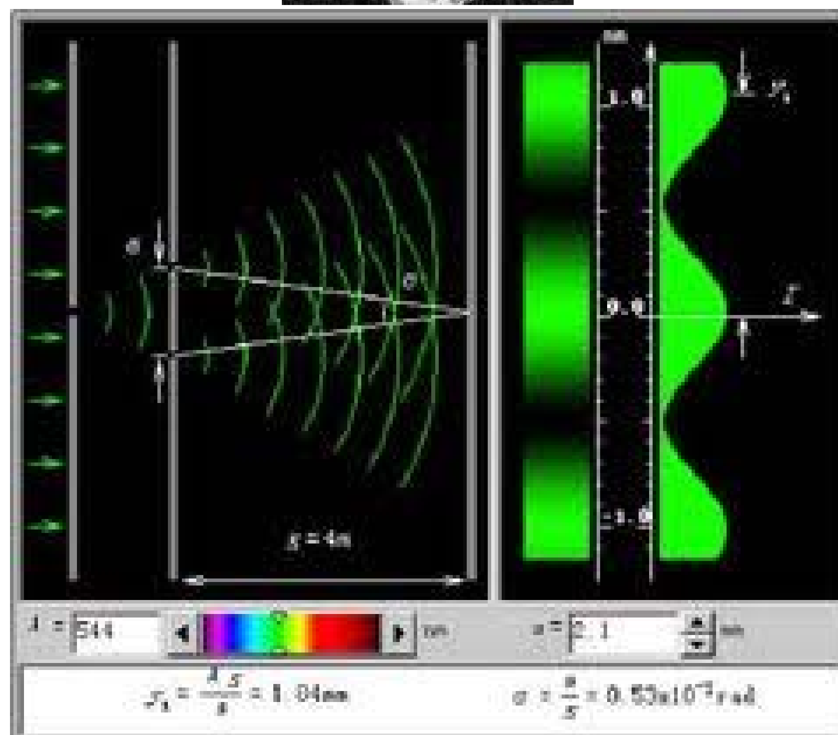
Algebra-based Physics II

Thomas Young (1773-1829)

Nov. 5th: Chap 27. 1-3

- Principle of linear superposition
- Young's double-slit experiment
- Thin-film interference

$$\boxed{d\sin\theta = \begin{cases} m\lambda \\ (m+\frac{1}{2}) \end{cases}}$$



Ch. 27. Interference and the Wave Nature of Light

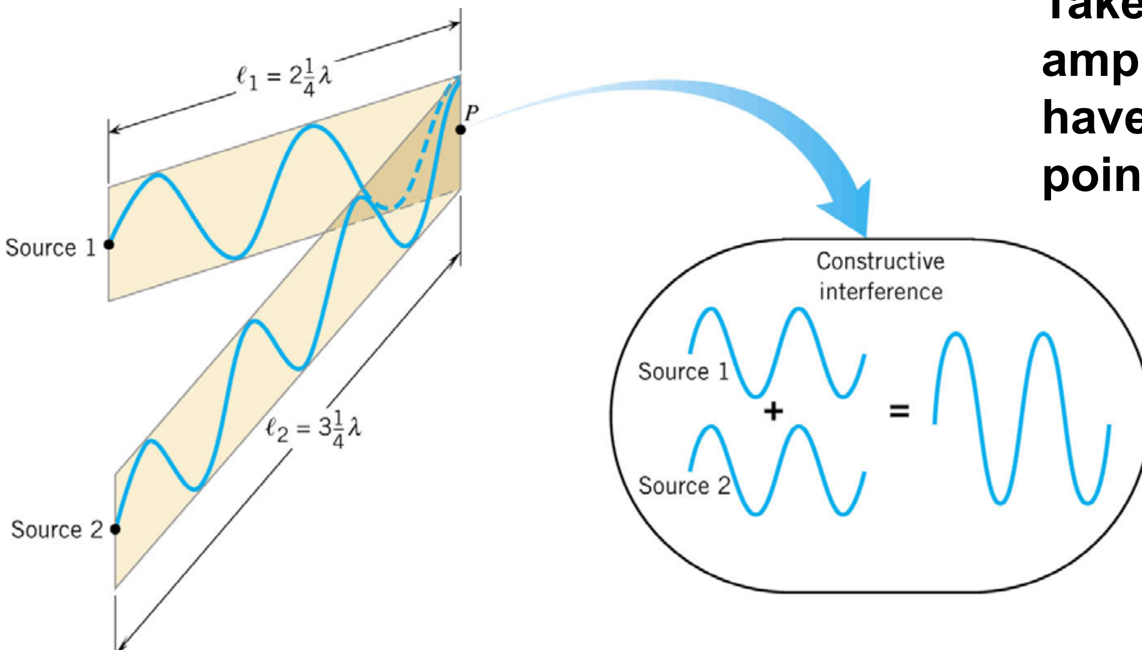
Up to now, we have been studying geometrical optics, where the wavelength of the light is much smaller than the size of our mirrors and lenses and the distances between them.

→ The propagation of light is well described by linear rays except when reflected or refracted at the surface of materials.

Now we will study wave optics, where the wavelength of the light is comparable to the size of an obstacle or aperture in its path.

→ This leads to the wave phenomena of light called interference and diffraction.

27.1 The Principle of Linear Superposition



Take two waves of equal amplitude and wavelength and have them meet at a common point:

If the two waves are in-phase, then they meet crest-to-crest and trough-to-trough.

Their two amplitudes add to each other. In this case, the resulting wave would have an amplitude that doubled.

This is called Constructive Interference (CI).

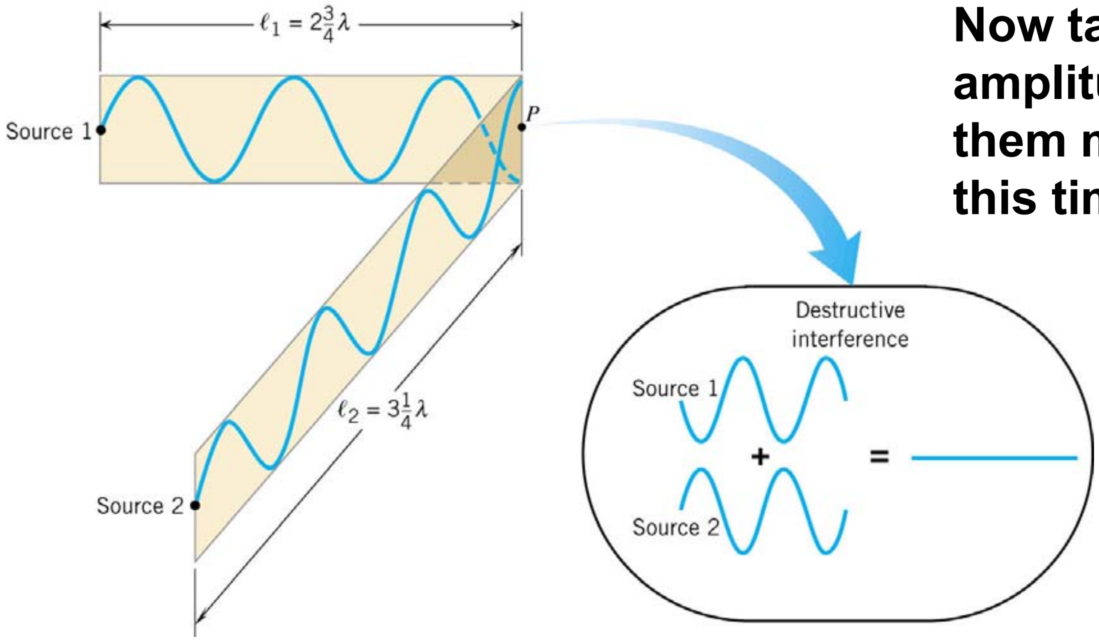
Define Optical Path Difference (OPD):

OPD = The difference in distance that two waves travel.

For CI to occur, we need the waves to meet crest-to-crest, thus the waves must differ by an integer multiple of the wavelength λ:

$$\boxed{\mathrm{OPD} = \mathrm{m}\lambda,\ \mathrm{m} = 0, 1, 2, \ldots}$$

Constructive Interference



Now take two waves of equal amplitude and wavelength and have them meet at a common point, but this time have them be out-of-phase.

Thus, they now meet crest-to-trough.

The resulting wave has zero amplitude. The two waves cancel out.

This is called Destructive Interference (DI).

For DI to occur, we need the waves to meet crest-to-trough, thus the waves must differ by any odd integer number of ½λ :

$$\boxed{\mathrm{OPD} = (\mathrm{m} + \tfrac{1}{2})\lambda,\ \mathrm{m} = 0, 1, 2, \ldots}$$

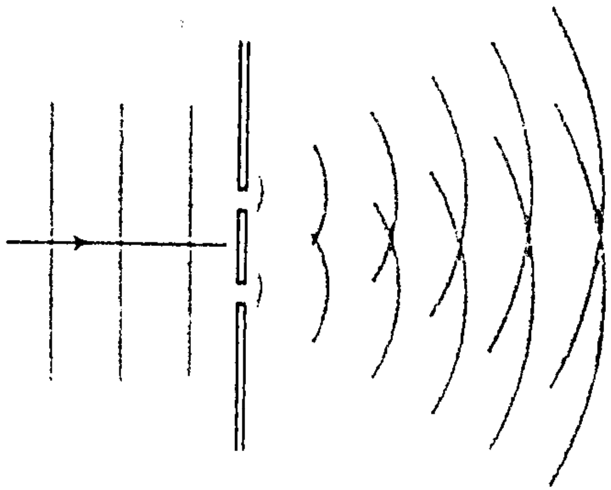
Destructive Interference

All waves do this, including EM waves, and since light is an EM wave, light waves do this too.

For interference to continue at some point, the two sources of light producing the waves must be coherent, which means that their phase relationship relative to each other remains constant in time.

27.2 Young's Double Slit Experiment (http://www.youtube.com/watch?v=AMBcgVlamoU)

Look what happens when water waves strike a barrier with two slits cut in it:

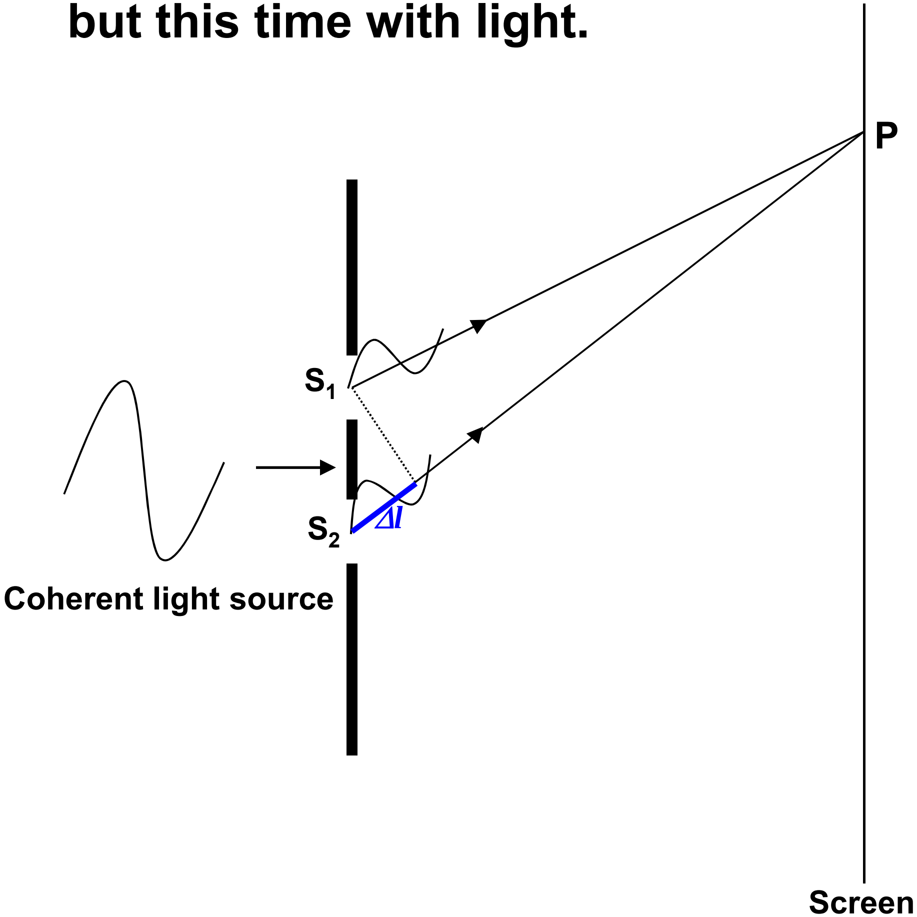


Each slit acts like a coherent point source of waves.

The waves diverge from each slit and interfere with each other.

The bright regions are areas of constructive interference, and the dark regions are areas of destructive interference.

In 1801 an English scientist Thomas Young repeated the double slit experiment, but this time with light.



Each slit acts like a coherent light source.

The two waves meet at point P on a screen.

Δl is the optical path difference of the two light waves coming from S_1 and S_2.

The two waves interfere with each other, and if:

$$\Delta l = m\lambda$$

Constructive interference, and we see a bright spot.

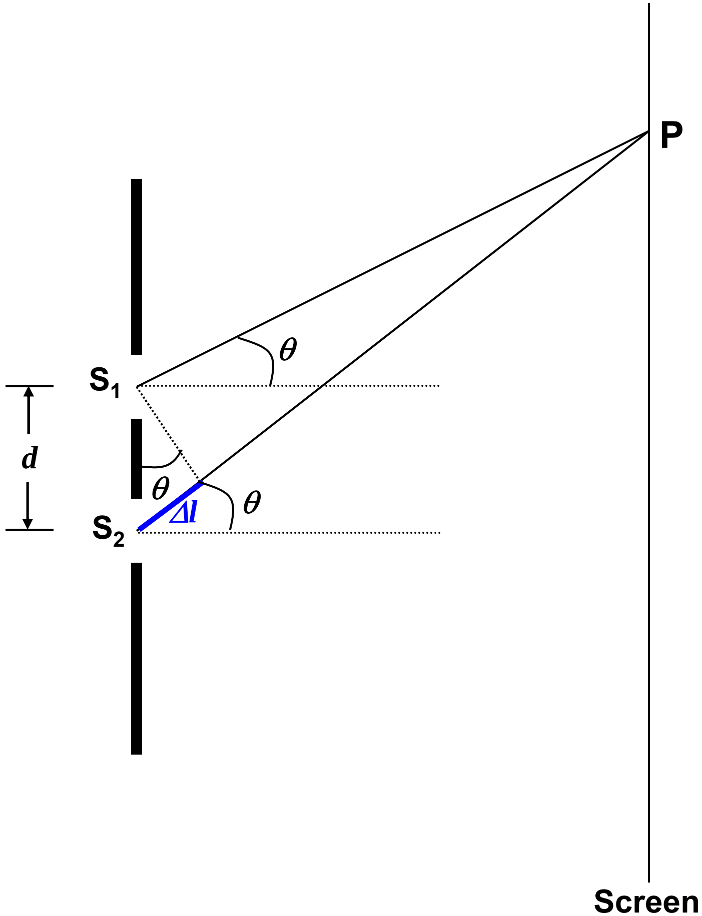
$$\Delta l = (m + \tfrac{1}{2})\lambda$$

Destructive interference, and we see a dark spot.

Thus, we should see alternating bright and dark regions (called fringes) as we move along the screen and the above two conditions are satisfied.

Can we find a relationship between the fringes and the wavelength of the light?

The answer is yes……………



Assume the screen is far away from the slits which are small. This is called the Fraunhofer approximation.

Thus, since the slits are very close together, θ is the same for each ray.

From the figure we see that:

$$\sin\theta = \frac{\Delta l}{d} \quad \Rightarrow \Delta l = d\sin\theta$$

We know that for constructive interference: $\Delta l = m\lambda$

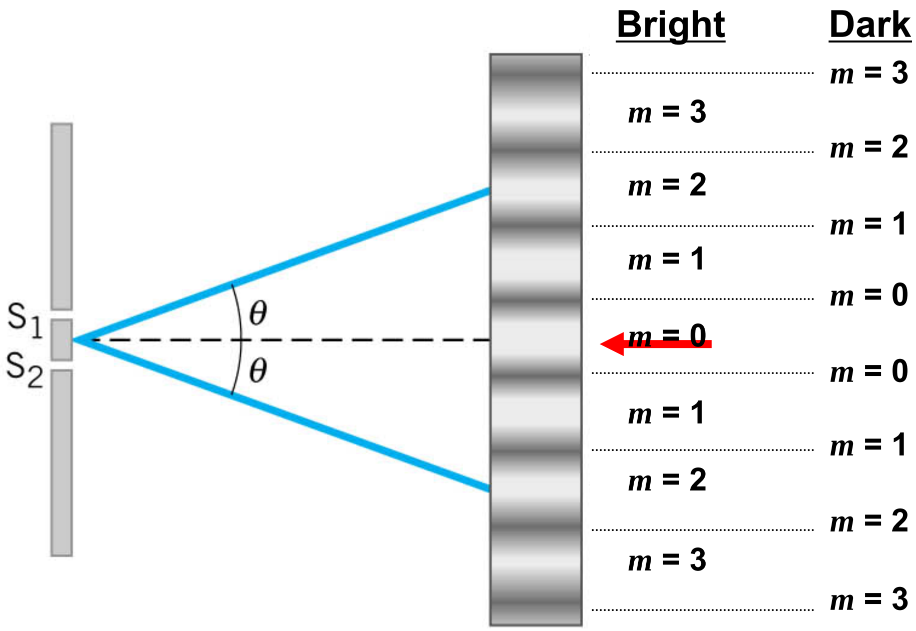
Thus, $\boxed{d\sin\theta = m\lambda}$ **for constructive interference.**

and...

m is the order of the fringe.

$\boxed{d\sin\theta = (m + \tfrac{1}{2})\lambda}$ **for destructive interference.**

These are the interference conditions for the double slit.



This is what a typical double slit interference pattern would look like.

Notice there are alternating light and dark fringes.

Also note that the central fringe at $\theta = 0$ is a bright fringe.

It is also the brightest of the bright fringes.

The order of the bright fringes starts at the central bright fringe.

The order of the dark fringes starts right above and below the central bright fringe.

So, the second dark fringe on either side of the central bright fringe is the 1st order dark fringe, or $m = 1$.

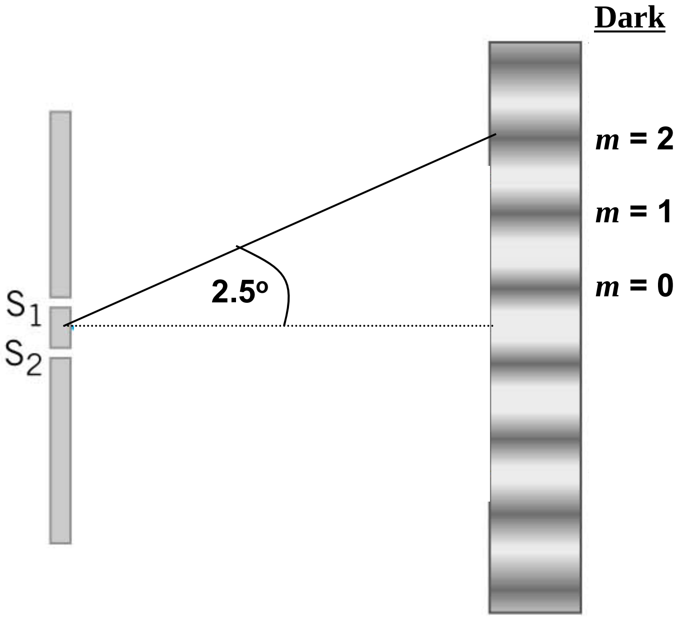
Remember, order means *m*.

Young's experiment provided strong evidence for the wave nature of light.

If it was completely particle like, then we would only get two fringes on the screen, not an interference pattern!

Example

In a Young's double-slit experiment, the angle that locates the 3rd dark fringe on either side of the central bright maximum is 2.5°. The slits have a separation distance d = 3.8 × 10^{-5} m. What is the wavelength of the light?



What is the order?

It is the 2nd order dark fringe, or *m* = 2.

Since it's a dark fringe, we know it must be destructive interference:

$$d\sin\theta = (m + \tfrac{1}{2})\lambda$$

$$\lambda = \frac{d\sin\theta}{m+\frac{1}{2}} \Rightarrow \lambda = \frac{(3.8\times10^{-5})(\sin 2.5^{o})}{2+\frac{1}{2}}$$

$$\Rightarrow \lambda = 6.63\times10^{-7}\ \text{m} = \boxed{663\ \text{nm}}$$

Algebra-based Physics II

Nov. 7th: Chap 27. 3-5

- Thin-film interference
- Diffraction

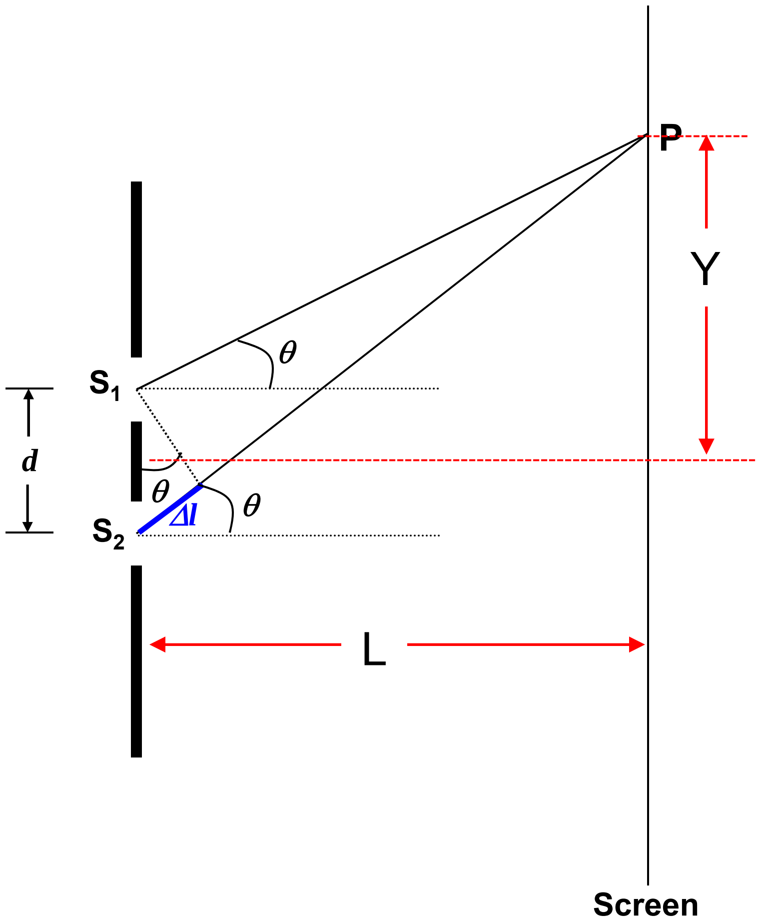
Reflection pattern from a soap film

Announcements:

- HW9 is posted
- 3rd-exam: Nov. 15 -17

Reflection pattern from a CD

Double slit interference



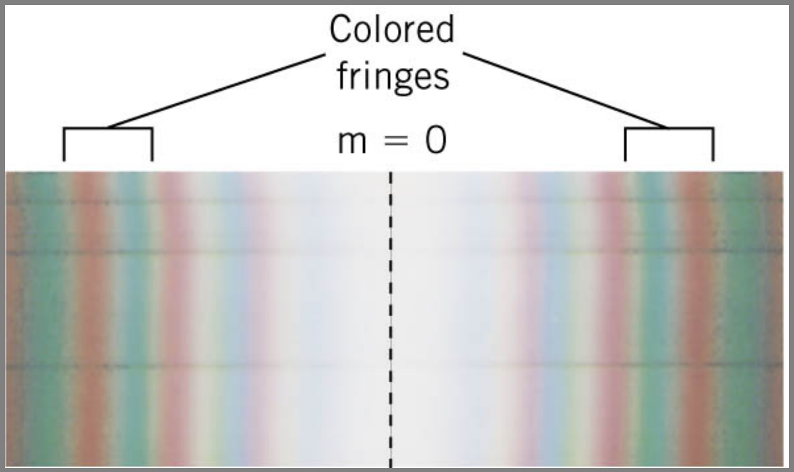
$$d\sin\theta = \begin{cases} m\lambda & \text{Constructive} \\ (m+\frac{1}{2}) & \text{Destructive} \end{cases}$$

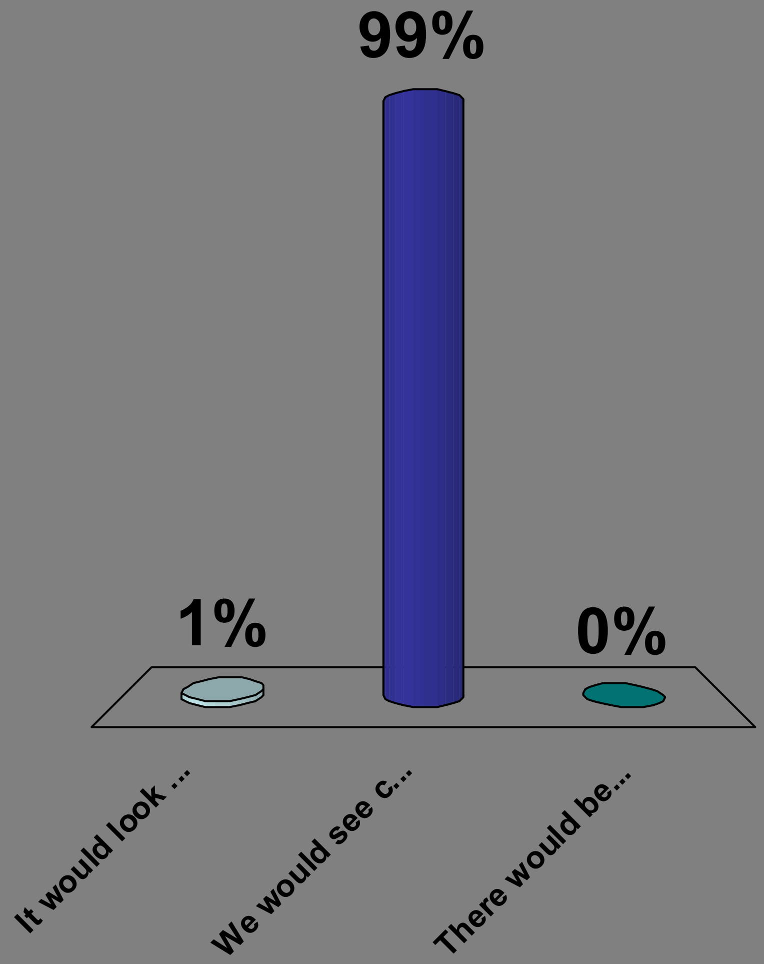
$$Y = L\tan\theta \approx L\sin\theta = \begin{cases} L\dfrac{m\lambda}{d} & \text{bright} \\ L\dfrac{(m+\frac{1}{2})\lambda}{d} & \text{dark} \end{cases}$$

Clicker Question 27 - 1

In our discussion of Young's double slit experiment, we only considered monochromatic light (light of one color). What would the interference pattern on the screen look like if we used white light instead?

1. It would look the same.
2. We would see colored fringes.
3. There would be no fringes.

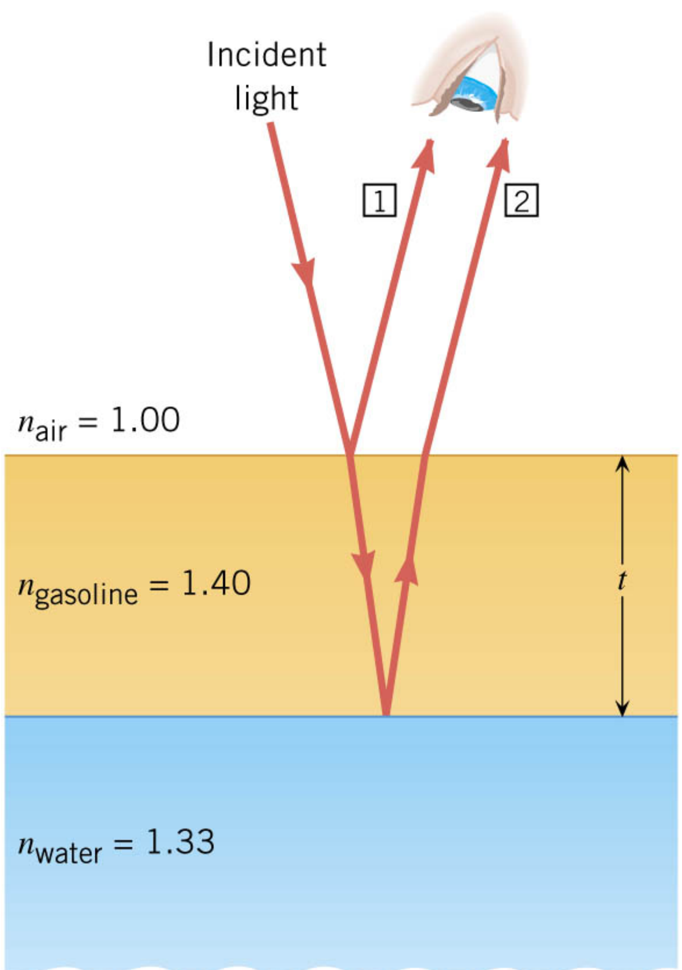




27.3 Thin Film Interference

Light waves can interfere in many situations. All we need is a difference in optical path length.

As an example, let's consider a thin film of oil or gasoline floating on the surface of water:



Part of a light ray gets reflected (1) from the surface of the film, and part gets refracted (2).

Then the refracted ray reflects back off the film/water interface and heads back into the air toward our eye.

Thus, two rays reach our eyes, and ray 2 has traveled farther than ray 1. Thus, there is a difference in the <u>optical path length</u>.

If the film is thin, and the ray strikes almost perpendicularly to the film, then the OPD is just twice the film thickness, or $\Delta l = 2t.$

If $2t = m\lambda$ **, we have Constructive Interference and the film appears bright.**

If $2t = (m + \frac{1}{2})\lambda$ **, we have Destructive Interference and the film appears dark.**

The optical path difference occurs inside the film, so the index of refraction that is important here is n_{film}.

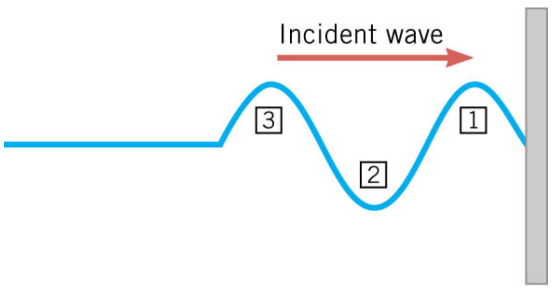
What is the wavelength of the light in the film (λ_{film})?

$$n_{film} = \frac{c}{v_{film}} = \frac{c}{f} \cdot \frac{f}{v_{film}} = \frac{\lambda_{vac}}{\lambda_{film}}$$

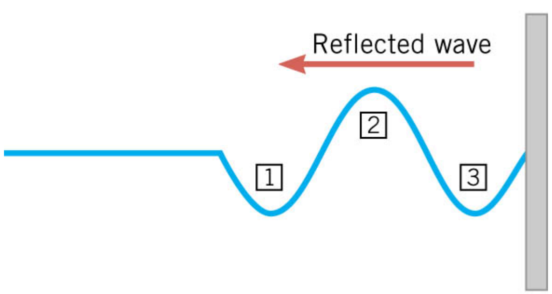
$$\boxed{\lambda_{film} = \frac{\lambda_{vac}}{n_{film}}}$$

One more important point:

When waves reflect from a boundary, it is possible for them to change their phase.



(a)



(b)

1. Light rays will get phase shifted by ½λ upon reflection when they are traveling from a smaller index of refraction to a larger index of refraction.

Smaller n → larger n → Phase shift!

2. Light rays will experience no phase shift upon reflection when they are traveling from a larger index of refraction to a smaller index of refraction.

Larger n → smaller n → No phase shift!

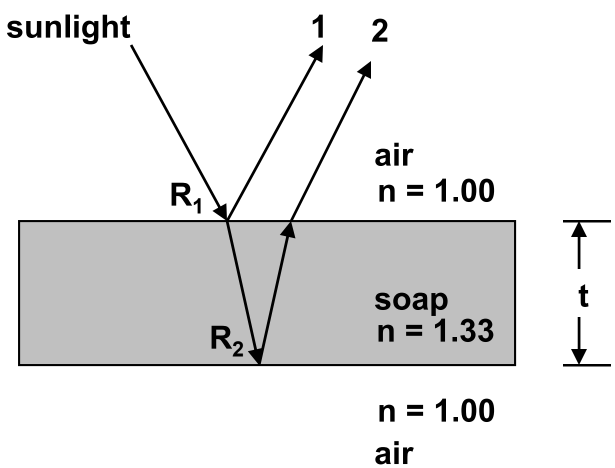
So a phase shift can occur upon reflection.

For thin films then the following is used:

$$\boxed{\Delta l + (\text{any phase shifts}) = \text{Interference condition}}$$

$$2t + (\text{phase shifts}) = \begin{cases} m\lambda, & \textbf{Film appears bright} \\ (m + \frac{1}{2})\lambda, & \textbf{Film appears dark} \end{cases}$$

Example A soap film (n = 1.33) is 375 nm thick and is surrounded on both sides by air. Sunlight, whose wavelengths (in vacuum) extend from 380 nm to 750 nm strikes the film nearly perpendicularly. For which wavelength(s) in this range does the film look bright in reflected light?



We want the film to appear bright, which means constructive interference:

$$\Delta l + (\text{phase shifts}) = m\lambda$$

Do we have any phase shifts?

At R_1 we are going from a smaller n to a larger n $\rightarrow$ ½λ phase shift.

At R_2 we are going from a larger n to a smaller n $\rightarrow$ No phase shift.

So now we have 1 phase shift: $$2t + \tfrac{1}{2}\lambda = m\lambda \quad \Rightarrow 2t = (m + \tfrac{1}{2})\lambda$$

This λ is the λ_{film}: $$\lambda_{film} = \frac{2t}{m + \frac{1}{2}}$$ But $\lambda_{vac} = n_{film}\lambda_{film}$

$$\Rightarrow \boxed{\lambda_{vac} = \frac{n_{film}2t}{m + \frac{1}{2}}}$$

$$\Rightarrow \lambda_{vac} = \frac{(2)(375\,\text{nm})(1.33)}{m+\frac{1}{2}} = \frac{997.5\,\text{nm}}{m+\frac{1}{2}}$$

m	λ_{vac}
0	**1995 nm**
1	**665 nm**
2	**399 nm**
3	**285 nm**

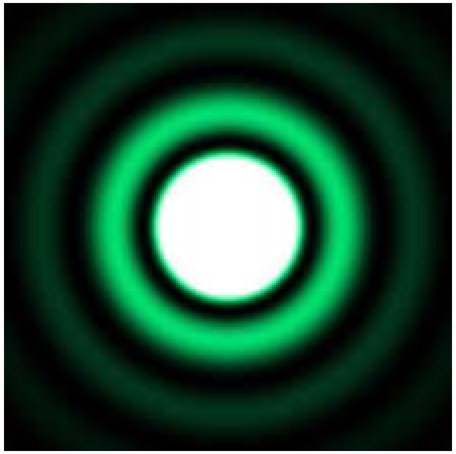
These two are in the visible spectrum (red and violet).

Thus, the film appears redish/violet.

Algebra-based Physics II

Nov. 10th: Chap 27. 5-7

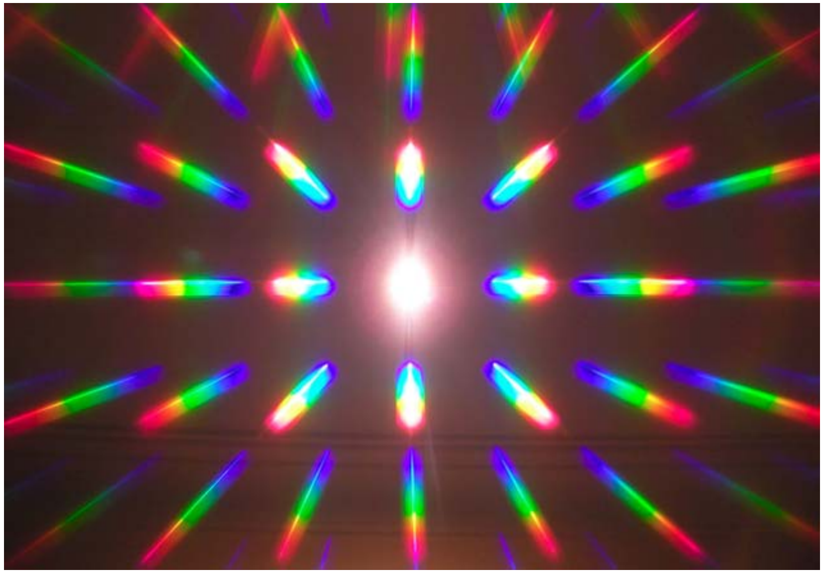
- Diffraction
- Resolving power
- The diffraction grating



Hole diffraction

Announcements:

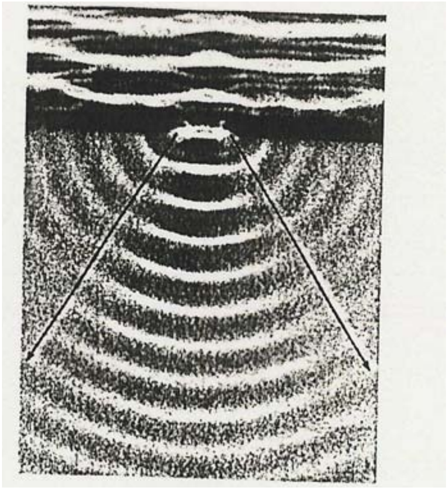
- HW10 for Chap. 27 is posted
- 3rd-exam: Nov. 15 -17



Halogen light diffraction

27.5 Diffraction

Diffraction is the bending of waves around obstacles or around the edges of openings.

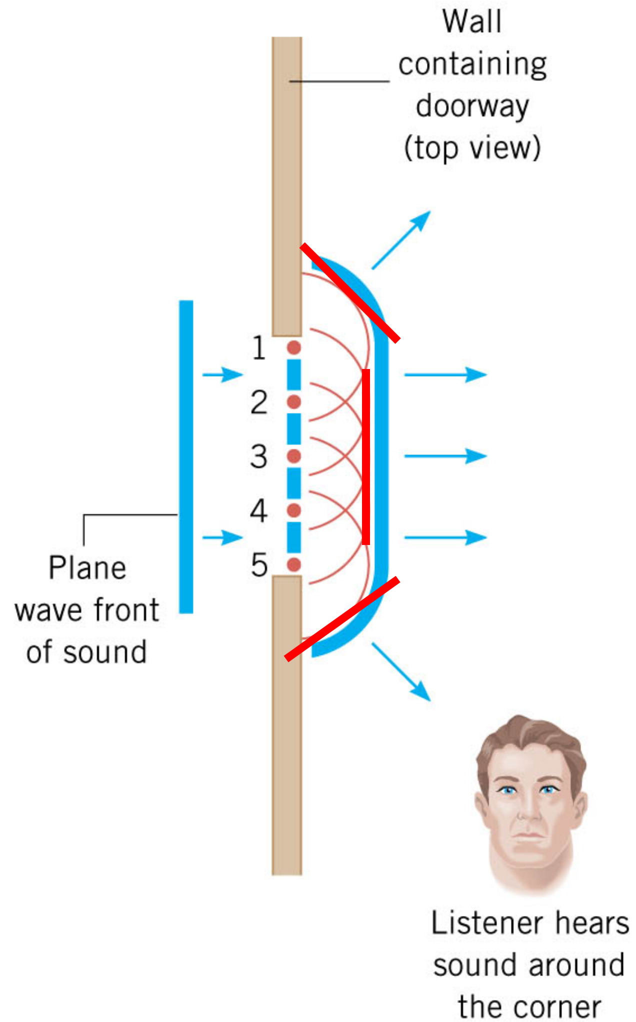


It is an interference effect – explained by Dutch scientist Christian Huygens.

Huygens Principle:

Every point on a wave front acts as a tiny source of wavelets that move forward with the same speed as the wave. At a later time, the new wave front is the surface that is tangent to the wavelets.

As an example, consider sound waves diffracting thru an opening.



Divide the opening up into equally spaced points, here we've chosen 5.

Each of these points acts like a source of waves or wavelets.

The wave front is always tangent to the waves.

The direction of propagation of the wave is always perpendicular to the wave front.

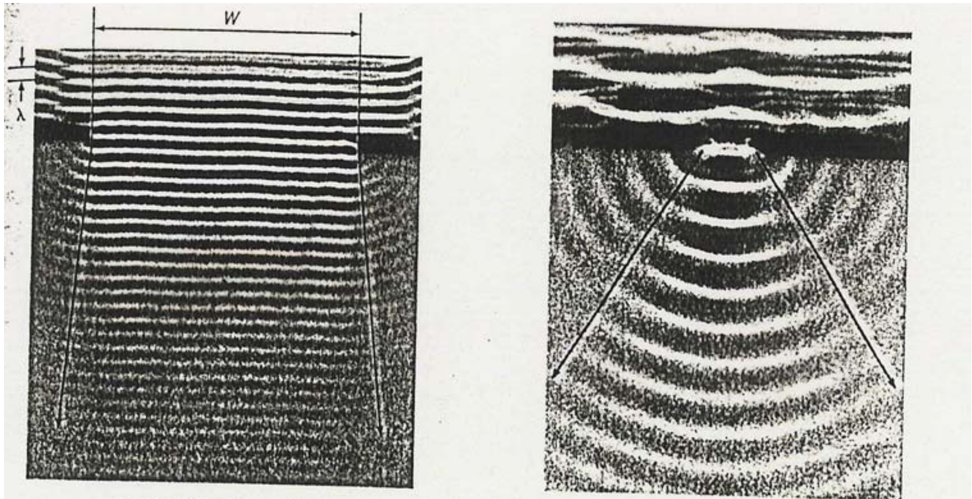
Thus, the wave can bend (or diffract) thru an opening or around a corner.

So what determines the degree of the diffractive bending?

It is determined by the ratio of the wavelength of the wave to the size (width) of the opening or obstacle.

$$\boxed{\text{Diffraction} \approx \frac{\lambda}{W}}$$

Thus, for more diffraction (or bending) of the waves, we want longer wavelengths and smaller openings.



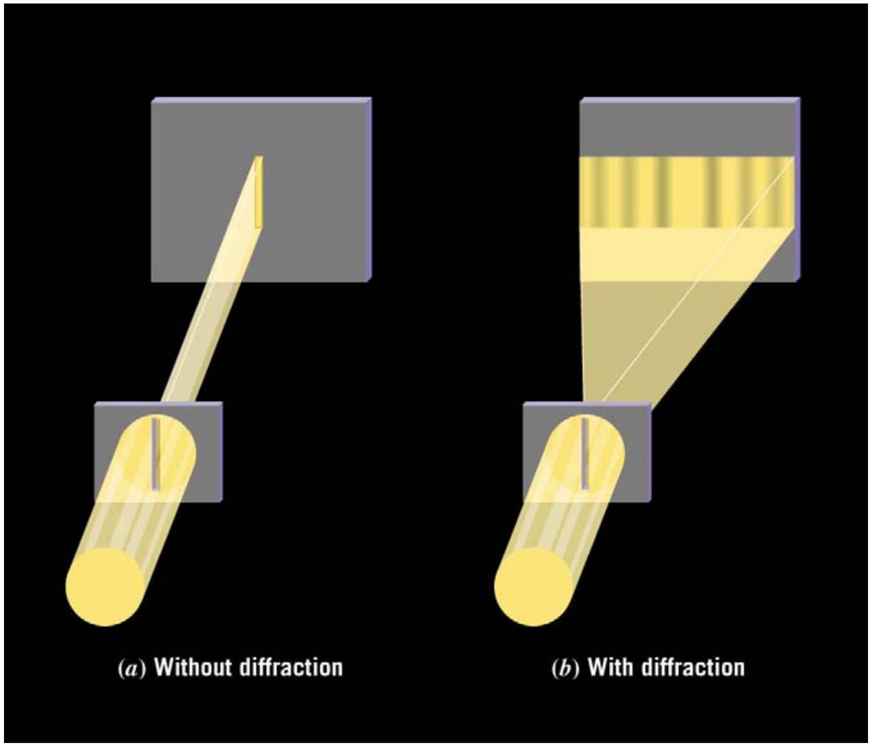
(a) Smaller value for λ/W, less diffraction.

(b) Larger value for λ/W, more diffraction.

We get more diffraction for the situation on the right, where the ratio λ/W is larger.

Single-slit Diffraction

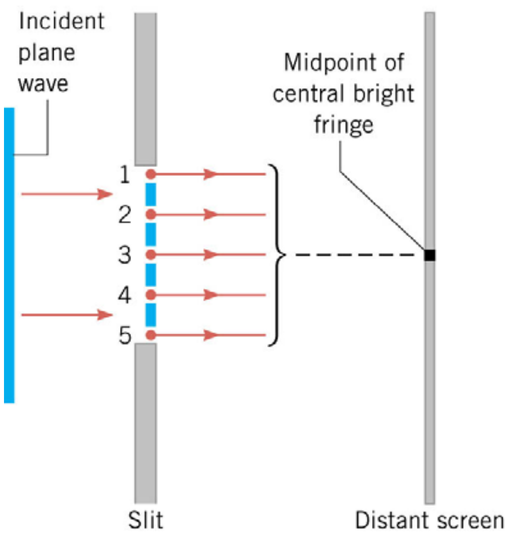
Consider monochromatic light of wavelength λ passing thru a single, narrow slit of width W.



If no diffraction occurred, we would just see one bright fringe directly behind the slit.

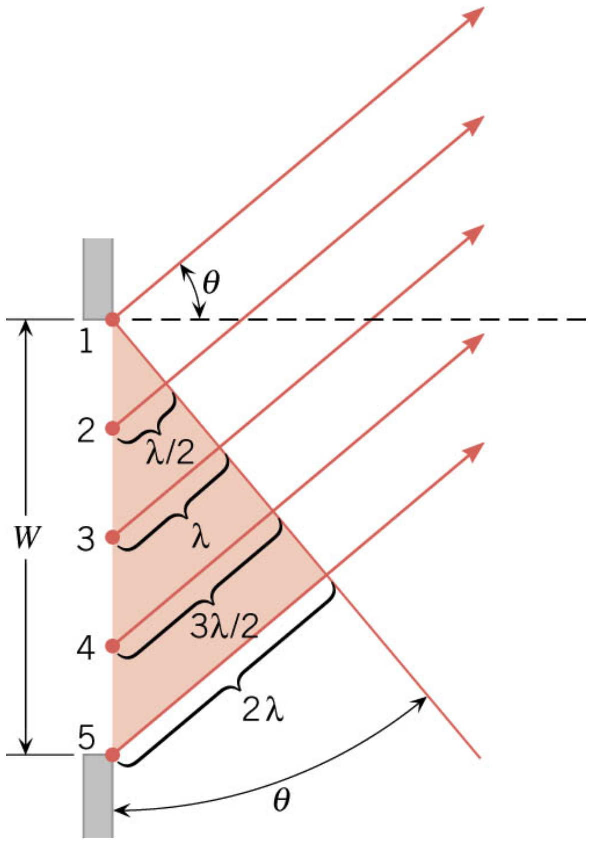
But, due to diffraction effects, we see an interference pattern – alternating bright and dark fringes.

As in the case for the double slit, here too do we have a central bright maximum. Why?



Huygens: Wavelets arriving at the center of the screen are traveling parallel and essentially the same distance. Since the OPD is zero, they arrive in phase and interfere constructively.

But, farther up the screen from the central bright maximum, the OPD between Huygen sources is not zero.



Here, for example, wavelets from sources 1 and 2 are out of phase and would interfere destructively on the screen, which is located a far distance away.

From the figure, we see that:

$$\boxed{\sin\theta = \frac{m\lambda}{W}}$$

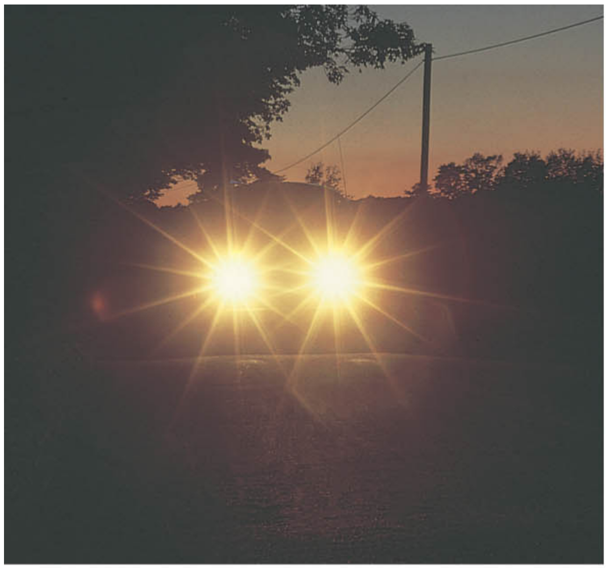
Width

This is the condition for dark fringes for single-slit diffraction, with m = 1, 2, 3,…

27.6 Resolving Power

Resolving Power: The ability of an optical instrument to distinguish (or resolve) two closely-spaced objects.

Look at the headlights of a car as it backs away from you to a far distance:



(a)

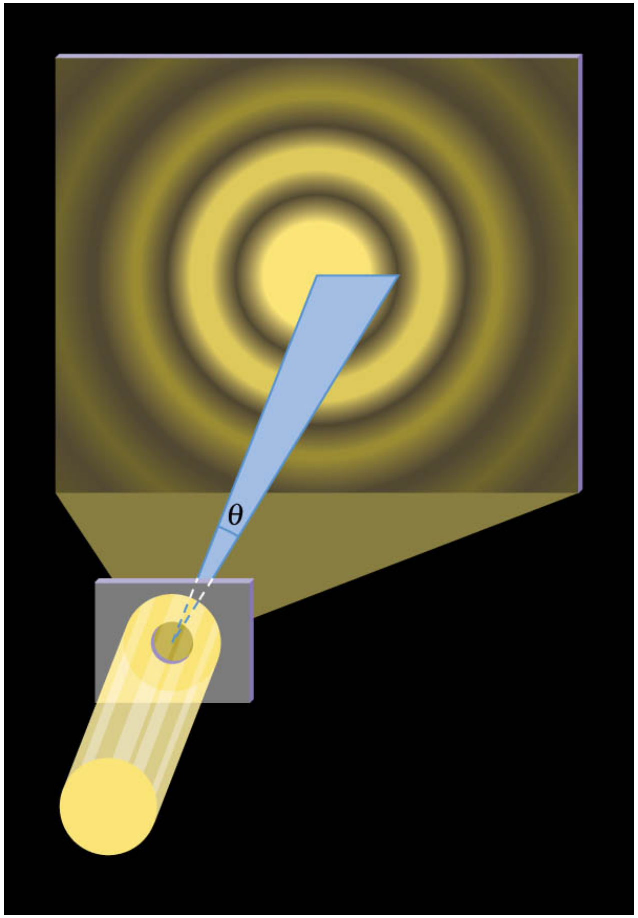
When the car is close, it is easy to distinguish two separate headlights.

But as it gets farther away, it's harder to resolve the two headlights.

Finally, there is a certain point when the car gets even farther away, that we can't distinguish the two headlights clearly.

This inability to resolve two closely-spaced objects is due to diffraction.

We have light passing thru openings, i.e. my eyes, a telescope, a microscope – <u>any optical instrument</u> – and it's diffraction thru these apertures that limits my resolution.



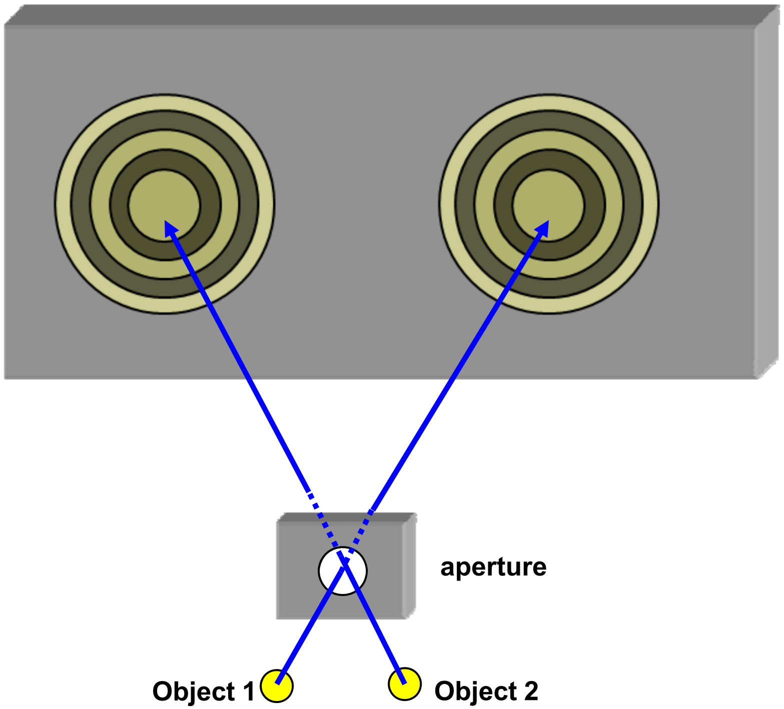
The screen to the left shows the diffraction pattern for light from <u>one</u> object passing thru a small circular opening. Notice there is a central bright fringe and alternating bright and dark fringes.

θ locates the angle from the central bright fringe to the first dark fringe.

If the screen distance is much larger than the width of the circular aperture (D), then:

$$\sin\theta = \frac{1.22\lambda}{D}$$

Now, if we have two objects, we would get the following:



Each object creates a diffraction pattern on the screen.

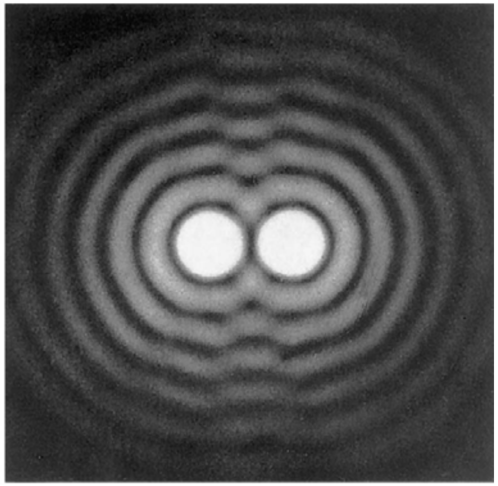
I can distinguish the two objects now, since their diffraction patterns are widely separated.

But look what happens if I move them closer together:

Now their diffraction patterns overlap, and I am unable to distinguish two separate objects.

It is useful to have a criterion for judging whether or not two objects are resolved.

We use the Rayleigh Criterion for Resolution: We say that two closely-spaced objects are just resolved when the first dark fringe of one image falls on the central bright fringe of the other.



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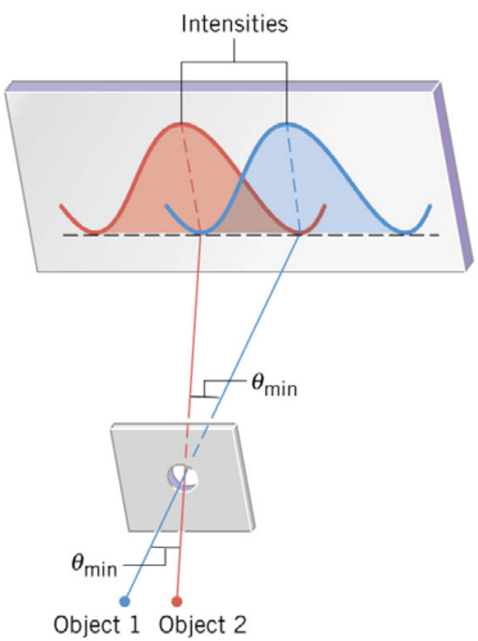


(a)

Here is an image of two objects that are just resolved.

Notice, the first dark fringe of one image is right at the edge of the central bright fringe of the other.

This sets a condition on the minimum angle between the two objects being resolved.

Thus, if $\theta < \theta_{min}$, then we won't be able to resolve the two objects.

Since θ_{min} is small, then $\sin\theta_{min} \approx \theta_{min}$.

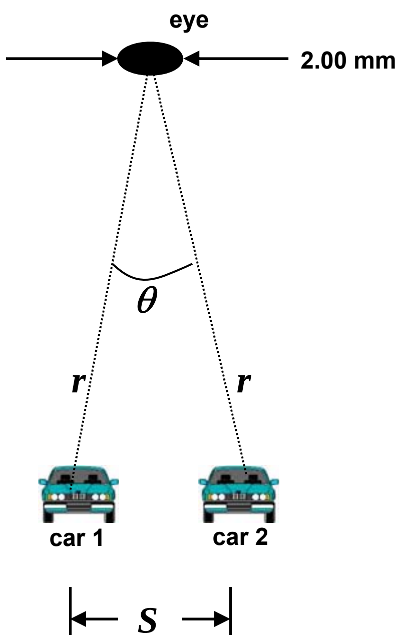
Thus, the Rayleigh Criterion for Resolution becomes:

$$\boxed{\theta_{min} \approx 1.22\frac{\lambda}{D}}$$

Example

You are looking down at earth from inside a jetliner flying at an altitude of 8690 m. The pupil of your eye has a diameter of 2.00 mm. Determine how far apart two cars must be on the ground if you are to have any hope of distinguishing between them in red light (wavelength = 665 nm in vacuum). Take into account the index of refraction in the eye.

Solution:



$$s = r\theta$$ **Since *r* >> *D*.**

$$\theta_{\min} \approx \frac{1.22\lambda}{D} = \frac{s}{r} \qquad s = \frac{1.22\lambda r}{D}$$

$$s = \frac{1.22\lambda_{eye} r}{D} \qquad \lambda_{eye} = \frac{\lambda_{vac}}{n_{eye}}$$

$$\boxed{s = \frac{1.22\lambda_{vac} r}{D n_{eye}}}$$

$$s = \frac{1.22(665\times10^{-9})(8690)}{(0.002)(1.36)} = \boxed{2.59\ \mathrm{m}}$$

27.7 The Diffraction Grating

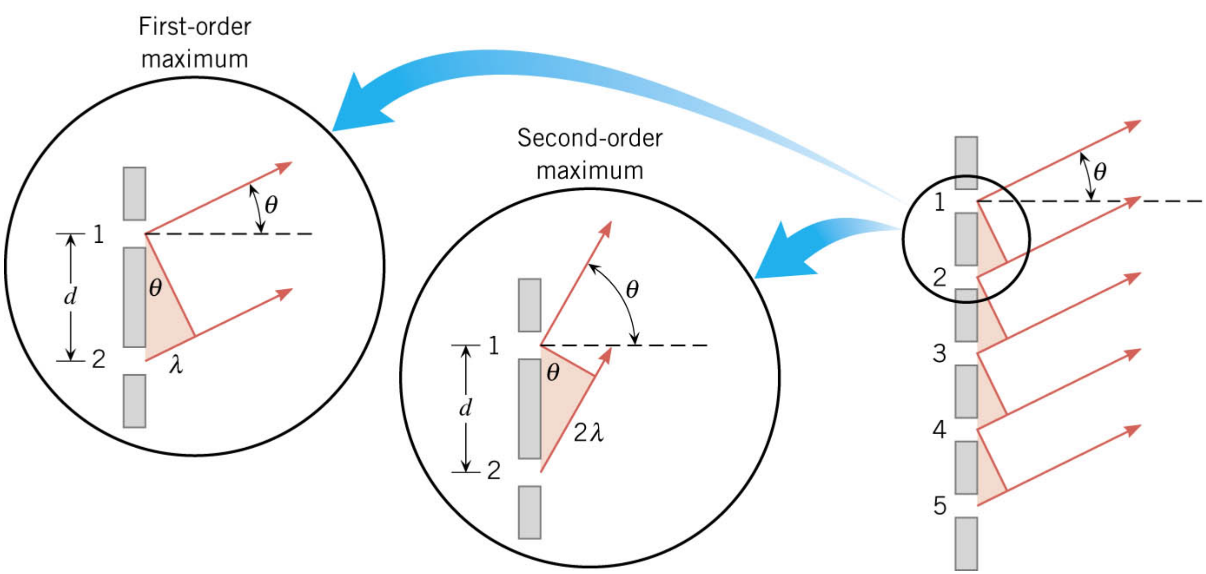
We see diffraction patterns of alternating bright and dark fringes when monochromatic light is shined on a single or double slit.

What if we shined light on many close-spaced slits? What would we expect to see?

Such an instrument is called a Diffraction Grating.

Some of them can have tens of thousands of slits per cm.

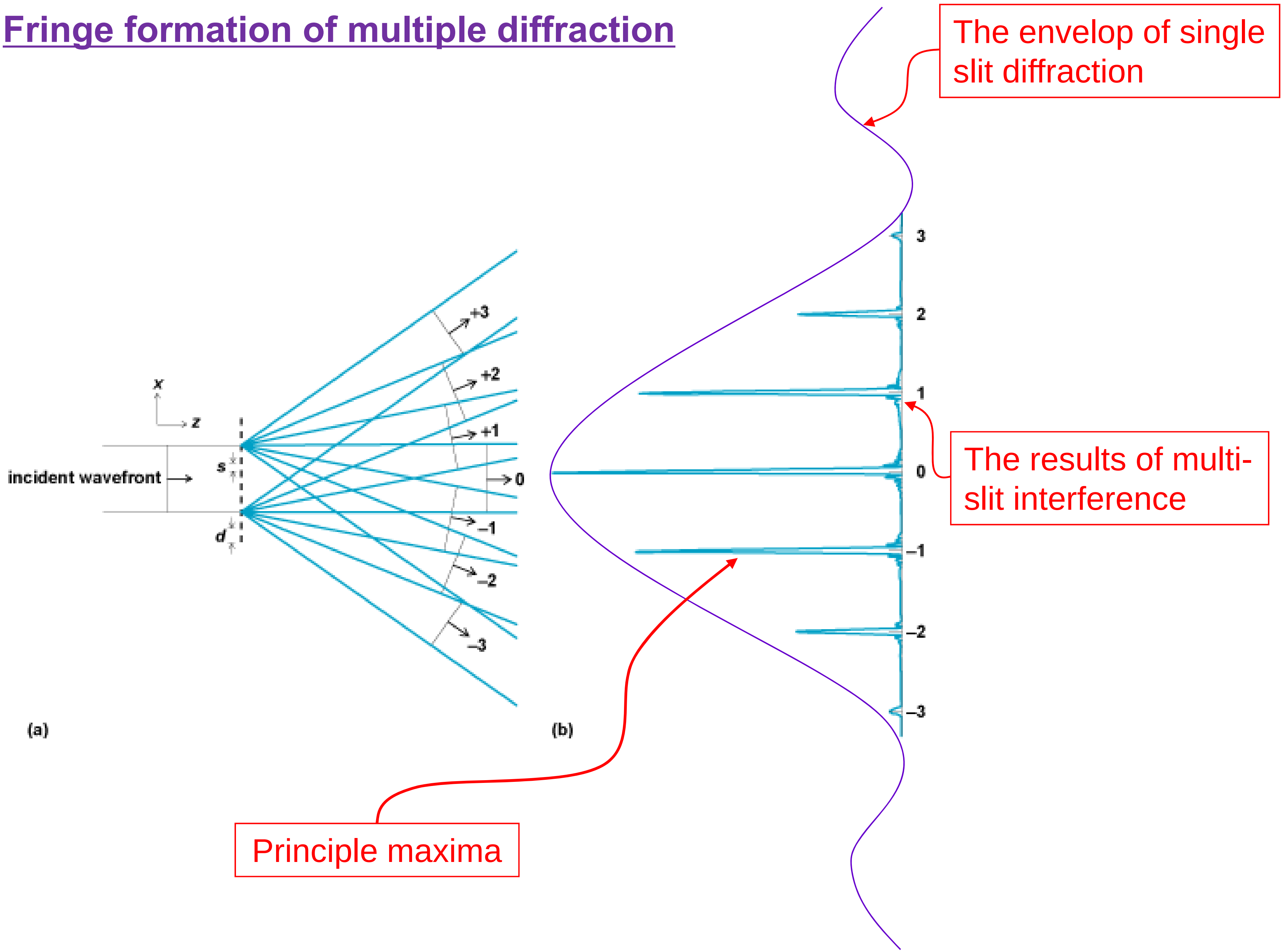
Again we see alternating bright and dark fringes:



Each slit acts as a source of wavelets in accord with Huygens.

The figure to the left shows have the first and second order (m = 1 and 2) maxima (bright fringes) develop.

Fringe formation of multiple diffraction



diffraction pattern of a grating

So, the principal maxima of the diffraction grating are given by:

$$\boxed{\sin\theta = \frac{m\lambda}{d}} \qquad m = 0, 1, 2, 3, \ldots$$

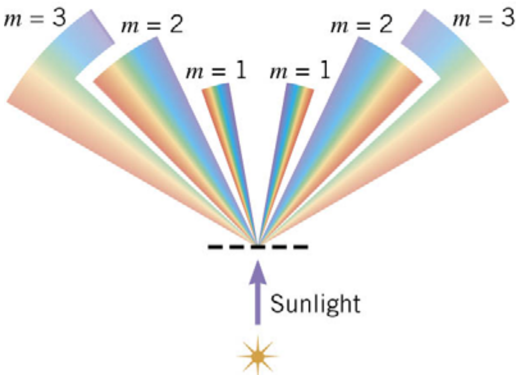
d **is the slit separation distance. It can be calculated by knowing the # of slits per cm.**

For example, let's say we have a diffraction grating with 7500 slits/cm, then

$$d = \frac{1}{7500}\text{ cm} = 1.33\times10^{-4}\text{ cm}$$

What if we shined white light on a diffraction grating?

Just like the double slit, we would see multiple colored bright fringes:



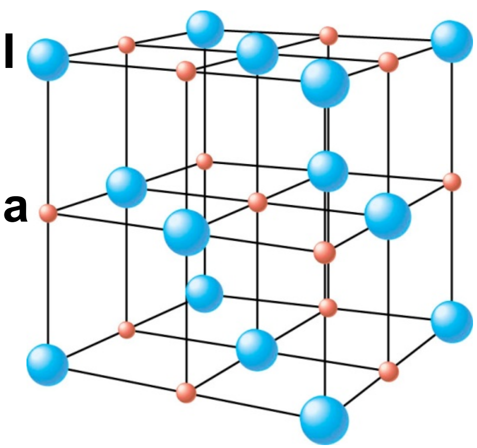
A diffraction grating separates light according to color (wavelength) much the same way a prism disperses light, but there is a difference:

In a prism, it's the longer wavelengths that are bent the least, whereas the diffraction grating bends the longer wavelengths the most.

27.9 X-ray Diffraction

Not all diffraction gratings are artificially made. Some are made by Mother Nature.

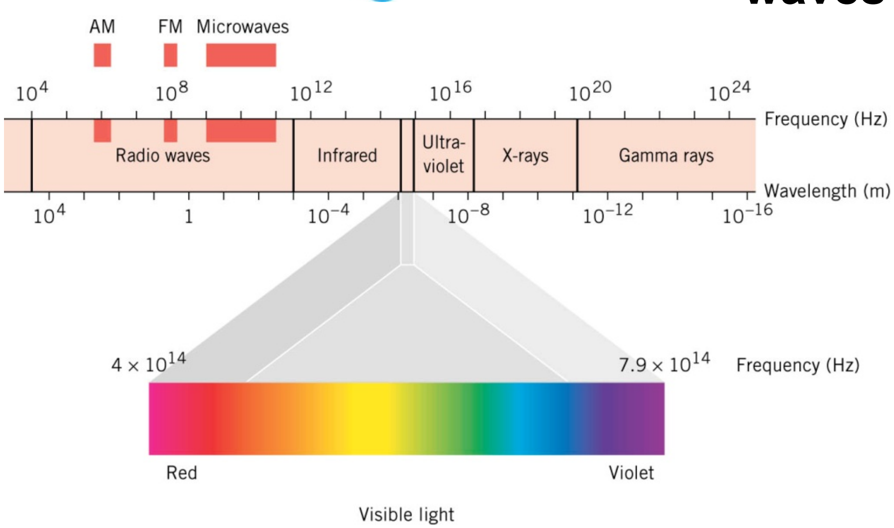
Take table salt, for example, NaCl.



The Na and Cl ions form a 3-dimensional periodic array of atoms called a crystal.

The distance between the atoms (interatomic spacing) is about 1 Å (1×10^{-10} m).

Thus, this distance could act like the slit separation distance on a diffraction grating for waves with an appropriate wavelength.

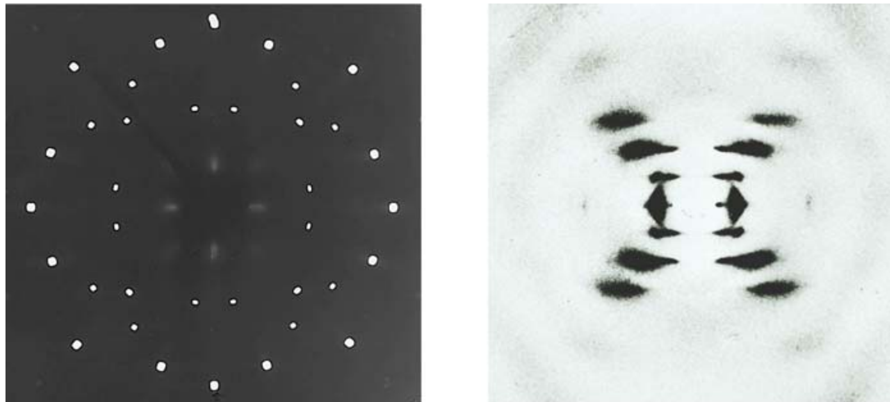


Thus, let's choose light whose wavelength is ~ 1×10^{-10} m.

Light of this wavelength resides in the X-ray part of the EM spectrum.

By shining X-rays on the crystal, we should see diffraction effects.

Indeed, a diffraction pattern does result when you shine X-rays on a crystalline material.



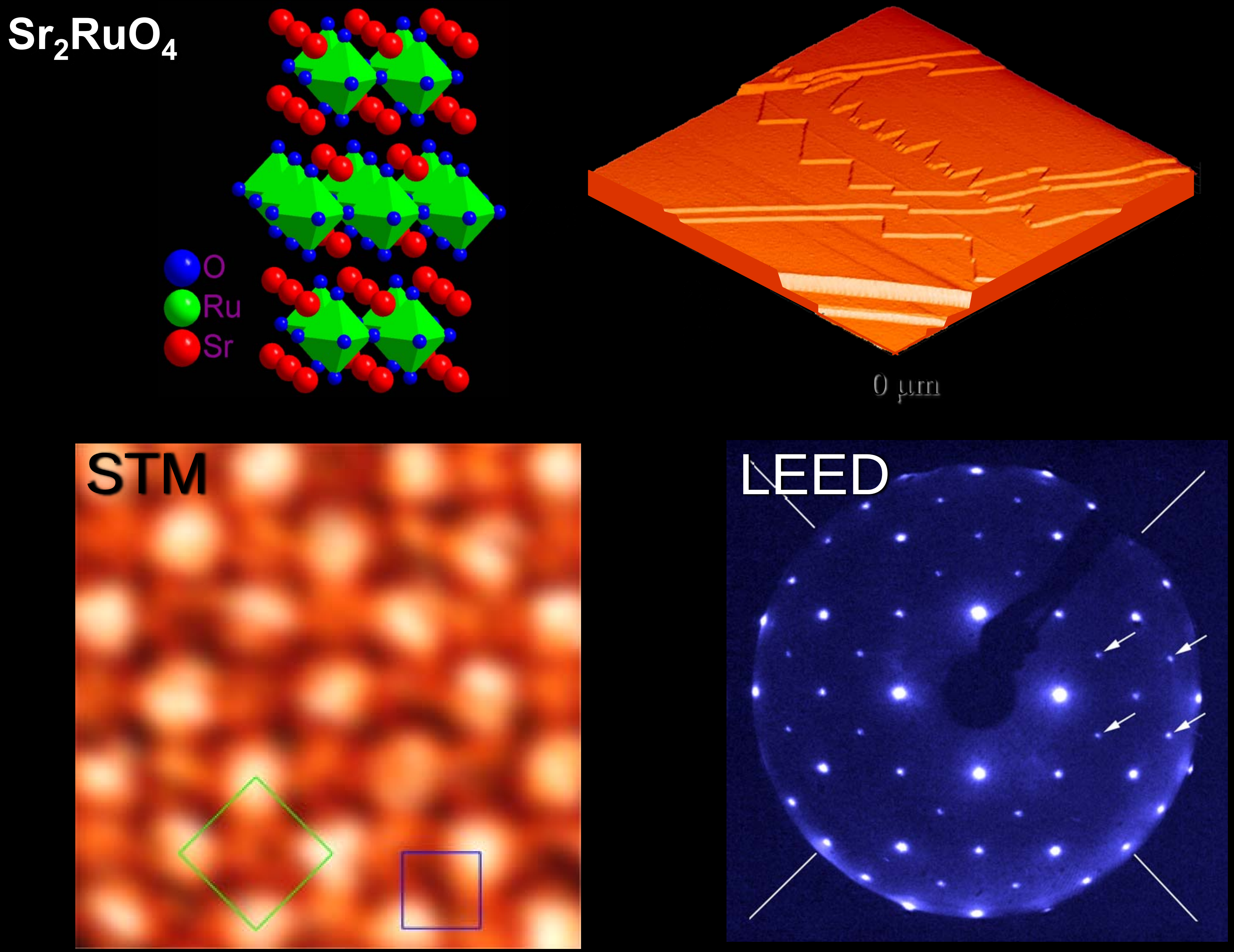
(a) (b)

Cubic NaCl

DNA Double Helix

The diffraction pattern gives us information about the structure of the crystal and the distance between its atoms.

X-ray diffraction is also vitally important to determining the structures of biological molecules, like proteins.



Sr2RuO4
O
Ru
Sr
0 µm
STM
LEED