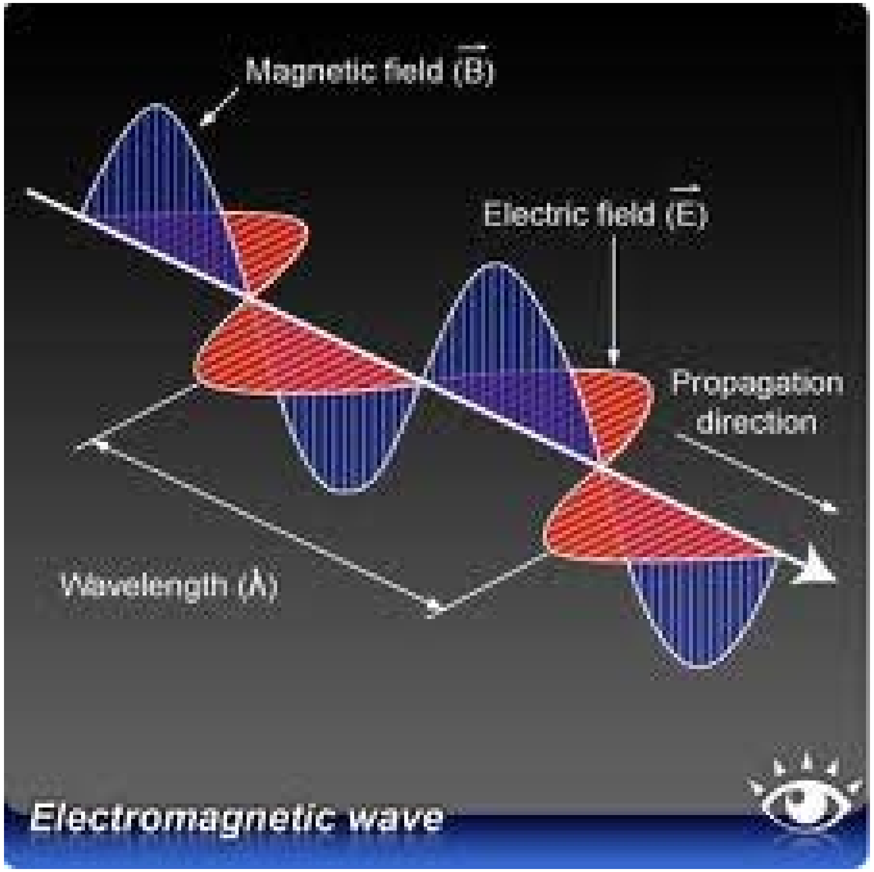
Algebra-based Physics II

Oct 11th: Chap 24.1-4
Electromagnetic (EM) waves

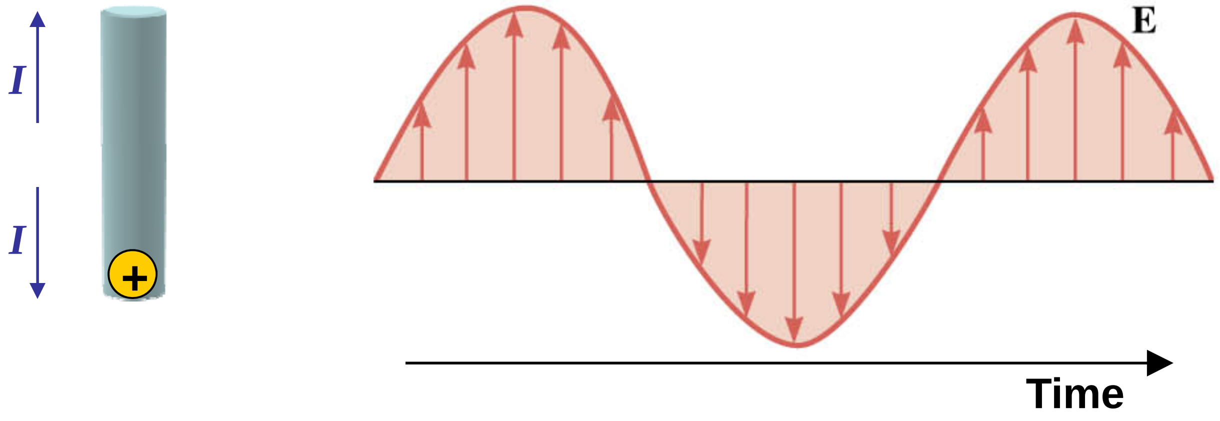
- Nature of the EM waves
- EM spectrum: wave length (λ)
- Energy carried by EM waves, speed (c)



Ch. 24 Electromagnetic Waves

Take a single positive charge and wiggle it up and down:

The charge changes position as a function of time.



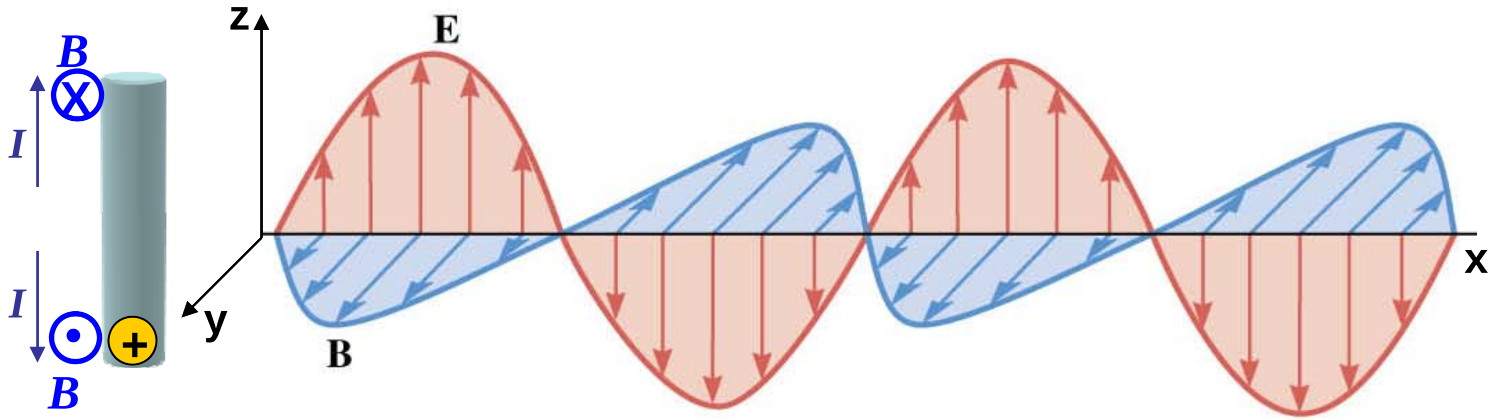
Thus, the electric field it creates changes in time.

But since the charge is moving, it constitutes a current:

The current points up when the charge moves up, and the current points down when the charge moves down.

This current, like all currents, creates a magnetic field!

The direction of the field is given by RHR-2.



By RHR-2, we see that when the current points up, the mag. field points into the screen, and when the current points down, the mag. field points out of the screen.

Thus, I have a changing magnetic field and a changing electric field which are oriented at right angles to each other!

Here, the electric field is in the xz-plane, and the magnetic field is in the xy-plane.

The fields move out away from the source (our accelerating charge):

Propagation of Electromagnetic (EM) Waves

An EM wave is a transverse wave:

The wave motion is at right angles to the direction of propagation.

It was James Clerk Maxwell (1831-1879) who worked out the mathematics of the wave propagation:

In words: The changing electric field induces a magnetic field (which also changes), and this changing magnetic field induces an electric field, etc.

This is how the wave propagates!

EM waves don't need a medium to travel through. They can propagate through a vacuum.

How fast do EM waves travel?

We can answer this question by looking at Maxwell's wave equation:

$$\nabla \times B = \mu_o J + \mu_o \varepsilon_o \frac{dE}{dt}$$

From partial differential equations, we identify the speed^2 of the wave as one over the coefficient on dE/dt.

Thus, we could write the wave equation as:

$$\nabla \times B = \mu_o J + \frac{1}{v^2}\frac{dE}{dt}$$

So $$v^2 = \frac{1}{\mu_o \varepsilon_o} \Rightarrow v = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$$

$$\Rightarrow v = \frac{1}{\sqrt{(4\pi \times 10^{-7})(8.85 \times 10^{-12})}} \Rightarrow v = \frac{1}{3.334 \times 10^{-9}}$$

$$\Rightarrow \boxed{v = 2.999 \times 10^{8} \text{ m/s}}$$

Wow! EM waves propagate at the speed of light!!!!

Review:

1. **Stationary charges create electric fields.**
2. **Moving charges (constant velocity) create magnetic fields.**
3. **Accelerating charges create electromagnetic waves.**

24.2 Electromagnetic Spectrum

Since Maxwell discovered that EM waves move at the speed of light, he hypothesized that light itself must be an EM wave!

Like any wave, EM waves have a frequency, a period, and an amplitude.

From Ch. 16, we know that: $v = f\lambda$

And since $v = c$**, we get for EM waves:** $c = f\lambda$

Speed of light (c)

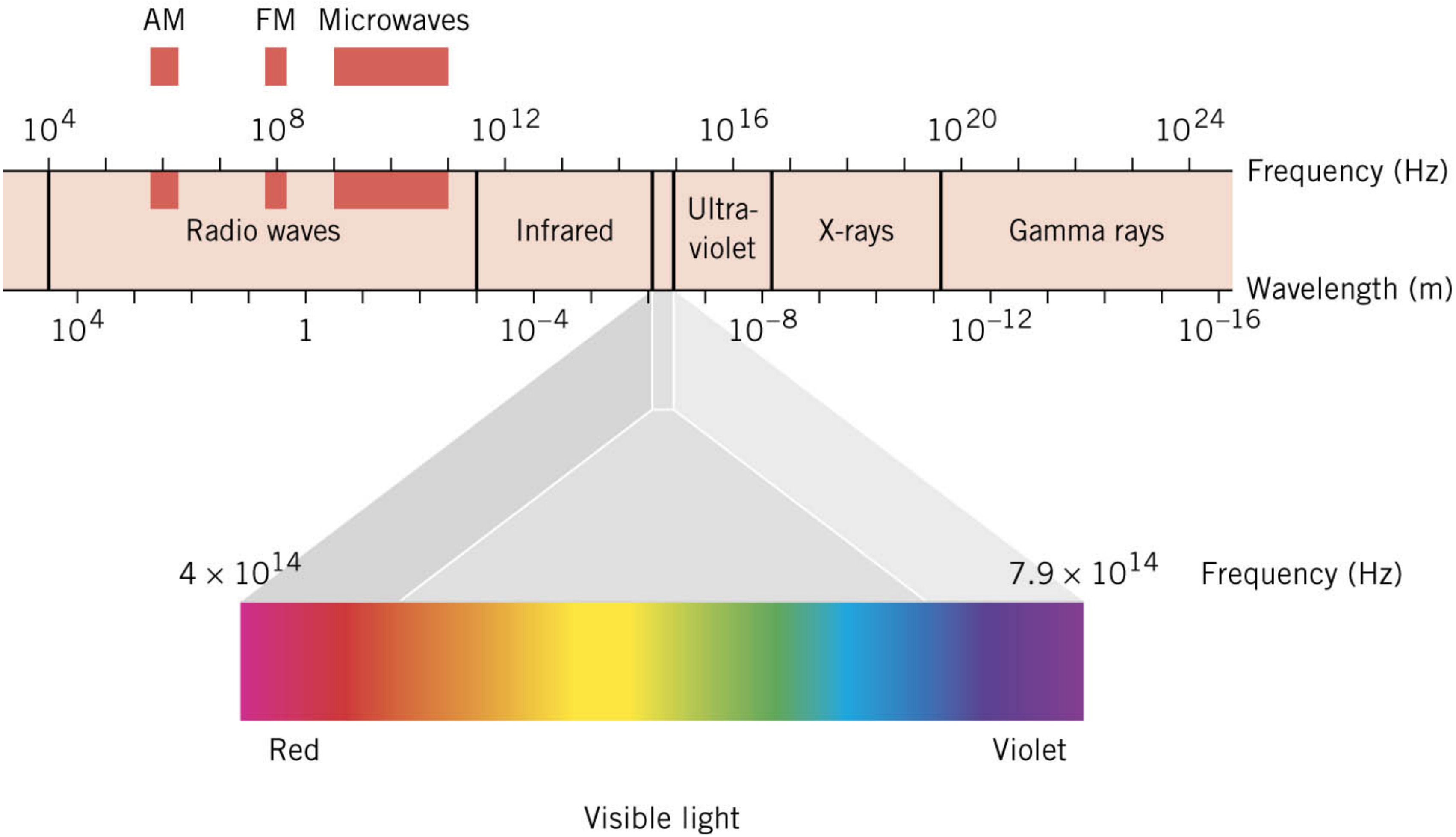
Frequency (Hz) (f)

Wavelength (m) (λ)

$$f = \frac{c}{\lambda}$$

Higher frequencies mean shorter wavelengths!

The Electromagnetic Spectrum



24.3 The Speed of Light

Very fast!!!!.....but finite.

$$c = 3.00 \times 10^8 \text{ m/s}$$

Moon to Earth → 1.3 seconds

Sun to Earth → 8 minutes

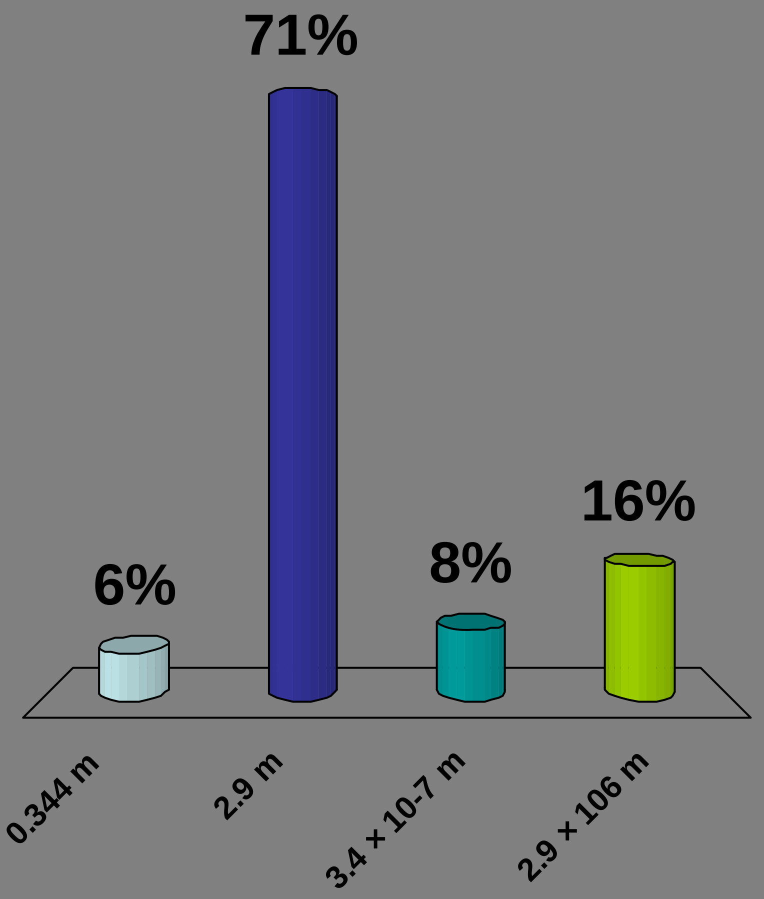
Distant stars and other astronomical objects are so far away that astronomers use a unit of distance called the light year (ly).

1 ly = The distance light travels in 1 year = 9.5×10^{15} m

Clicker question 24 - 1

The FM station broadcasting at 103.3 MHz plays music for the "Diva in all of us". What is the wavelength of these radio waves?

1. 0.344 m
2. 2.9 m
3. 3.4×10^{-7} m
4. 2.9×10^{6} m



24.4 Energy Carried by EM Waves

EM waves carry energy just like any other wave.

An EM wave consists of both an electric and magnetic field, and energy is contained in both fields.

A measure of the energy stored in an electric field is given by the energy density:

$$\text{Electrical Energy Density} = \frac{\text{Electrical Energy}}{\text{Volume}} = \tfrac{1}{2}\kappa\varepsilon_o E^2$$

κ **is the dielectric constant, and it equals 1 for a vacuum.**

So in a vacuum,

$$\boxed{\text{Electrical Energy Density} = \tfrac{1}{2}\varepsilon_o E^2}$$

We find a similar expression for the energy density in the magnetic field:

Notice that the energy goes as the square of the fields.

$$\boxed{\text{Magnetic Energy Density} = \frac{1}{2\mu_o}B^2}$$

So the total energy density in an EM wave is the sum of these two:

$$u = \tfrac{1}{2}\varepsilon_o E^2 + \frac{1}{2\mu_o} B^2$$

But in an EM wave, the electric field and magnetic field carry the same energy.

Thus, $$\tfrac{1}{2}\varepsilon_o E^2 = \frac{1}{2\mu_o} B^2$$

This allows me to write the total energy density in terms of just E or just B:

$$u = \varepsilon_o E^2 = \tfrac{1}{\mu_o} B^2$$

Since, $$\tfrac{1}{2}\varepsilon_o E^2 = \frac{1}{2\mu_o} B^2 \Rightarrow E^2 = \frac{1}{\mu_o \varepsilon_o} B^2 \Rightarrow E^2 = \frac{1}{c^2} B^2$$

$$\Rightarrow \boxed{E = cB}$$

So in an EM wave, the magnitude of the electric field is proportional to the magnitude of the magnetic field, and the proportionality constant is c, the speed of light!

The magnitude of the electric and magnetic fields in an EM wave fluctuate in time. It is useful to consider an *average value* of the two fields:

This is called the rms or root-mean-square value of the fields:

$$E_{rms} = \frac{E_o}{\sqrt{2}}$$

$$B_{rms} = \frac{B_o}{\sqrt{2}}$$

Here, E_o and B_o are the peak values of the two fields.

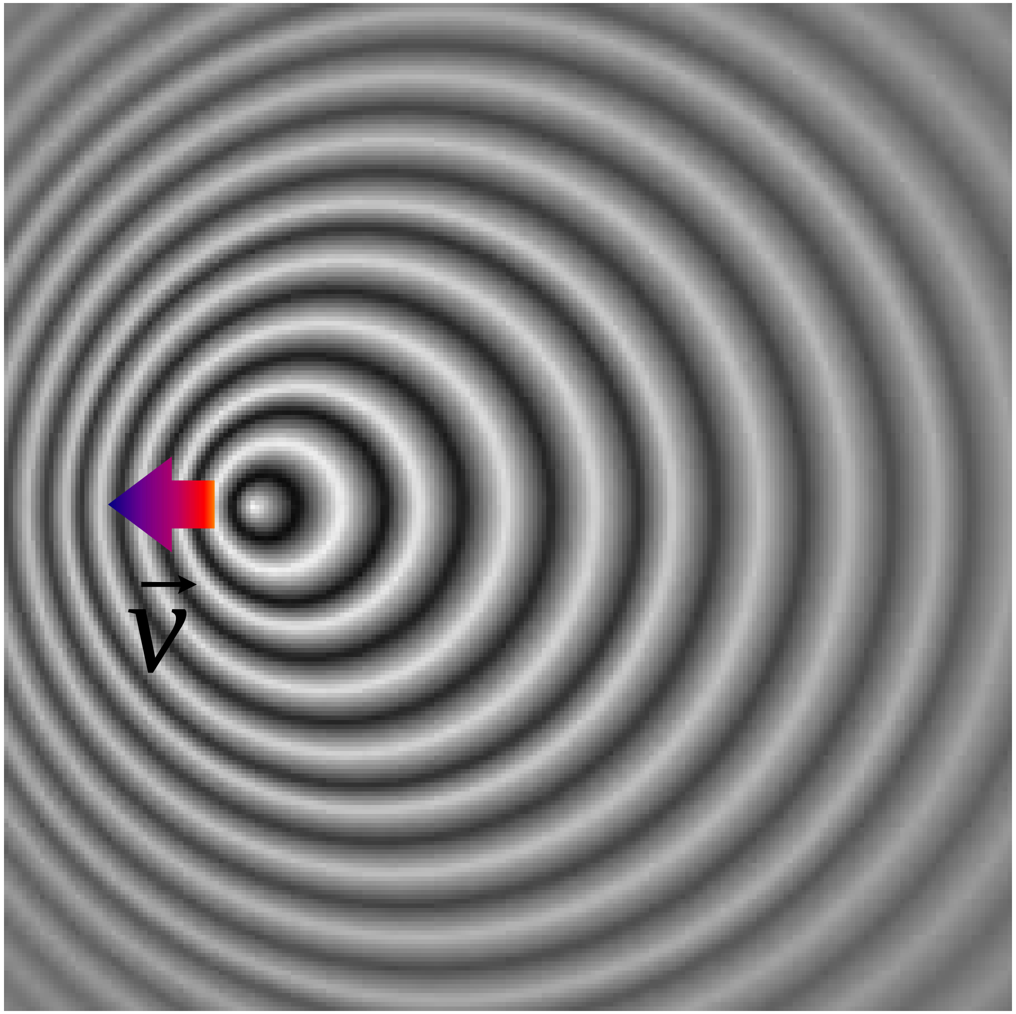
Now we can calculate an average energy density using the rms values:

$$\bar{u} = \tfrac{1}{2}\varepsilon_o E_{rms}^2 + \frac{1}{2\mu_o} B_{rms}^2$$

Algebra-based Physics II

Oct 11th: Chap 24.4-6

- Intensity of light or EM wave
- Doppler effect
- Polarization of light



Announcement

- HW6 is due on Friday
- 2nd-exam is coming

So as the EM waves propagate thru space, they carry energy along with them.

The transport of energy is related to the intensity of the wave.

Back in Ch. 16 we defined the intensity of a wave as the power per unit area:

$$S = \frac{P}{A} \Rightarrow \frac{W}{tA} \Rightarrow \frac{E}{tA}$$

Intensity = energy flows through a unit area and per unit time

What is the relationship between the intensity, S, and the energy density, u?

Sparing you the derivation, we find that

$$S = cu$$

So the intensity and energy density are just related by the speed of light, c.

In terms of the fields then:

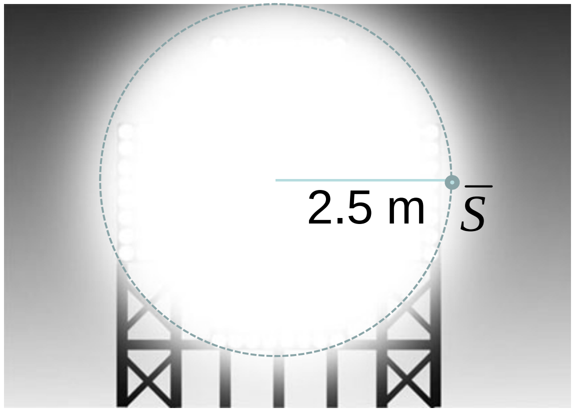
$$S = cu = \tfrac{1}{2} c\varepsilon_o E^2 + \frac{c}{2\mu_o} B^2$$

$$S = c\varepsilon_o E^2$$

$$S = \frac{c}{2\mu_o} B^2$$

If we use the rms values for the fields, then $S \rightarrow \bar{S}$, the average intensity.

Example: A point light source is emitting light uniformly in all direction, At a distance of 2.5 m from the source, the *rms* electric field strength of the light is 19.0N/C. Assuming that the light does not reflect from anything in the environment, determine the average power of the light emitted by the source.



What do we know: E_{rms}; r

Average light intensity at the imaginary spherical surface:

$$\overline{S} = c\overline{u} = c\varepsilon_0 E_{rms}^2$$

Power of the source: $\overline{P} = \overline{S} \cdot A = \overline{S} \cdot (4\pi r^2)$ $\quad \overline{S} = \dfrac{\overline{P}}{4\pi r^2}$

$$\overline{P} = \overline{S} \cdot A = \overline{S} \cdot (4\pi r^2)$$

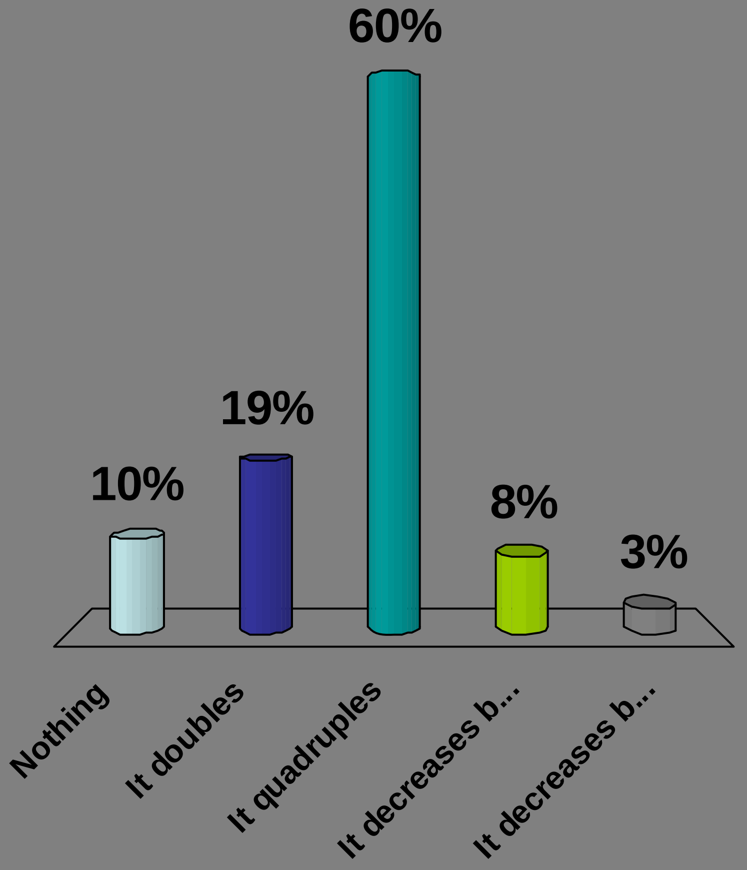
$$= c\varepsilon_0 E_{rms}^2 \cdot (4\pi r^2)$$

$$= 75.3\, W$$

Clicker question 24 - 2

Both the electric and magnetic field of an EM are doubled in magnitude. What happens to the intensity of the wave?

1. Nothing
2. It doubles
3. ✓ It quadruples
4. It decreases by a factor of 2
5. It decreases by a factor of 4



24.5 The Doppler Effect

When the observer of a wave, or source of the wave (or both) is moving, the observed wave frequency is different than that emitted by the source.

EM waves also exhibit a Doppler effect. But….

1. **They don't require a medium thru which to propagate, and..**
2. **Only the relative motion of the source to the observer is important, since the speed at which all EM waves move is the same, the speed of light.**

So how do we calculate the shift in frequency?

If the EM wave, the source, and the observer all travel along the same line, then:

$$f_o = f_s\left(1 \pm \frac{v_{rel}}{c}\right)$$

f_o **is the observed frequency**

f_s **is the frequency emitted by the source**

v_{rel} **is the relative velocity between observer and source**

The + sign is used when the object and source move toward each other.

The – sign is used when the object and source move away from each other.

(*This is valid for speeds v_{rel} << c.)

Astronomers can use the Doppler effect to determine how fast distant objects are moving relative to the earth.

Example

A distant galaxy emits light that has a wavelength of 500.7 nm. On earth, the wavelength of this light is measured to be 503.7 nm. (a) Decide if the galaxy is moving away from or toward the earth. (b) Find the speed of the galaxy relative to earth.

Solution

We start with the Doppler equation: $$f_o = f_s\left(1 \pm \frac{v_{rel}}{c}\right)$$

The light is shifted to longer wavelengths, which means smaller frequencies: $f = c/\lambda$

Thus, $f_o < f_s$. Which means that the parenthesis $\left(1 \pm \frac{v_{rel}}{c}\right)$ must be < 1.

Therefore, the correct sign in the parentheses is the – sign: the galaxy is moving away from earth.

(b) From the Doppler equation: $v_{rel} = c\left(1 - \frac{f_o}{f_s}\right)$. But $f = c/\lambda$.

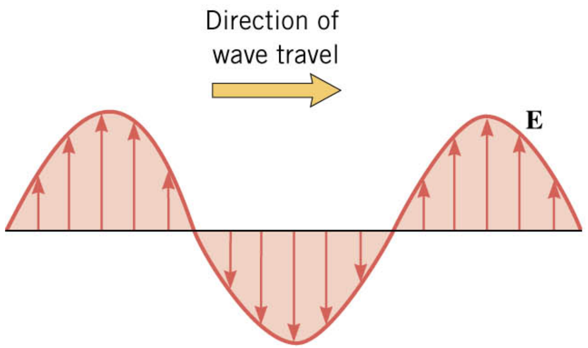
Thus, $$v_{rel} = c\left(1 - \frac{\lambda_s}{\lambda_o}\right) = 3\times10^8\left(1 - \frac{500.7\text{ nm}}{503.7\text{ nm}}\right) = \boxed{1.8\times10^6\text{ m/s}}$$

24.6 Polarization

EM waves are transverse waves and can be polarized.

Consider the electric field part of an EM wave.

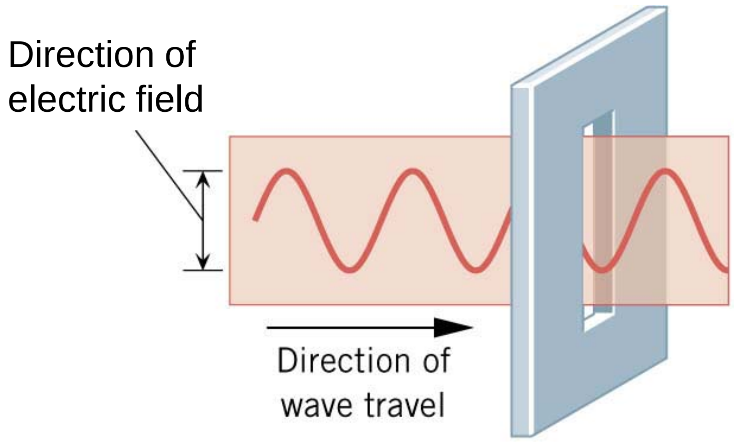
It oscillates up and down as the wave propagates:



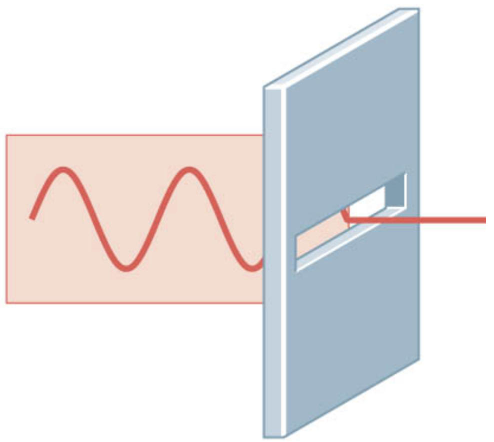
Thus, the wave oscillations are perpendicular to the direction of travel and occur in only one direction.

We refer to this wave as linearly polarized.

A vertical slit would allow the wave to pass through, since the slit is parallel to the oscillations:

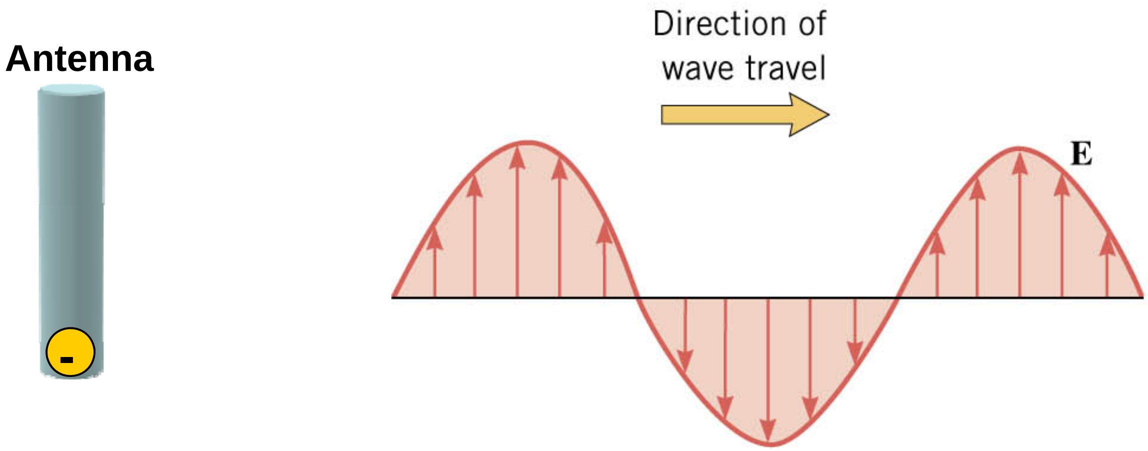


A horizontal slit would block the wave and not allow it to pass, since the slit is perpendicular to the oscillations:



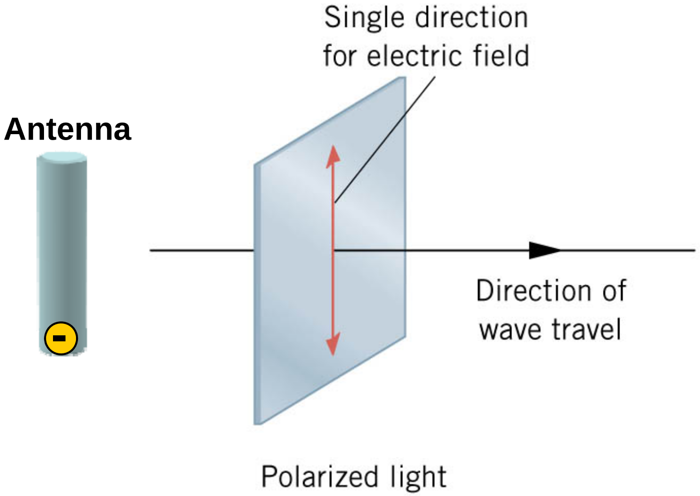
So how is polarized light produced?

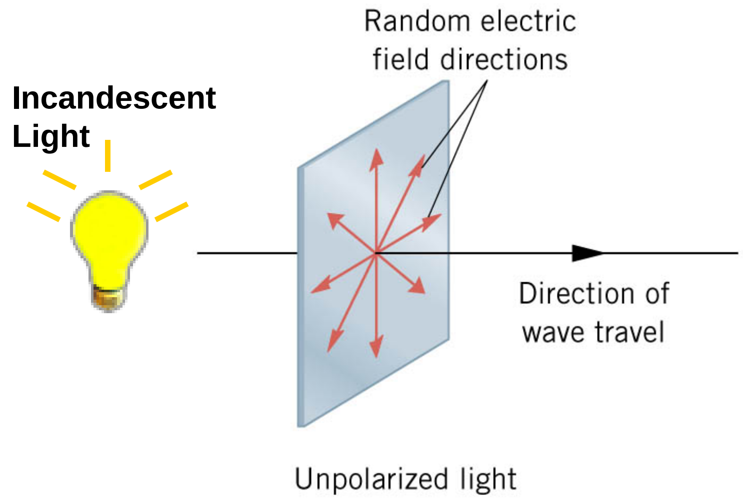
We could use the antenna on a radio station:



The up and down vibrations of the electrons in the antenna produce polarized waves whose direction is determined by the orientation of the antenna.

Incandescent light is unpolarized, resulting from many atoms vibrating in all possible orientations.



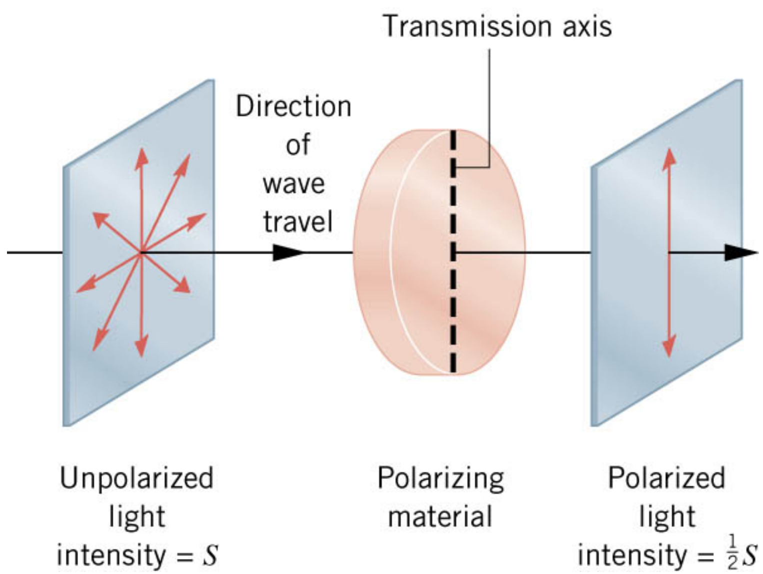


We can convert unpolarized light into polarized light by blocking all but one of the electric field orientations.

A device that does this is called a polarizer or polaroid.

The one direction that a polarizer allows light to pass thru it is called the transmission axis.

Let's start with unpolarized light and pass it thru a polarizer:



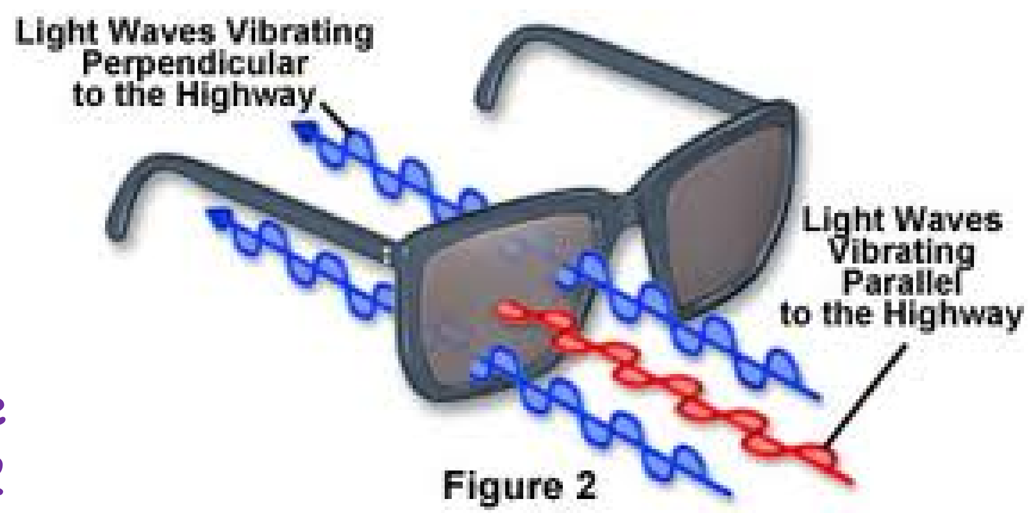
If the intensity of the unpolarized light is S before it passes thru the polarizer, then the intensity of the polarized light coming out will be ½S.

Algebra-based Physics II

Oct 15th: Chap 24.4 & Review

Announcement

- 2nd-exam: Oct. 18-20
- Formula sheet is ready
- Arrange your exam schedule
- Exam covers up to Chap. 22

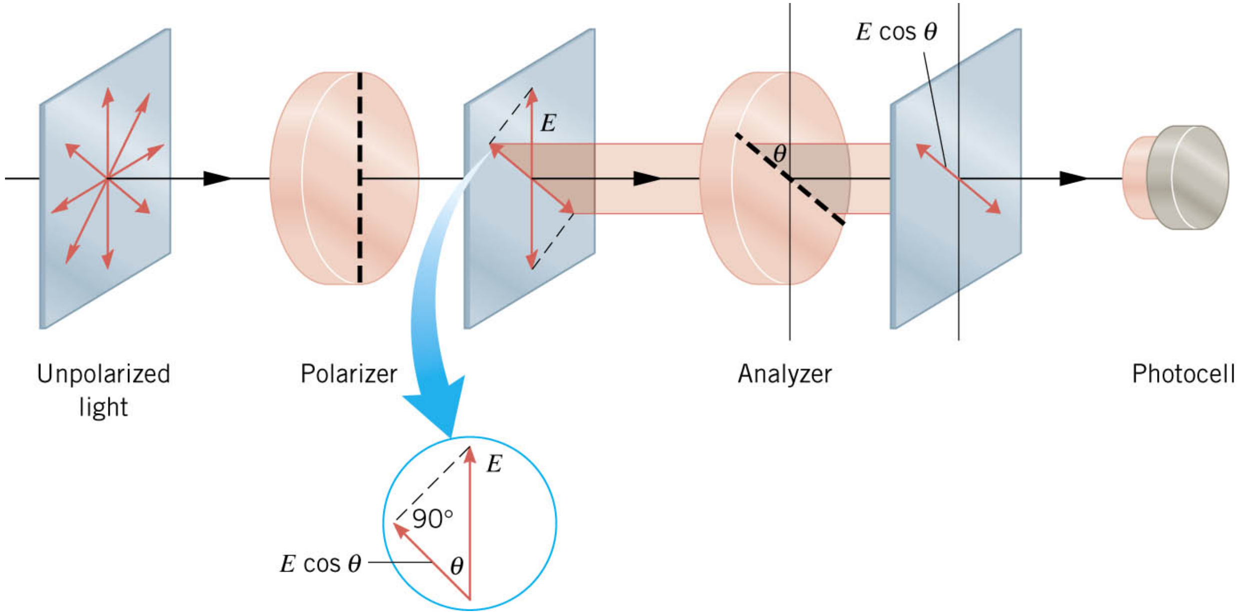


<u>Class Website</u>:

http://www.phys.lsu.edu/~jzhang/teaching.html

Malus' Law

Once the light has been polarized, it's possible for another polarizer (called the analyzer) to change the direction and intensity of the polarized light.



We know from our earlier discussion on intensity that: $S = c\varepsilon_o E^2$

Therefore: $S \propto E^2.$ **Out of the analyzer then,** $S \propto E^2 \cos^2\theta.$

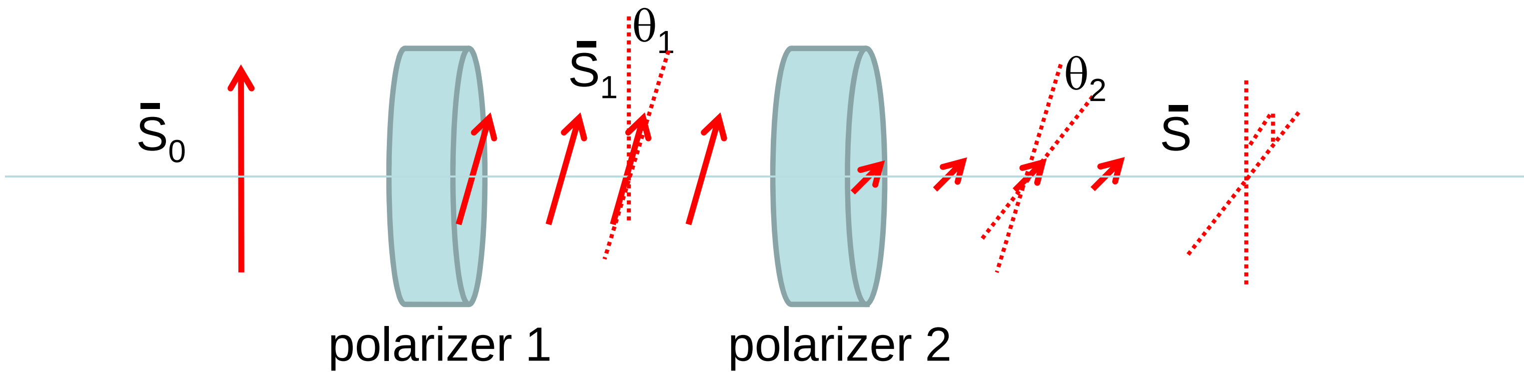
So both the intensity and polarization direction can be adjusted by changing the angle of the analyzer.

The average intensity of the light leaving the analyzer is: $\bar{S} = \bar{S}_o \cos^2\theta$

$\bar{S}_o$ **is the average intensity of the light entering the analyzer.**

Malus' Law

Example: How can you turn the polarization of a linear polarized light by 90° with a minimum number of polarizers? If the initial intensity is $\bar{S}_0$, what is the final intensity after the polarizers?



At least need two polarizers

$$\bar{S}_1 = \bar{S}_o \cos^2\theta_1$$ through 1st-polarizer

$$\bar{S} = \bar{S}_1 \cos^2\theta_2 = \bar{S}_0 \cos^2\theta_1 \cos^2\theta_2 \qquad (\theta_2 = \frac{\pi}{2} - \theta_1)$$ through 2nd-polarizer

$$= \bar{S}_0 \cos^2\theta_1 \sin^2\theta_1$$

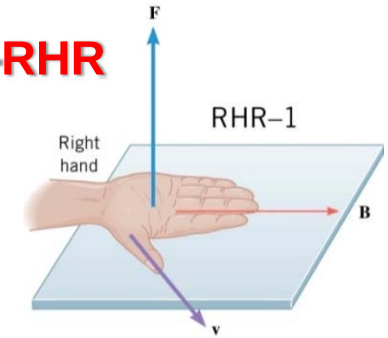
$$= \frac{\bar{S}_0}{4} \sin^2 2\theta_1$$

Review

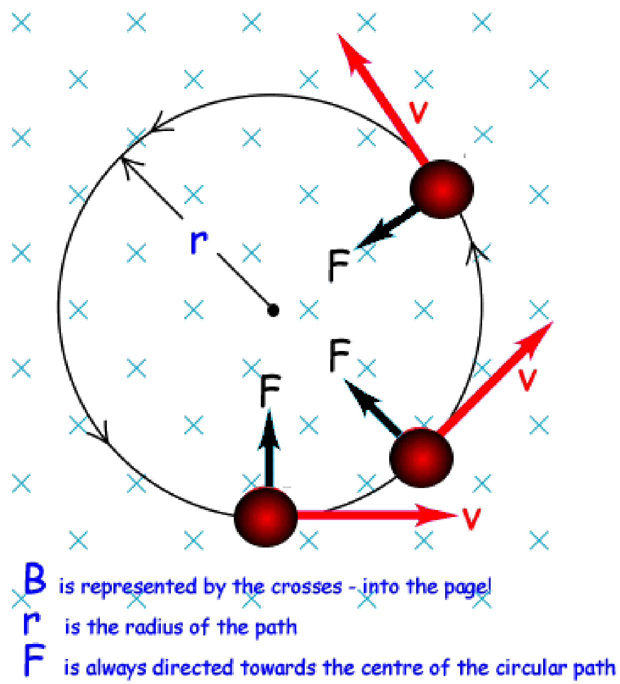
How to resolve the force acting on a moving change by a fielf and related motion?

$$F = qvB\sin\theta$$

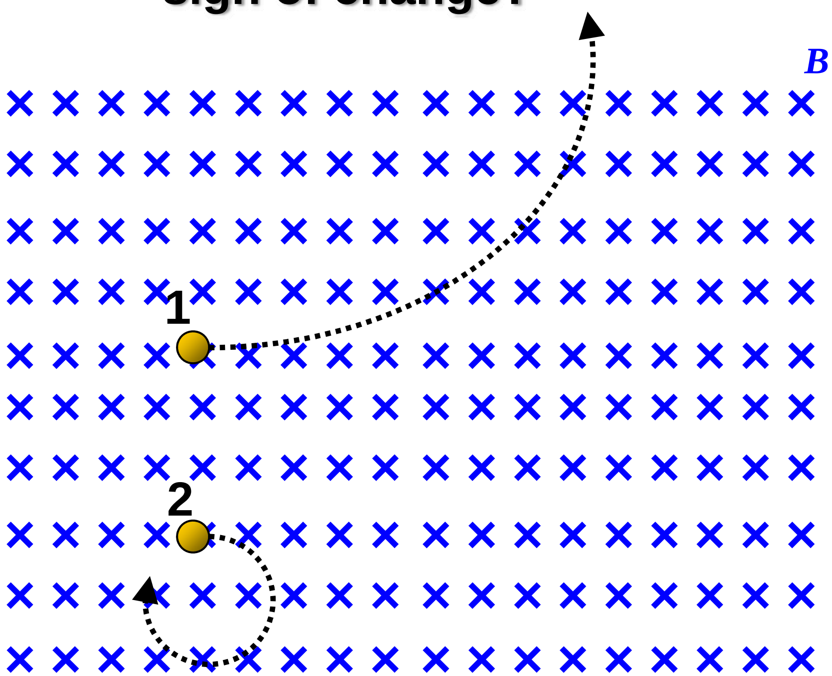
1st-RHR



- **circular motion** $r = \dfrac{mv}{qB}$



- **sign of change?**

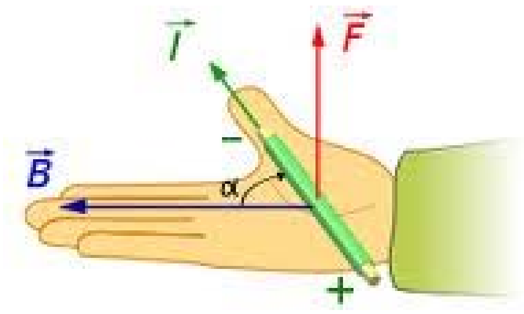


- **mass spectroscopy**

$$m = \left(\frac{qr^2}{2V}\right)B^2$$

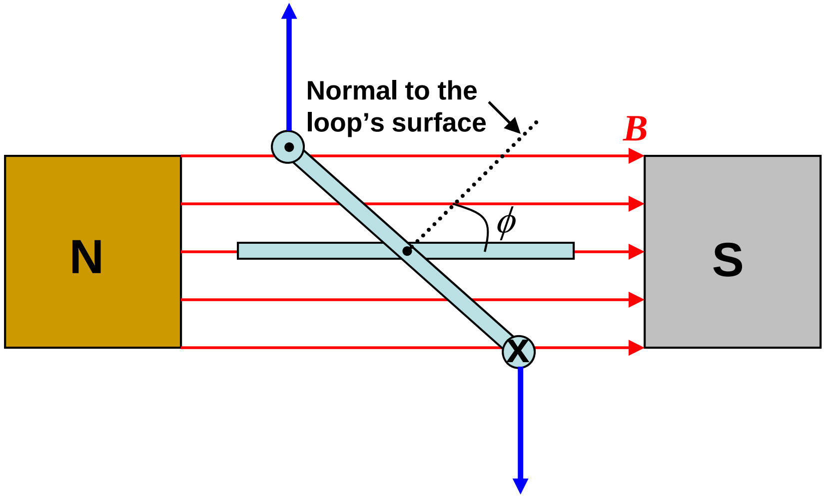
How to calculate the force acting on a piece of current by a uniform field?

$$F = ILB\sin\theta$$

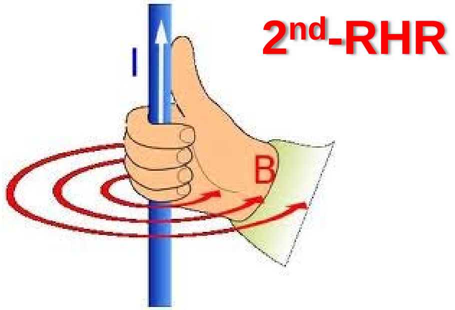


How to find the torque acting on a coil by a uniform field?

$$\tau_{Net} = NIAB\sin\phi$$



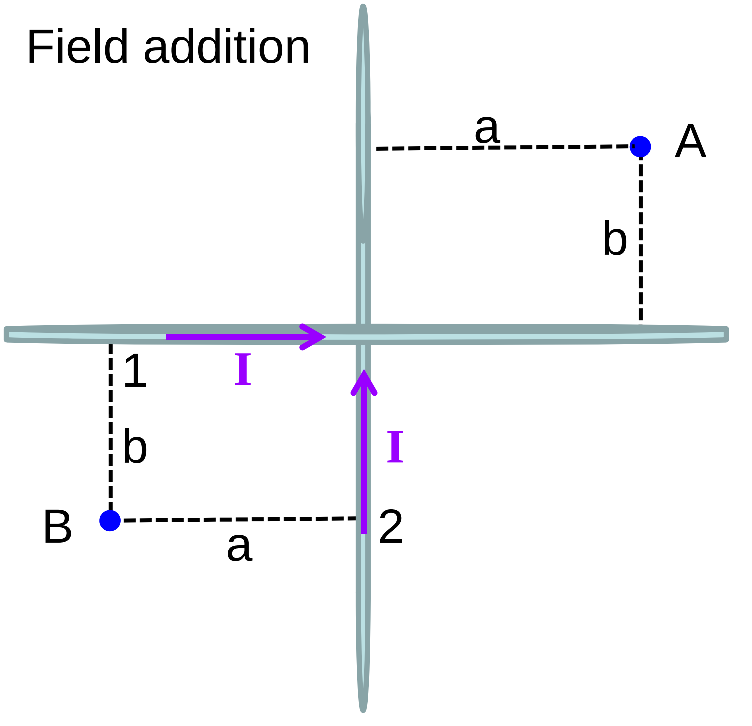
How to find the magnetic field created by a current ?

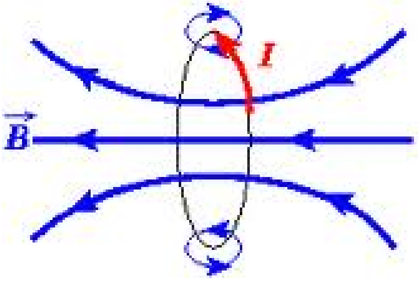


Straight wire

$$B = \frac{\mu_o I}{2\pi\, r}$$

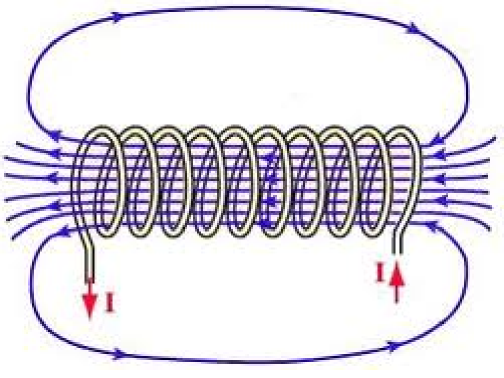
Field addition





Coil with N loop

$$B_{center} = N \frac{\mu_o I}{2R}$$

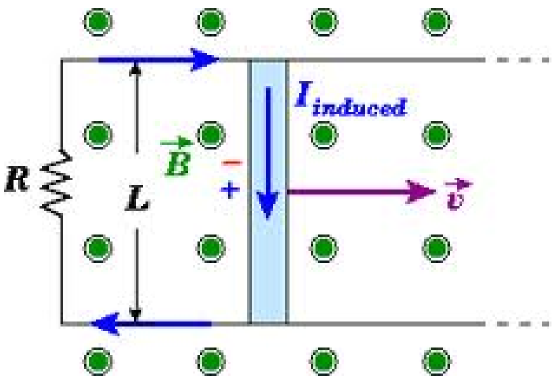


Solenoid

$$B = \mu_o I n$$

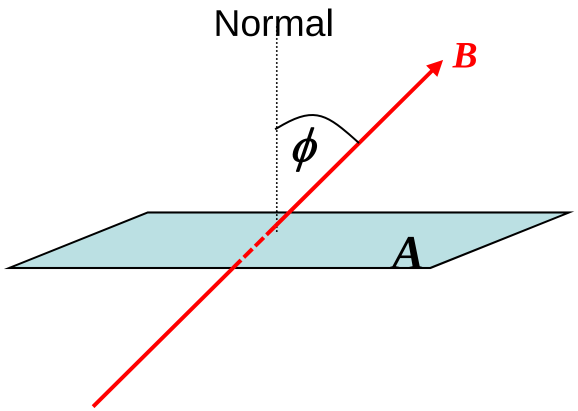
A uniform field inside (inductor!)

How to find motional emf?

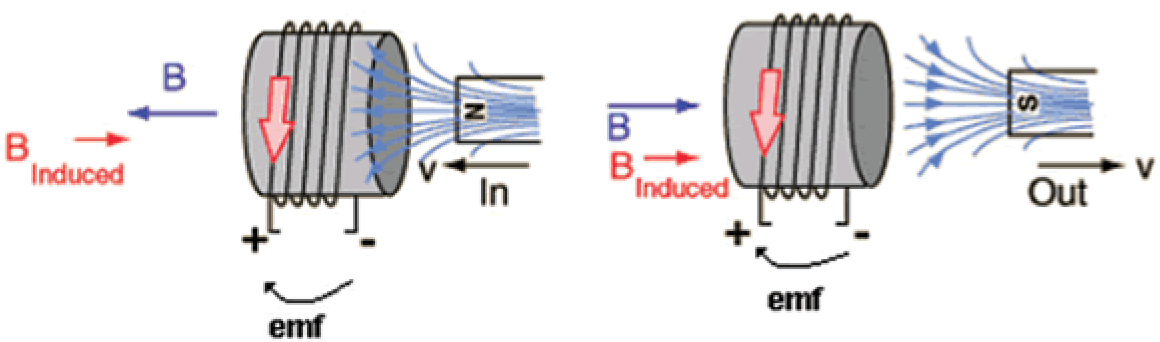


$$\mathcal{E} = vBL$$

How to calculate magnetic flux through a coil and induced emf ?



$$\Phi_M = BA\cos\phi$$

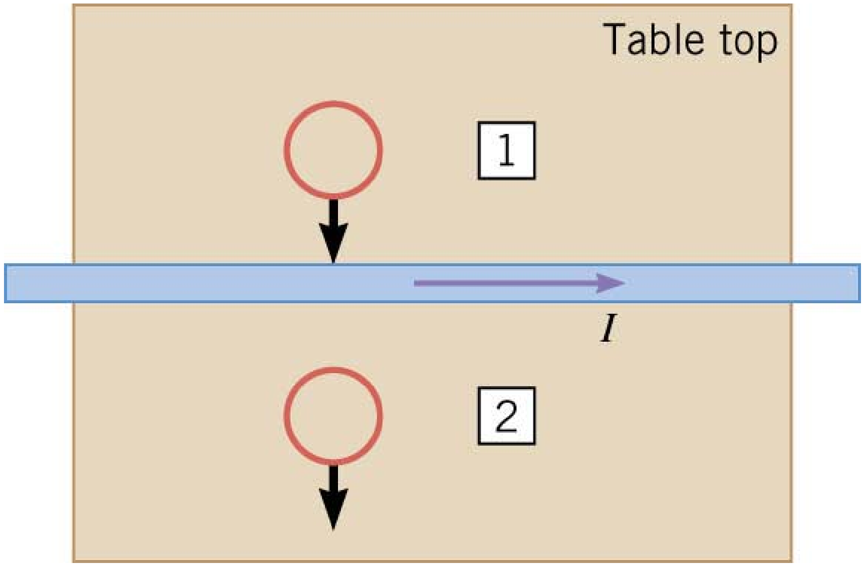


$$\mathcal{E} = -N\frac{\Delta\Phi_M}{\Delta t}$$

Faraday's law

How to use Lenz's law to figure out the induced current?

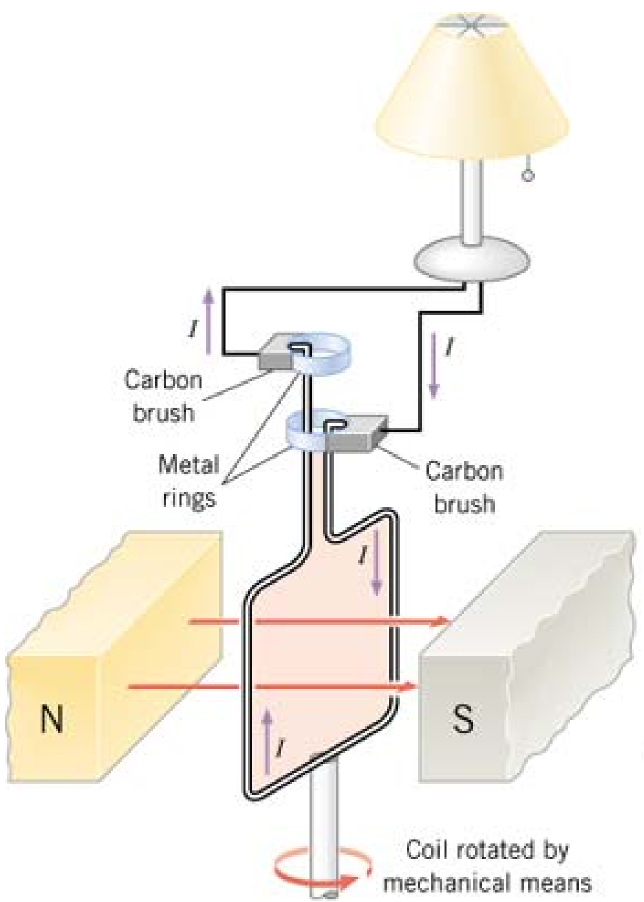
The induced emf resulting from a change in magnetic flux leads to an induced current which produces a magnetic field to oppose the change in flux.



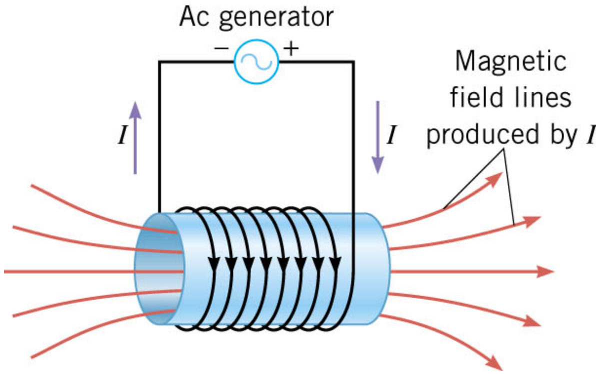
AC electric generator:

$$\mathcal{E} = NAB\omega \sin \omega t = \mathcal{E}_0 \sin \omega t$$

$$\mathcal{E}_0 = NAB\omega$$



How to figure out the self inductance of a coil or solenoid?



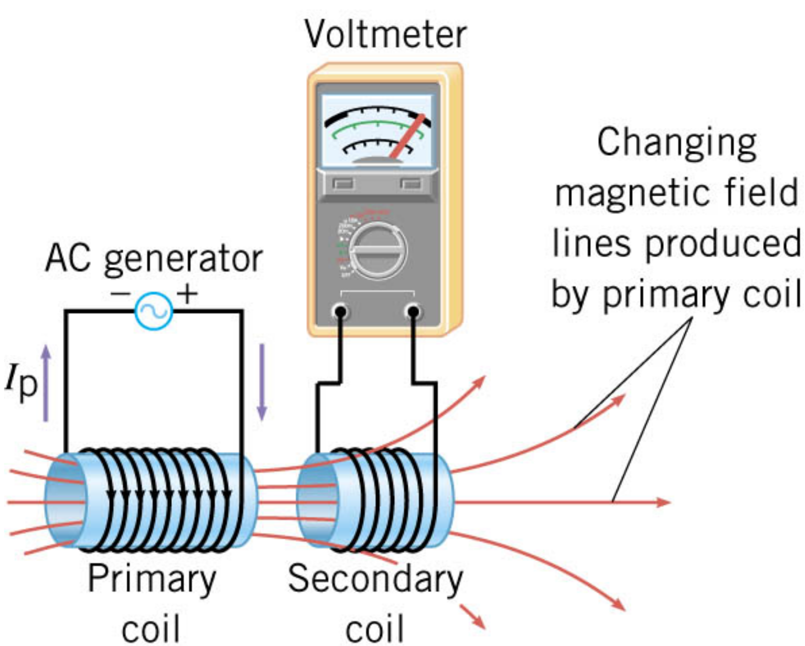
$$L = \frac{N\Phi}{I} = \mu_0 n^2 A l$$

$$Energy = \frac{1}{2} L I^2$$

$$\mathcal{E} = -L \frac{\Delta I}{\Delta t}$$

$$Energy\ density = \frac{1}{2\mu_0} B^2$$

How to find mutual inductance?



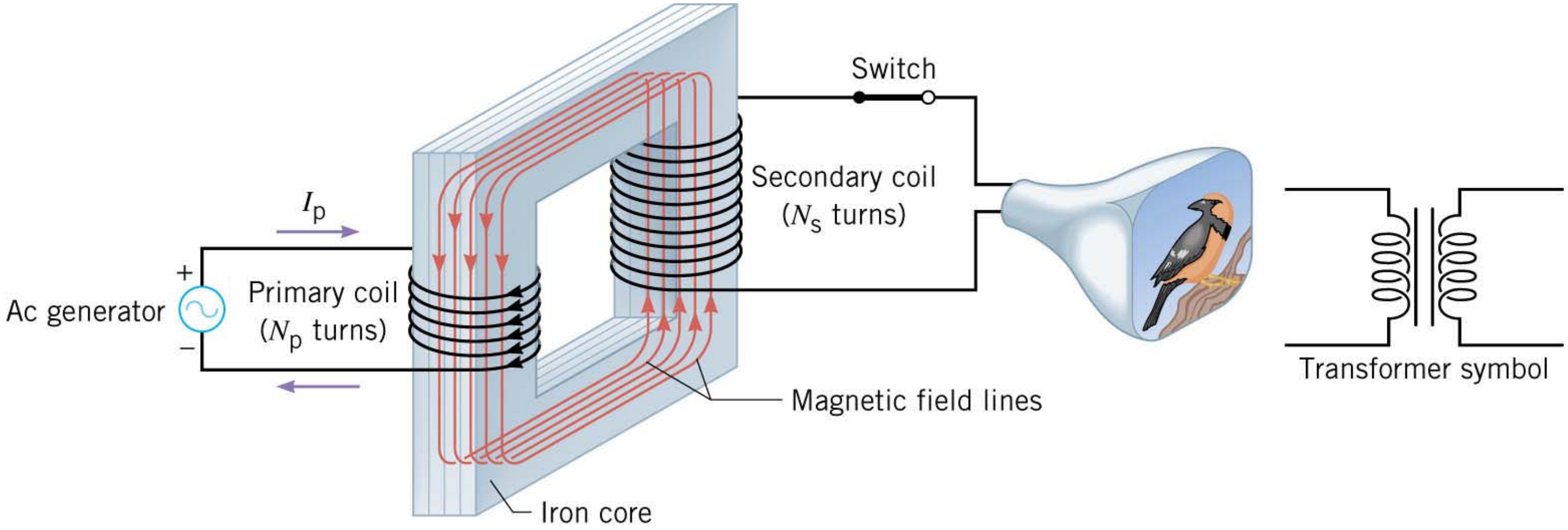
$$M = \frac{N_s \Phi_s}{I_P}$$

$$\varepsilon_s = -M \frac{\Delta I_P}{\Delta t}$$

$$\frac{V_s}{V_P} = \frac{N_s}{N_P}$$

$$I_P V_P = I_S V_S$$

How to voltage or current change in a transformer?



$$\frac{V_s}{V_P} = \frac{N_s}{N_P}$$ **transformer equation**

$$I_P V_P = I_S V_S \quad (P_p = P_s)$$ **energy conservation**