Dimensions and on the Motion of Small Particles Suspended in a Stationary Liquid

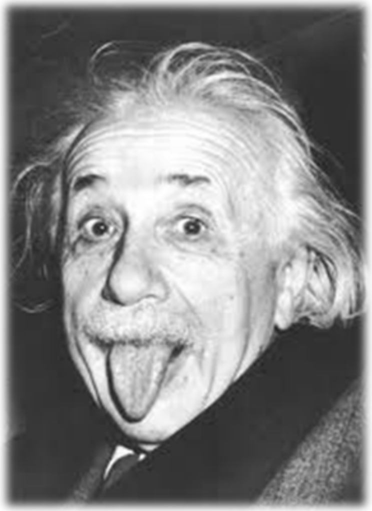
The arguments and calculations are among the most difficult in hydrodynamics and could only be approached by someone who possesses understanding and talent for the treatment of mathematical and physical problems, and it seems to me that Herr Einstein has provided evidence that he is capable of occupying himself successfully with scientific problems. I have examined the most important part of the calculations and have found them to be correct in every respect, and the manner of the treatment testifies to mastery of the mathematical methods concerned.

-Professor Kleiner
University of Zurich

3. On the Electrodynamics of Moving Bodies

4. Does an Object's Inertia Depend on its Energy Content?

It is the topic of Einstein's first paper that we will focus on next.



He worked with Planck's assumption of quantized energy and assumed that it was a universal characteristic of light.

He postulated that the energy of light, instead of being equally distributed throughout space, consisted of discrete quanta of energy called photons.

The energy of light (or a photon) is given by:

$$E = hf = \frac{hc}{\lambda}$$

Thus, if you know the frequency or wavelength of light, you can calculate its energy.

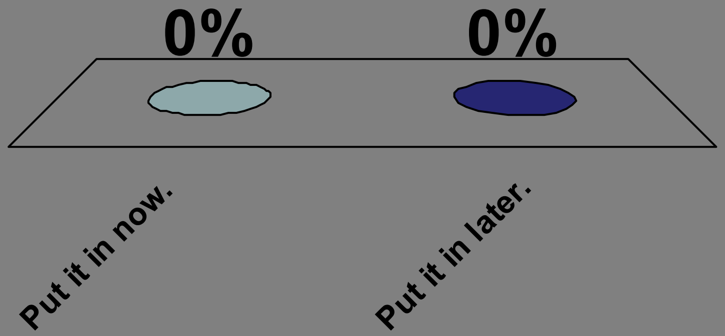
The greater the intensity of the light, the more photons it contains, but each photon has an energy, $E = hf$.

Einstein will use these ideas to explain something called the Photoelectric Effect.

Clicker question 29 - 1

Let’s say you enjoy a cup of coffee with non-dairy creamer after you eat. But the waitress brings your coffee with your food. Should you put the creamer in now, or wait until after you eat to put it in, if you want to maximize your chance at enjoying a hot cup of coffee?

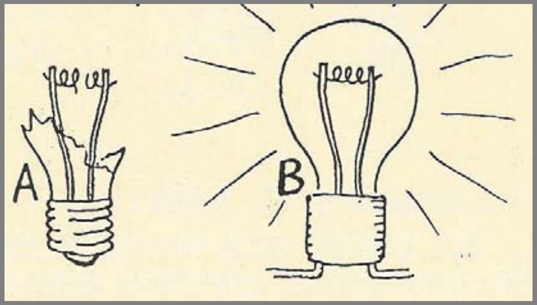
1. Put it in now. ✓
2. Put it in later.

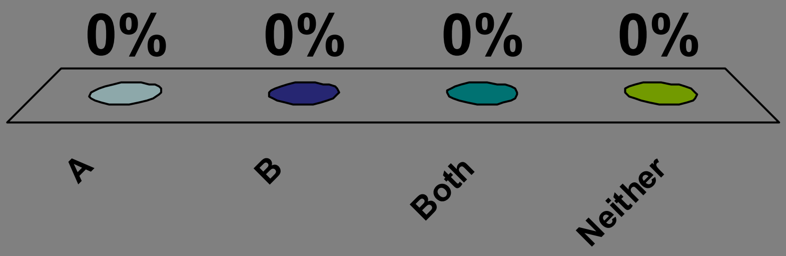


Clicker question 29 - 2

Which light bulb is emitting radiation?

1. A
2. B
3. Both
4. Neither





Algebra-based Physics II

Nov. 17th: Chap 29. 3-4

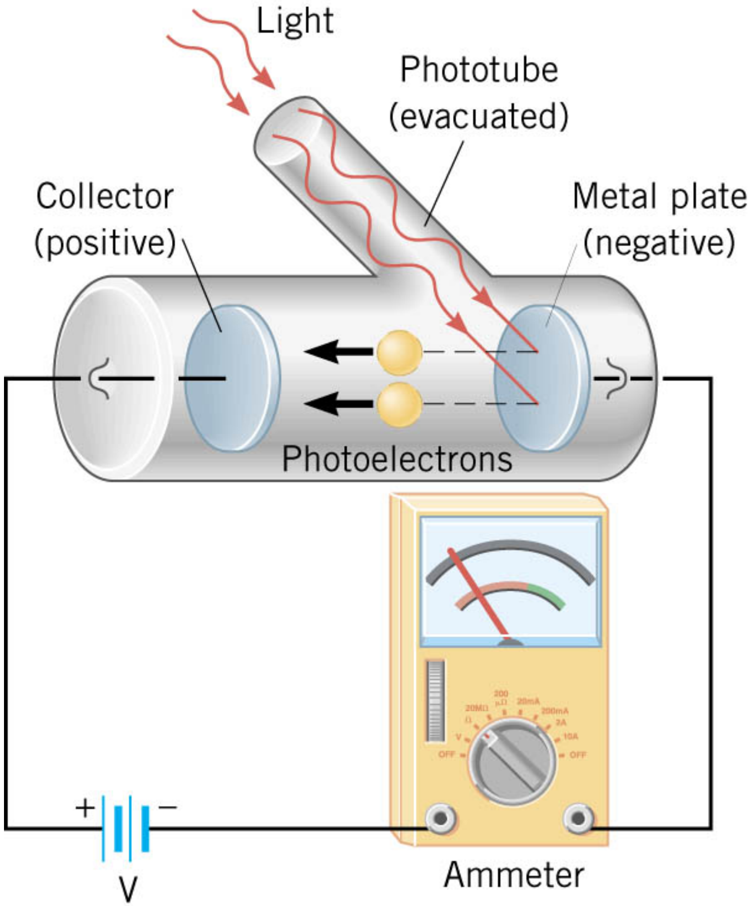
- Photons & the photoelectric effect
- Compton Effect

QUANTUM MECHANIC

29.3 The Photoelectric Effect

In 1887 Heinrich Hertz produced and detected electromagnetic waves, thus proving Maxwell's theory.

He also discovered something called the Photoelectric Effect.



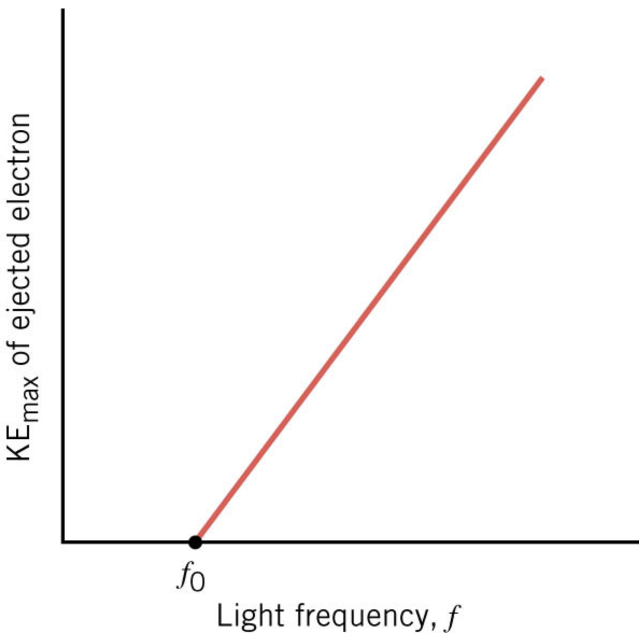
When light shines on a metal plate, some of the electrons in the metal get ejected from its surface and then are accelerated by a potential difference.

This results in a current flow in the circuit as shown.

Important Characteristics of the photoelectric effect

1. Only light with a frequency above some minimum value, f_o, will result in electrons being ejected – regardless of the light's intensity.

Let's look at a plot of the KE of the ejected electrons vs. the frequency of the light shining on the metal:



Notice: No electrons are ejected from the metal for frequencies below some f_o.

$f < f_o$, no ejected electrons.

$f \geq f_o$, electrons are ejected from the metal's surface.

f_o is called the Threshold Frequency.

Now choose some constant value for the frequency $f \geq f_o$, so that electrons are being ejected from the metal.

2. The maximum KE of the ejected electrons remains constant, even if the intensity of the light is increased.

Classically, we would expect higher intensity light to eject electrons with greater KE. It doesn't happen.

Also, we would expect that if we used very low intensity light, that it would take a long time for the electrons to build up enough energy to be ejected from the metal's surface.

That doesn't happen either! Even if the light intensity is very low, electrons are still ejected from the metal's surface, almost instantaneously, as long as $f \geq f_o$.

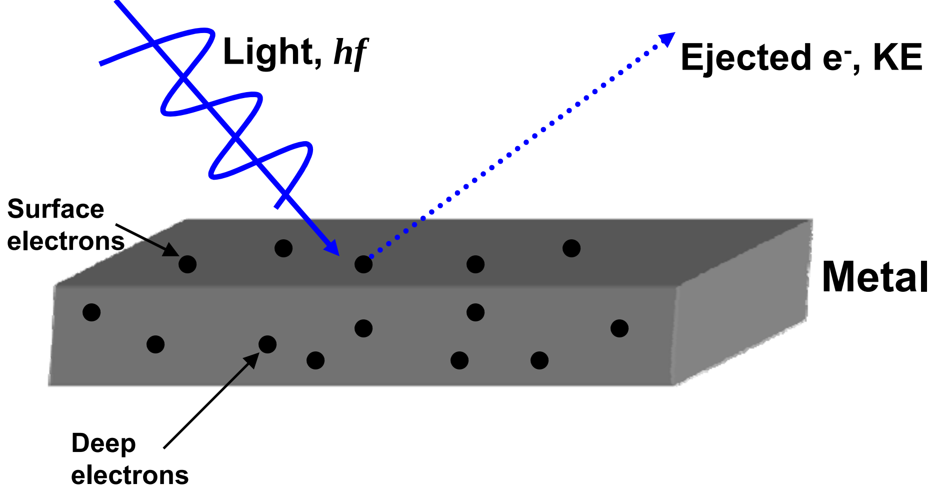
As we mentioned previously, Einstein assumed that light was composed of discrete packets (particles) of energy called photons.

And the photon energy is given by:

$$E = hf = \frac{hc}{\lambda}$$

The more intense the light is, the more photons it carries, but each photon still has an energy: $E = hf$.

Now let's examine the photoelectric effect in a little more detail.



Free electrons occupy the entire volume of the metal.

However, electrons close to the metal's surface (surface electrons) are more weakly bound to the metal than the deep electrons.

But even though the surface electrons are more weakly bound, there is still a minimum "binding energy" I must overcome to get them out of the metal.

This is called the Work Function (W_o) of the metal.

<u>It is an energy, and it is typically on the order of a few eV.</u>

During the effect, a photon of light ($f > f_o$) with energy hf strikes the metal and electrons are ejected with energy KE.

By conservation of energy, the following relationship must be true:

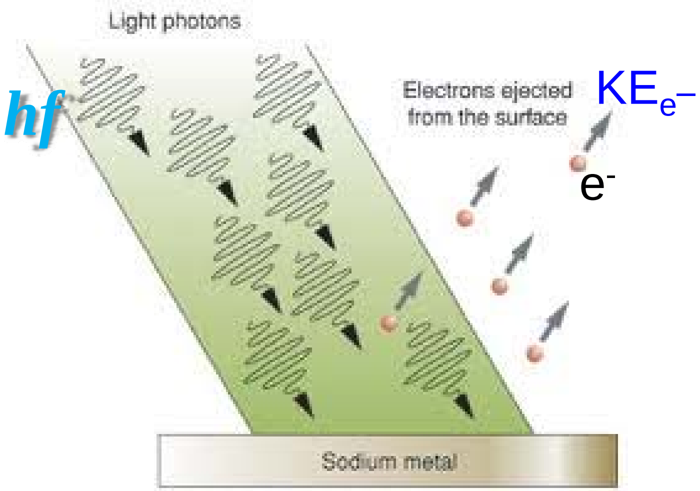
The photon energy – The binding energy = KE of the ejected electrons

or

$$\boxed{hf - W_o = KE_{e-}}$$

This is the Einstein equation for the Photoelectric Effect.

Simple picture view:

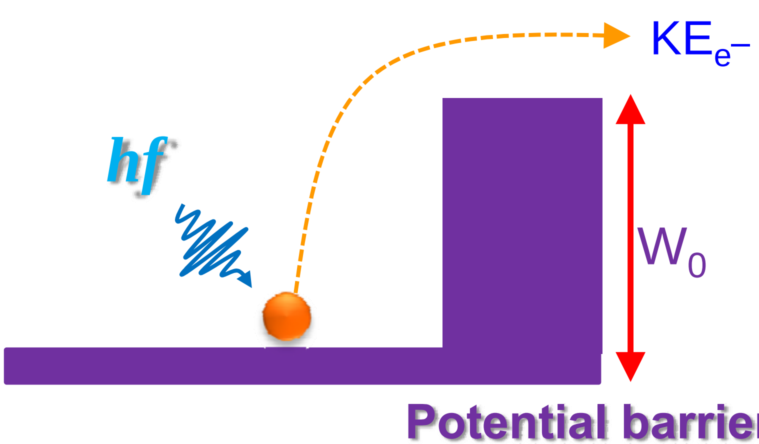


Einstein Theory:

Light consists of photons (*hf*)

One electron can discretely absorb one photon

Electron use photon energy to overcome the potential barrier



Energy conservation:

$$hf - W_o = KE_{e-}$$

Whether or not electrons can get out depends on the frequency of light not the intensity of light !!!!!

It was for this work on the photoelectric effect that Einstein received the Nobel Prize in Physics in 1921.

The wave description of EM radiation (light) fails to describe the photoelectric effect. We need to use the particle (photon) picture.

…But don't abandon the wave description yet!

The photoelectric effect was one of the earliest indications of the Particle/Wave Duality of Light.

So light is composed of particles called photons.

Einstein showed that the total energy of an object is: $E = \gamma mc^2$

Where: $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Notice, that if v = 0, then γ = 1, and $E = mc^2$

This is just his famous rest-mass equation.

Rewrite the general energy equation: $\dfrac{E}{\gamma} = mc^2 \Rightarrow E\left(\sqrt{1-\frac{v^2}{c^2}}\right) = mc^2$

What if we apply this equation to photons? **Well, for photons, $v = c$.**

$$E\left(\sqrt{1-\frac{c^2}{c^2}}\right)=mc^2 \Rightarrow 0=mc^2$$

Therefore, for the equation to be correct, *m* must equal 0 for the photon!

A photon is a massless particle that moves at the speed of light!

Radiation of a certain wavelength causes electrons with maximum kinetic energy of 0.68 eV to be ejected from a metal whose work function is 2.75 eV. What would be the maximum kinetic energy with which this same radiation ejects electrons from another metal whose work function is 2.17 eV?

Case 1:

$$W_0 = 2.75eV; \quad \mathrm{KE}_{e^-} = 0.68eV$$

$$hf = W_0 + \mathrm{KE}_{e^-} = 3.43eV$$

Case 2:

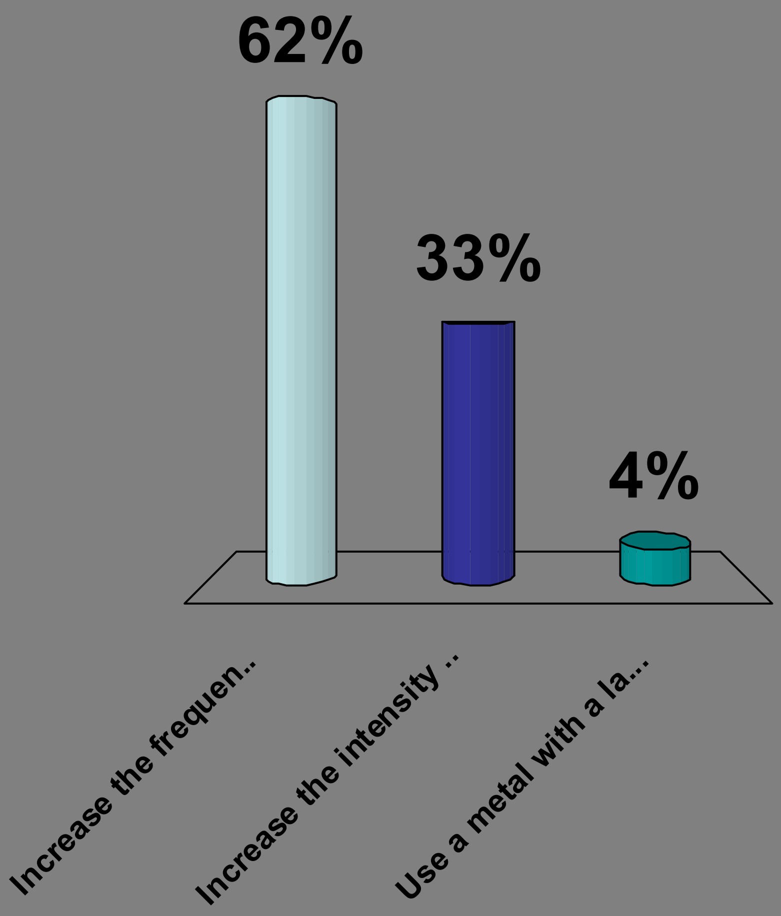
$$\mathrm{KE}_{e^-} = hf - W_0 = 3.43eV - 2.17eV = 1.26eV$$

Clicker question 29 - 3

In the photoelectric effect, electrons are ejected from the surface of a metal when light shines on it. Which of the following would lead to an increase in the maximum KE of the ejected electrons?

1. Increase the frequency of the light.
2. Increase the intensity of the light.
3. Use a metal with a larger work function.

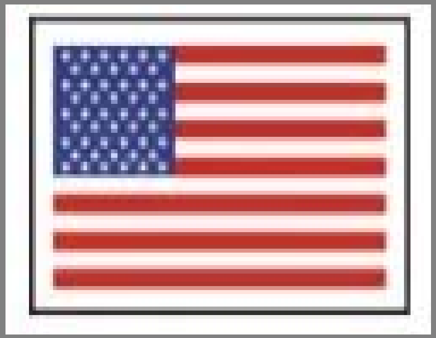
$$hf - W_o = KE_{e-}$$

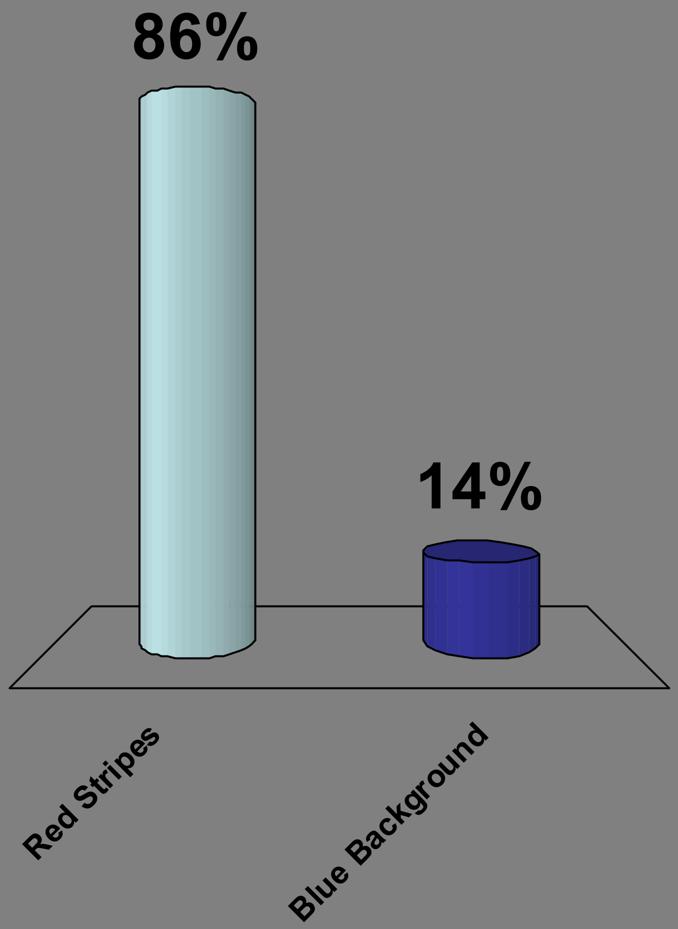


Clicker question 29 - 4

After 9/11 many patriotic Americans put little stickers of the American flag on their bumpers or in the rear window of their cars. Over a couple of years, the flags fade. Which will fade first, the red stripes or the blue star background?

1. Red Stripes ✓
2. Blue Background

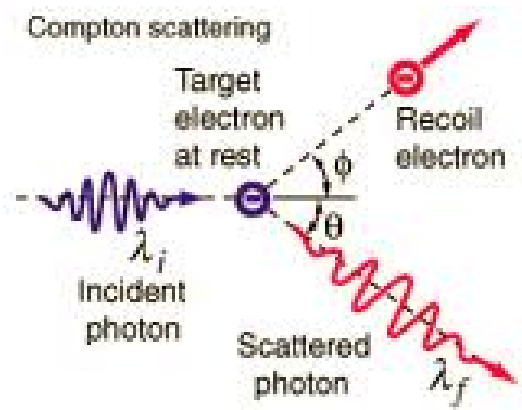




29.4 Compton Effect: A particle picture of light

Arthur Compton

Light behaves as particles which are scattered by electrons



$$hf = hf' + KE$$

Energy of incident photon

Energy of scattered photon

Kinetic energy of recoil electron

Energy conservation:

$$E = hf = h\frac{c}{\lambda}$$

Momentum conservation:

$$p = \frac{hf}{c} = \frac{h}{\lambda}$$

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

Algebra-based Physics II

Nov. 19th: Chap 29. 5-6:

- **Wave nature of matter**

$$\lambda = \frac{h}{p}$$

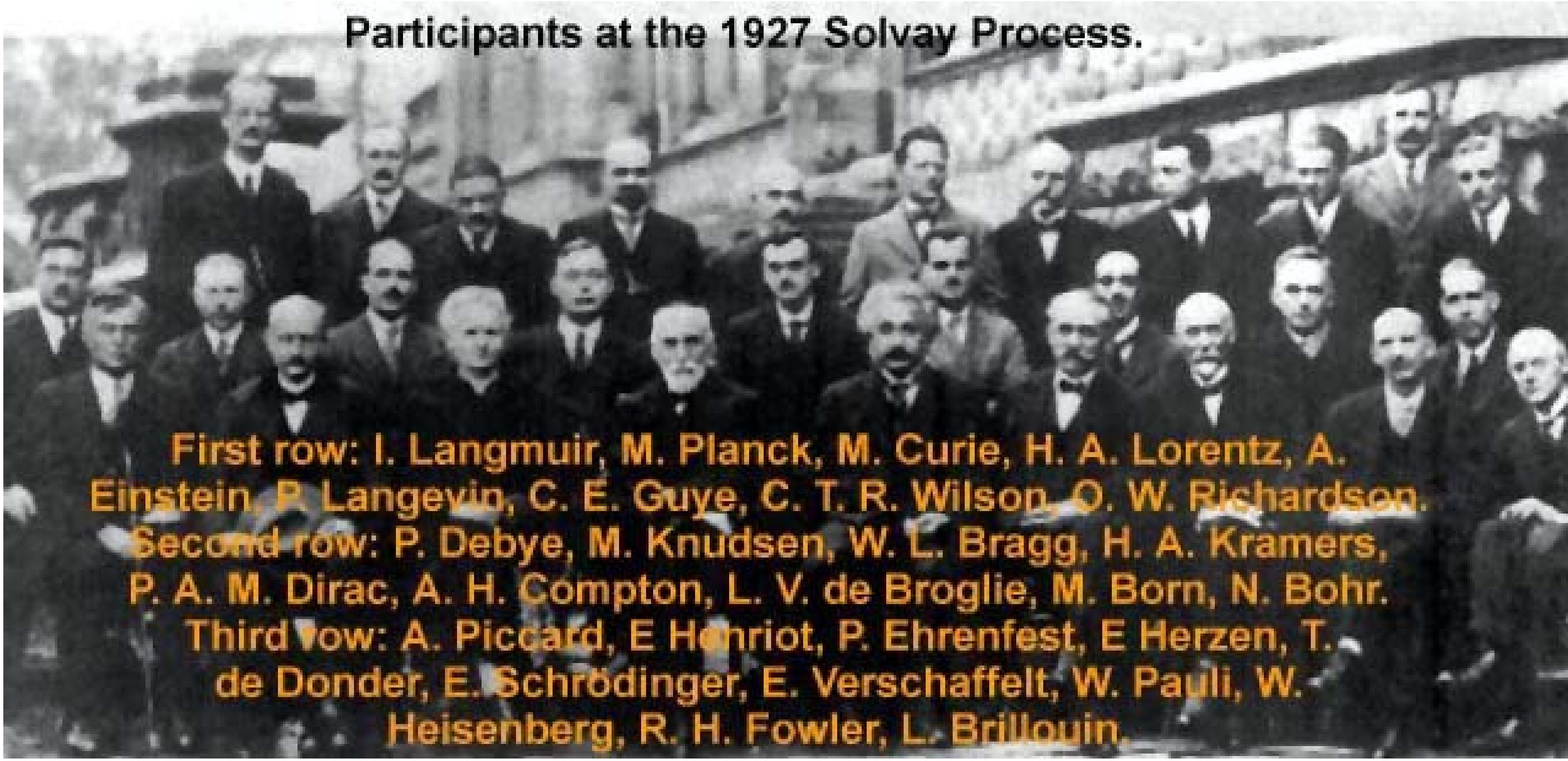
- **Heisenberg uncertainty principle**

$$\Delta\chi\Delta\rho \geq \frac{\hbar}{2}$$

George W Bush jnr
Obviously a Quantic!

"I have learned from mistakes I may or may not have made"

The birth of quantum epoch



Example: An X-ray photon is scattered at an angle of θ = 180° from an electron that is initially at rest. After scattering, the electron has a speed of 4.67 × 106 m/s. Find the wavelength of the incident X-ray photon.

What do we know: θ = 180°; v = 4.67 × 106 m/s for electron with m=m_e

Compton scattering: $\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta) = \frac{h}{m_e c}(1-\cos 180^\circ) = \frac{2h}{m_e c}$

$$\Downarrow$$

$$\lambda' = \lambda + \frac{2h}{m_e c}$$

Energy conservation: $hf = hf' + KE \xrightarrow{f = \frac{c}{\lambda}} \frac{hc}{\lambda} = \frac{hc}{\lambda'} + KE = \frac{hc}{\lambda'} + \frac{1}{2}m_e v^2$

$$\Downarrow$$

$$\lambda^2 + \frac{2h}{m_e c}\lambda - \frac{4h^2}{m_e^2 v^2} = 0 \Leftarrow 2hc(\lambda' - \lambda) = m_e v^2 \lambda \lambda'$$

29.5 Particle Waves – de Broglie

Louis de Broglie

Working as a graduate student in 1923, de Broglie hypothesized that if light waves can have particle-like properties, then maybe particles (i.e. electrons) can have wave-like properties.

This was a very radical assumption, since at this time, there was no evidence to support this hypothesis.

From Einstein's relativity equations, it is found that the momentum of a photon is:

$$p = \frac{hf}{c} = \frac{h}{\lambda}$$

de Broglie rewrote this as:

$$\lambda = \frac{h}{p}$$

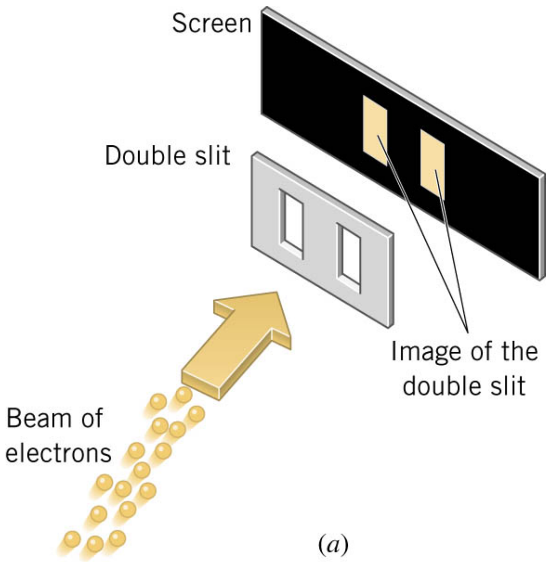
p is the relativistic momentum.

This is known as the de Broglie wavelength of a particle.

This allows us to calculate the wavelength for any object with nonzero momentum, p.

Now let's repeat Young's double-slit experiment, but this time lets shoot electrons (particles) at the slits instead of light.

What would we expect to see???



(*a*)

Well, we might expect the screen to appear as it does to the left – two bright fringes, one directly behind each slit.

What we actually see is shown in the figure at the lower left – alternating dark and bright fringes.

In other words, the electrons have acted like waves and interfered with each other to produce the classic interference pattern!

Our notion of the electron as being a tiny discrete particle of matter does not account for the fact that the electron can behave as a wave in some circumstances.

It exhibits a dual nature – behaving sometimes like a particle, and sometimes like a wave.

Things are even weirder than this!!!.....

The previous scenario is also true for light. Sometimes it's wave-like and sometimes it's particle-like.

This is referred to as the Particle-Wave Duality of Light.

The first experiment to show wave-like properties of particles, and thus prove de Broglie's hypothesis, was a diffraction experiment.

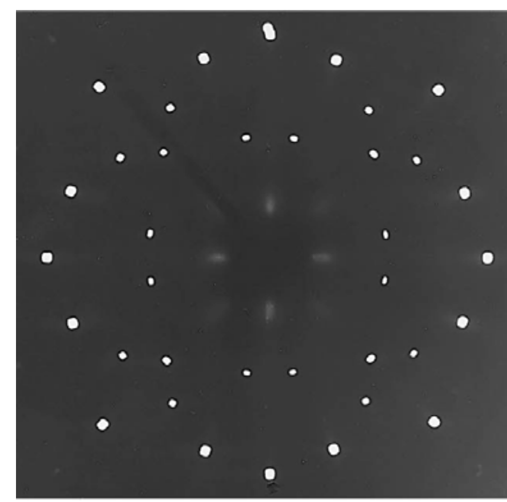
Two American physicists (Davisson and Germer) in 1927 demonstrated electron diffraction with a crystal of nickel.

Wave properties are displayed by all particles, such as protons and neutrons too.

Davisson and Germer

FIG. 1. Laue photograph showing neutron diffraction by NaCl.

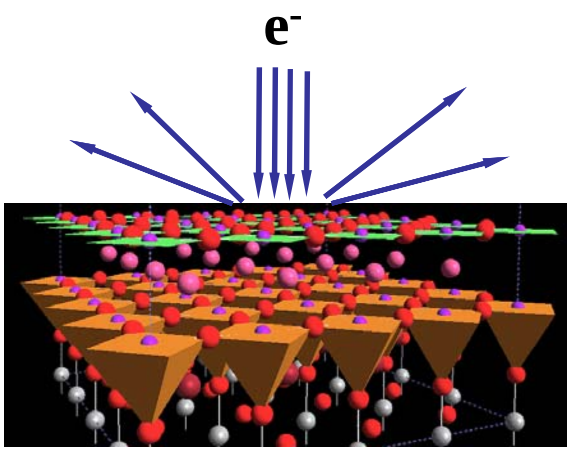
NaCl – Neutron diffraction



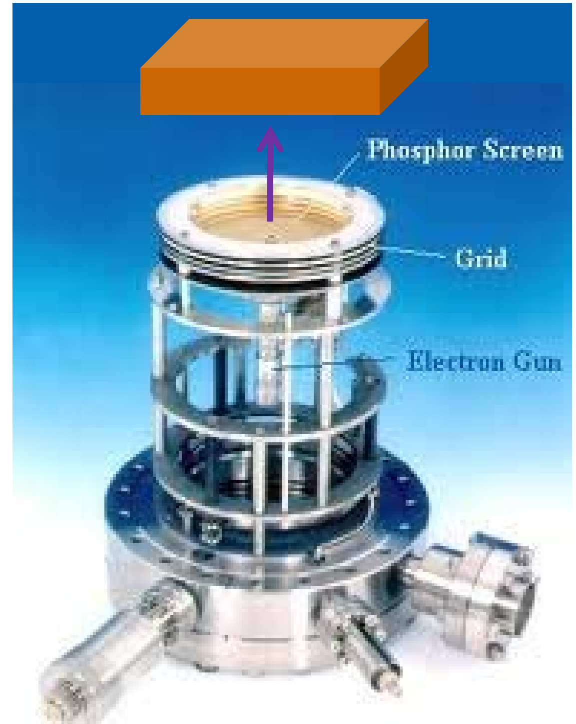
(a)

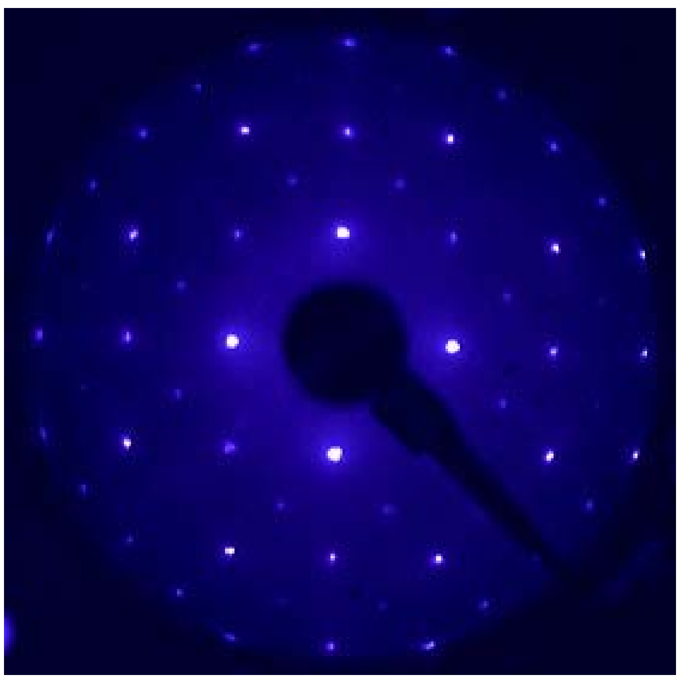
NaCl – X-ray diffraction

Electron diffraction from a crystalline surface: modern version



Low energy electron diffraction





Electrons behave like waves and the crystal plays like grating thus causing diffraction pattern

So all objects have a de Broglie wavelength – baseballs, cars, even you and me!

But remember, in order for wave effects to be seen, such as interference and diffraction, the wavelength must be comparable to the size of the opening or obstacle.

For fun, let's calculate human body de Broglie wavelength:

$$\lambda_{\text{Human}} = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(90.7\ \text{kg})(6.7\ \text{m/s})} = \boxed{1\times10^{-36}\ \text{m}}$$

So what does this number mean???

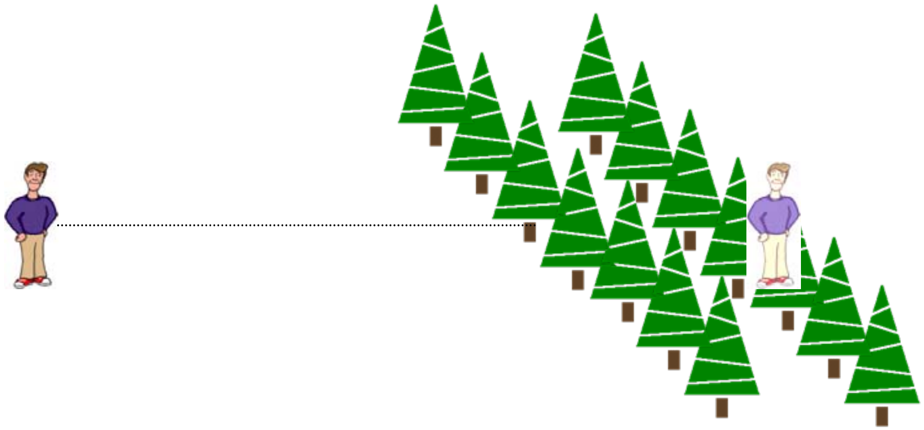
Well, the size of an atom is roughly 1×10^{-10} m. So my deBroglie wavelength is some 26 orders of magnitude smaller than the size of an atom!!!

Which means….we don't observe wave-like properties with everyday objects, baseballs, humans, etc.

Thus, we need really small masses and high speeds to observe the wave-like properties…..sub-atomic particles!

But, just for fun, consider if my de Broglie wavelength was say 1 meter.

What might happen if I ran into a forest of pine trees???



You could interfere with yourself! And diffract!

So what are these particle-waves really???

They are waves of probability!

In the double-slit experiment done with electrons, the bright fringes on the screen are regions of high probability.

In other words, it's more probable that the electrons will hit the screen there.

In the double-slit experiment done with light, the intensity of the bright fringes is proportional to either E^2 or B^2.

For particle waves, the intensity of the maxima (bright fringes) is proportional to:

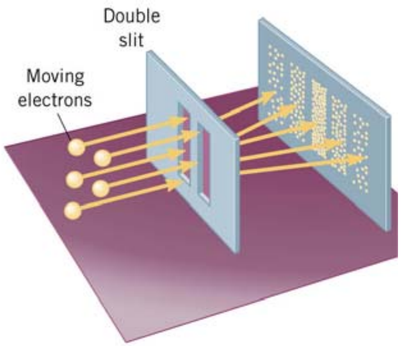
$$\Psi^2$$

Ψ represents the wave function of the particle.

It was Erwin Schrödinger and Werner Heisenberg who independently developed the theory of how to determine a particle's wave function.

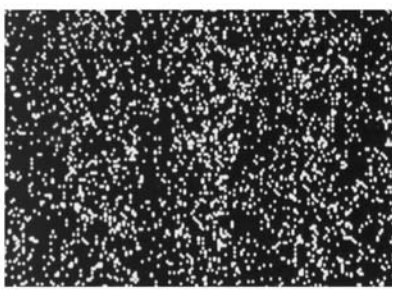
Once you have Ψ, you can predict how the particle will evolve over time.

They established a new branch of physics called Quantum Mechanics!



(*a*)

(*b*) After 100 electrons



(*c*) After 3000 electrons

(*d*) After 70 000 electrons

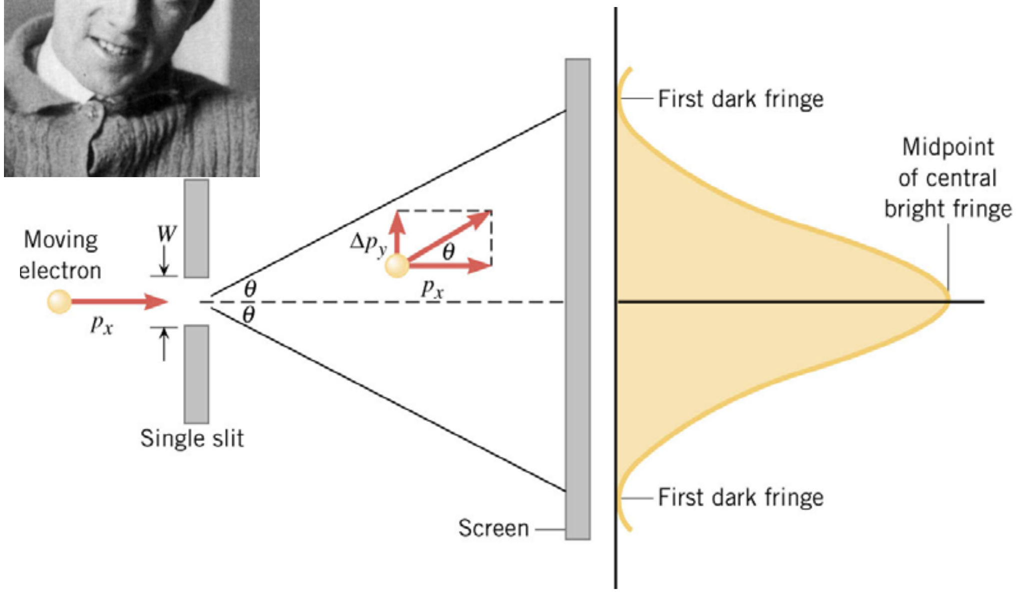
The characteristic interference pattern becomes evident after a sufficient number of electrons have struck the screen.

It (the double-slit experiment) encapsulates the central mystery of quantum mechanics. It is a phenomenon which is impossible, absolutely impossible, to explain in any classical way and which has in it the heart of quantum mechanics. In reality, it contains the only mystery…the basic peculiarities of all quantum mechanics.

-Richard P. Feynman

29.6 The Heisenberg Uncertainty Principle

consider single-slit diffraction with electrons, and concentrate on those electrons that form the central bright fringe.



The electrons enter the slit with momentum p_x.

Once they pass thru the slit, the electrons have probability to gain momentum in the y-direction.

The maximum any electron could gain would be $\Delta p_y = p_y - p_{y0} = p_y$.

Thus, Δp_y represents the uncertainty in the momentum in the y-direction.

From single-slit diffraction with light, we know that, $\sin\theta = \dfrac{\lambda}{W}$ **specifies the location of the first dark fringe.**

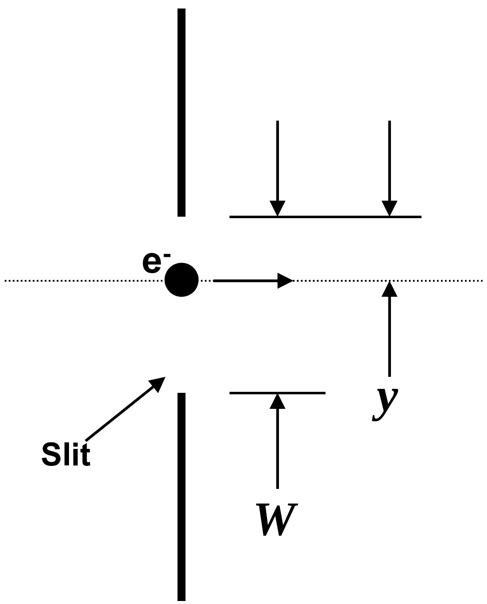
If the screen distance is large, then $\sin\theta \approx \tan\theta$**, or** $\tan\theta = \dfrac{\lambda}{W} = \dfrac{\Delta p_y}{p_x}.$

Thus, $\Delta p_y = \frac{\lambda p_x}{W}$ But, from de Broglie, we know that: $p_x = \frac{h}{\lambda}$

Thus, $\Delta p_y = \frac{h}{W}.$ So what does this equation tell us???

The smaller W is, the more accurately we know the y-value of the electron as it passes thru the slit.

W represents the uncertainty in the y-position of the electron: $W \equiv \Delta y.$



But, the smaller W is, the greater Δp_y becomes. In other words, the more accurately we know the particle's position in the slit, the larger the uncertainty in its momentum.

Plugging into above, we find: $\Delta y \Delta p_y \geq \frac{\hbar}{2}$ $\hbar = \frac{h}{2\pi}$

This is a statement of the Heisenberg Uncertainty Principle (HUP) for Momentum and Position:

It is impossible to specify precisely the position and momentum of a particle at the same time.