Announcements:

• HW5 is posted and due on Wed 11:59 PM, Oct. 6th.

• Revised lecture note for Chap. 21 is posted

Class Website:

http://www.phys.lsu.edu/~jzhang/teaching.html

Michael Faraday
22.1 Induced emf (electromotive force, driving voltage)

So electric currents (moving charges) create Magnetic Fields.

Is the reverse true? Can magnetic fields create currents???

Yes!!! But it takes a changing magnetic field to produce a current!!!

After Oersted’s discovery, it took 12 years to move the magnet!

Notice: A current would also be produced if I held the magnet stationary and moved the coil! The field in the coil is still changing!

The current that is produced in the coil is called an induced current.

The coil then acts like a battery, or a source of emf. This emf is called an induced emf.

The current and emf (voltage) are called induced, because they are due to the changing magnetic field..
Another way to induce an emf and produce a current in the coil is to change the area of the coil:

Either enlarging the coil or shrinking it will produce an induced current.

As long as the area of the loop keeps changing, an induced current will flow!

So why does this happen???

The coil is a conductor, and thus contains charges which can move easily.
Since I’m moving the conductor which contains charges, I’m moving the charges in a magnetic field. Thus they feel a force!

The phenomenon of producing induced emf’s with a changing magnetic field is called **Electromagnetic Induction**.

### 22.2 Motional emf

What happens if we move a conducting rod at right angles to a uniform magnetic field?

When the rod moves, the electrons in the rod feel a force, since they are charges moving in a magnetic field: \( F = qvB\sin \theta \)

By RHR-1, the electrons move to the bottom of the rod, and positive charges move to the top.

Thus, the rod acts as a source of emf, like a battery. The induced emf is called a **motional emf**.

(motion of charges through a mag. Field)
An electric field is setup within the rod, and the separation of charges along the rod continues until the attractive electrical force between the charges equals the magnetic force.

\[ F_E = F_M \]

Once \( F_E = F_M \) no further charge separation takes place.

The motional emf is maintained as long as the bar keeps moving.

If \( v = 0 \), then \( F_M = qvB \sin \theta = 0 \) and \( F_E \) will reunite the + and – charges, thus eliminating the emf.

What is the magnitude of the emf?

\[ \mathcal{E} = vBL \]

\( \mathcal{E} \) is the induced emf (voltage)

\( v \) is the speed of the bar

\( B \) is the magnetic field

\( L \) is the length of the bar in the field

This is true when \( v \), \( B \), and \( L \) are mutually perpendicular.
A conducting rod is pulled through the magnetic field shown. Which side of the rod becomes negatively charged?

1. Top
2. Bottom
3. Right
4. **Left**
5. None
So a motional emf occurs when a conductor is moved through a magnetic field.

Let’s form a complete circuit with the conducting rod, so that a current can flow.

When we slide the bar to the right at speed $v$, an induced emf is set up across the bar, and an induced current will flow.

What is the direction of the induced current?

The bar acts like a battery, so the current flows counter-clockwise.

But now the current in the moving bar feels a force from the magnetic field!

By **RHR-1** this force is directed to the left!

The force **opposes** the direction of motion of the rod.

The rod will stop, unless a larger force keeps pulling it to the right.

As the rod slides, the light bulb uses energy. Where does the energy come from???

**It comes from the force pushing the rod to the right!**
We must have conservation of energy.

So the work done on the rod in sliding it to the right is equal to the energy consumed by the light bulb!

And………More importantly:

When a motional emf leads to an induced current, a magnetic force will act to oppose the motion in accord with the Principle of Conservation of Energy.

This is the foundation for Lenz’s Law which we will get to in 22.5
Clicker question 22 -2

The conducting rod is given one shove to the right and then released. What happens?

1. The rod continues to move at constant speed.

2. The rod slows down and stops.

3. The rod slows down, stops, and then returns to its original position.

Light bulb

Conducting rails

(frictionless)

B

0% 0% 0%
The conducting rod is given one shove to the right and then released. What happens?

1. The rod continues to move at constant speed.
2. The rod slows down and stops.
3. The rod slows down, stops, and then returns to its original position.
Eddy currents are electric currents that can arise in a piece of metal when it moves thru a region where the magnetic field is not the same everywhere. The picture shows a metal sheet moving to the right at a velocity $v$ and a magnetic field $B$ exists that is directed into the page. At the instant shown, the field only extends over the left half of the sheet. An emf is induced that leads to the eddy current shown. What happens to the sheet?

1. Nothing. It continues at constant speed.
2. **It slows down**
3. It speeds up
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Example (How to use $\varepsilon = vBL$?)
A conducting rod slides down between two frictionless vertical copper tracks at a constant speed of 4 m/s perpendicular to a 0.50-T magnetic field. The length of the rod is 1.3 m. A 0.75 $\Omega$ resistor is connected across the top of the track.
(a) What is the mass of the bar?
(b) What is the change in the gravitational potential energy that occurs in a time of 0.2 s?
(c) How much electrical energy is dissipated in the resistor in 0.2 s?

Speed is constant $\Rightarrow F_{net} = 0 \Rightarrow F_{grav} = F_{mag}$

$mg = ILB \sin \theta = ILB$ but $I = \frac{\varepsilon_{induced}}{R} = \frac{vBL}{R} \Rightarrow$ (a) $m = \frac{vB^2L^2}{gR} = 0.23 kg$

(b) $\Delta U = mg\Delta h = -mg \cdot v\Delta t = -1.8 J$

(c) Power: $P = \frac{V^2}{R} = \frac{\varepsilon_{induced}^2}{R} = \frac{(vBL)^2}{R} = 9 W$

$E_{electric} = P\Delta t = 1.8 J$

Nothing but energy conservation!
Magnetic field induced emf

Magnetic Induction occurs as long as the total magnetic field flux through a coil changes!!!!!!

Case one: flux changes due to the variation of the coil size
Case two: flux changes due to the variation of the B-field
22.3 Magnetic Flux

In ch. 18 we defined the electric flux through a surface element \( A \) as:

\[
\Phi_E = E \cdot A = E A \cos \phi
\]

We can do a similar thing for magnetic fields:

\[
\Phi_M = B \cdot A = B A \cos \phi
\]

This is the magnetic flux through surface \( A \).

Units?

(Magnetic Field) \times (Area) = \left[ T \cdot m^2 \right] = \left[ \text{Weber} \right] = \left[ \text{Wb} \right]

So, the magnetic flux will be maximum when the field is perpendicular to the surface!
\[ \Phi_M = BA \cos \phi \]

Notice that \( \Phi_M \propto B \)

Thus, if \( B \) increases then the magnetic flux \( \Phi_M \) also increases.

Remember that the magnitude of \( B \) is represented by the density of field lines. The more field lines I have per unit area, the stronger the field. Thus, the more field lines I have per unit area, the bigger the flux.

\[ \Phi_M \propto B \propto (\text{the \# of field lines thru a surface}) \]

More field lines thru the same area \( \rightarrow \) greater flux!
Now let’s reexamine our sliding conducting bar in terms of Magnetic Flux:

Let the bar start at $t = 0$.

Now let the bar move to its position when $t = t_o$.

The bar moves a distance $x_o$ and sweeps out an area $A_o$.

Now let the bar slide for a longer time to $t_f$.

It moves to position $x_f$ and sweeps out an area $A_f$.

We know the emf across the bar is:

$$\mathcal{E} = vBL = \frac{\Delta x}{\Delta t} BL = \frac{\Delta(xL)}{\Delta t} B = \frac{\Delta(A)}{\Delta t} B = \frac{\Delta(B \cdot A)}{\Delta t} = \frac{\Delta(\Phi_M)}{\Delta t}$$

Thus, the induced emf is equal to the change in the magnetic flux divided by the change in time!
This is usually written as:

$$\mathcal{E} = -\frac{\Delta \Phi_M}{\Delta t}$$

The minus sign is here, because the induced current flows in a direction such that the magnetic field it creates opposes the change in the magnetic flux.

22.4 Faraday’s Law of Electromagnetic Induction

Often times the magnetic flux will pass thru a coil with multiple turns:

$$\mathcal{E} = -N \frac{\Delta \Phi_M}{\Delta t}$$

This is Faraday’s Law. 

$N$ is the # of turns in the coil.

Units? Volts [V]

So an emf is induced whenever $\Phi_M$ changes in time for any reason!

Since $\Phi_M = BA \cos \phi$,

So, we could write Faraday’s Law as:

$$\mathcal{E} = -\frac{\Delta (BA \cos \phi)}{\Delta t}$$

That means that if $B$, $A$, or $\phi$ changes in time, then an emf is induced!
Example

A rectangular loop of wire with sides of 0.20 and 0.35 m lies in a plane perpendicular to a constant magnetic field of 0.65 T. In a time of 0.18 s, one-half of the loop is folded back on the other half as shown. What is the average EMF induced in the loop?

\[ \mathcal{E} = N \frac{\Delta \Phi}{\Delta t} = \frac{NB\Delta A}{\Delta t} \]

Initially \( A = (0.2 \text{ m})(0.35 \text{ m}) = 0.07 \text{ m}^2 \)

In the final state the area is very small, i.e. \( A \approx 0 \)

\[ \mathcal{E} = -\frac{1 \cdot (0.65T)(0 - 0.07m^2)}{0.18s} = 0.25V \]
The drawing shows a copper wire bent into a circular shape with a radius of 0.5 m. The radial section BC is fixed in place, while the copper bar AC sweeps around at an angular speed of 15 rad/s. The bar makes electrical contact with the wire at all times. The wire and the bar have negligible resistance. A uniform magnetic field exists everywhere, is perpendicular to the plane of the circle and has a magnitude of 3.8 x 10⁻³ T. What is the magnitude of the current induced in the loop ABC?

\[ \mathcal{E} = N \frac{\Delta \Phi}{\Delta t} = \frac{NB \Delta A}{\Delta t} \quad \text{but} \quad \Delta A = \frac{\pi r^2}{2\pi} \cdot \Delta \theta \]

\[ \frac{\Delta A}{\Delta t} = \frac{\pi r^2}{2\pi} \cdot \frac{\Delta \theta}{\Delta t} = \frac{\pi r^2}{2\pi \cdot \text{rad}} \cdot \frac{15 \text{rad}}{1.0 \text{s}} = \frac{15 r^2}{2 \text{s}} \]

\[ \frac{\Delta A}{\Delta t} = \frac{15 (0.5 \text{m})^2}{2 \text{s}} = 1.88 \frac{m^2}{s} \]

\[ \mathcal{E} = 1 \cdot (3.8 \times 10^{-3} \text{T}) \frac{\Delta A}{\Delta t} = (3.8 \times 10^{-3} \text{T}) \left(1.88 \frac{m^2}{s}\right) \]

\[ \mathcal{E} = 7.1 \text{mV} \]

\[ I = \frac{V}{R} = \frac{\mathcal{E}}{R} = \frac{7.1 \times 10^{-3} \text{V}}{3 \Omega} = 2.4 \text{mA} \]
Announcements:

• 2010 Nobel Prize in Physics “for groundbreaking experiments regarding the two-dimensional material graphene”

Andre Geim  Konstantin Novoselov
22.5 Lenz’s Law

Remember Faraday’s Law: \( \mathcal{E} = -N \frac{\Delta \Phi_M}{\Delta t} \), where \( \Phi_M = BA \cos \phi \).

So a changing magnetic field can produce a current. What is the direction of this current?

To get the direction, we must consider TWO magnetic fields:

- The first is the original one that produced the induced current.
- The second is the field created by the induced current itself.

To determine the direction of the induced current flow, we use Lenz’s Law:

The induced emf resulting from a change in magnetic flux leads to an induced current which produces a magnetic field to oppose the change in flux.
Example: Consider a single loop of wire sitting in the plane of the paper:

Now turn on a magnetic field everywhere into the page:

Then, in all of these Lenz's Law problems, you have to ask yourself two questions:

First, what is the direction of the magnetic flux? In other words, what direction does the original magnetic field point. Here, it points into the page.

Second: Is the flux increasing or decreasing? Here, it is increasing. So, we have an increasing magnetic flux directed into the page.

Lenz's Law tells us the induced current will flow in the direction that will create a new magnetic field to oppose this change. Thus, if we have an increasing magnetic flux into the page, then we should create a field pointing out of the page to oppose this change.

By using RHR-3, we see that to create a magnetic field out of the page, my induced current in the loop must flow ccw! This results in the proper direction for the induced field!
Now let’s take the same loop again and place it in a uniform magnetic field that exists everywhere out of the page:

Now decrease the field.

Will there be an induced current?

Yes, since we have a change in magnetic flux!

1. What is the direction of the flux? Out of the page

2. Is it increasing or decreasing? Decreasing

Thus, we have a decreasing magnetic flux that points out of the page.

To oppose this change, Lenz’s Law tells us an induced current will flow to create a induced field that tries to maintain the flux out of the page.

Thus, by RHR-3, we need a ccw current!

It always opposes the change!
Another way to remember this is the following:

If the original field is increasing, then the induced field must point in the opposite direction.

If the original field is decreasing, then the induced field must point in the same direction.

Now let’s slide a conducting ring through a region of space where the magnetic field points into the screen:

1. Before it enters the field.
2. As it enters the field.
3. While it’s completely in the field.
4. As it leaves the field.
5. After it has left the field.
Is there an induced current in the ring at position 1? No! No change in flux!

Is there an induced current in the ring at position 2? Yes! Change in flux! The flux is increasing into the page.
Lenz tells us we need an induced field that points out of the page.
By RHR-3, this is a ccw current.

Is there an induced current in the ring at position 3? No! No change in flux!

Is there an induced current in the ring at position 4? Yes! Change in flux! The flux is decreasing into the page.
Lenz tells us we need an induced field that points into the page.
By RHR-3, this is a cw current.

Is there an induced current in the ring at position 5? No! No change in flux!
A long, straight wire lies on a table and carries a current $I$. A small circular loop of wire is pushed across the top of the table from position 1 to position 2. What is the direction of the induced current flow in the loop when it is pushed past position 1?

1. CW
2. CCW
3. No induced current.
**Clicker question 22-5**

What is the direction of the induced current flow when the loop is pushed passed position 2?

1. CW
2. CCW
3. No induced current.
22.7 Electric Generator

Faraday’s Law: A changing magnetic flux through a loop of wire generates a potential difference (EMF) and current

\[ E = -N \frac{\Delta \Phi}{\Delta t} \]

Imagine a rotating loop in a constant magnetic field

Is the flux changing? \[ \Phi = AB \cos \phi \]

\[ \Phi = 0 \]
The AC Generator

Is the flux changing?
\[ \Phi = AB \cos \phi \]

Yes!

\[ \phi \quad \text{Angle between B and normal to loop} \]

\[ \omega = \frac{\Delta \phi}{\Delta t} = \frac{2\pi}{T} = 2\pi f \]

\[ f = \frac{1}{T} \]

\[ \mathcal{E} = -N \frac{\Delta \Phi}{\Delta t} \rightarrow \mathcal{E} = NAB \omega \sin \omega t \]

\[ \mathcal{E} = \mathcal{E}_0 \sin \omega t \]

Rotate with an angular speed of \( \omega \)
Almost all electricity is produced by water or steam turbines

US Electric Power Sources
US Dept. of Energy 1996

Hoover Dam (US Bureau of Reclamation)
Algebra-based Physics II

Oct 8th: Chap 22.8-9
   Mutual and self inductance
   Energy in magnetic field
   Power transformer
Chap 24.1-2
   Electromagnetic waves

• HW5 is due today
• HW6 is already posted

inductors

Power transformers
Example: Find the direction of net field

\[ B_1 = B_2 = B_3 = \frac{\mu_0 I}{2\pi r} \]

\[ B = 2B_1 = \frac{\mu_0 I}{\pi r} \]
22.8 Mutual and Self Induction

Let's place two coils side by side. Let's connect one to an AC generator (primary coil) and the other to a voltmeter (secondary coil):

The primary coil creates a magnetic field, and some of those field lines pass thru the secondary coil.

This produces a change in magnetic flux in the secondary coil, leading to an induced emf!

This is called **Mutual Inductance**.

From Faraday's Law: \[ \epsilon_s \propto \Delta \Phi_{M_s} \],

where \( \epsilon_s \) is the induced emf in the secondary coil, and \( \Delta \Phi_{M_s} \) is the change in mag. flux thru the secondary coil.
The net flux thru the secondary coil is: \( N_s \Phi_s \propto I_P \) since \( \Phi_s \propto B_P \propto I_P \)

Where \( N_s \) is the number of turns in the secondary coil.

The flux thru the secondary coil is proportional to the current in the primary.

Make this an equality:

\[
N_s \Phi_s = MI_P \implies M = \frac{N_s \Phi_s}{I_P}
\]

\( M \) is a quantity called the **Mutual Inductance**.

We can substitute this into Faraday’s Law:

\[
\mathcal{E}_s = -N \frac{\Delta \Phi_s}{\Delta t} = -\frac{\Delta (N_s \Phi_s)}{\Delta t} = -\frac{\Delta (MI_P)}{\Delta t} = -M \frac{\Delta I_P}{\Delta t}
\]

\[
\Rightarrow \mathcal{E}_s = -M \frac{\Delta I_P}{\Delta t}
\]

Now it’s easy to see that the induced emf in the secondary coil depends on the changing current in the primary coil.

**Units of \( M \)?**

\[
\left[ \frac{\text{V} \cdot \text{s}}{\text{A}} \right] \rightarrow [\text{Henry}] \rightarrow [\text{H}]
\]
**Example:**

Two coils of wire are placed close together. Initially a current of 2.5A exists in one of them but there is no current in the other. The current is then switched off in a time of $3.7 \times 10^{-2}$ s. During this time, the average emf induced in the other coil is 1.7 V. What is the mutual inductance of the two coils?

\[ \text{emf induced in the 2nd coil:} \]

\[ \varepsilon_s = -M \frac{\Delta I_P}{\Delta t} \]

\[ M = -\frac{\varepsilon_s}{\frac{\Delta I_P}{\Delta t}} = -\frac{1.7V}{0 - 2.5A} = \frac{2.516 \times 10^{-2} H}{3.7 \times 10^{-2} s} \]
Self Inductance

Consider just one coil connected to an AC generator:

The AC current produces a changing magnetic field which produces a change in mag. flux within the coil.

This leads to an induced emf in the coil!

This process is called **Self Induction.**

Let $\Phi$ be the flux thru one loop of the coil, so $N\Phi$ is the net flux.

$$\Phi \propto B \propto I \quad \text{So,} \quad N\Phi \propto I.$$ Make this an equality: $$N\Phi = LI$$
\[ L = \frac{N\Phi}{I} \]

L is a quantity called the **Self Inductance**.

Using Faraday’s Law again, like we did for mutual inductance, we find:

\[ E = -L \frac{\Delta I}{\Delta t} \]

**Energy store in an inductor:**

\[ \text{Energy} = \frac{1}{2} LI^2 \]

\[ \text{Energy} = \frac{1}{2} LI^2 = \frac{1}{2} \mu_0 n^2 AlI^2 \quad \text{(since} L = \mu_0 n^2 Al) \]

\[ = \frac{1}{2\mu_0} B^2 Al \quad (B = \mu_0 nI) = \frac{1}{2\mu_0} B^2 \cdot \text{field volume} \]

**Energy density**

\[ \text{Energy density} = \frac{1}{2\mu_0} B^2 \]
Example: A long solenoid of length $8.0 \times 10^{-2} \text{ m}$ and cross-sectional area $5.0 \times 10^{-5} \text{ m}^2$ contains 6500 turns per meter of length. Determine the emf and induced in the solenoid and energy stored when the current in it changes from 0 to 1.5 A during the time interval from 0 to 0.2 s.

What do we know?

Solenoid: $l = 8 \times 10^{-2} \text{ m}; \ A = 5 \times 10^{-5} \text{ m}^2; \ n = 6500 \ \text{turns/m}$

$\Delta t = 1.5 \text{ A}; \ \Delta I = 1.5 \text{ A}$

$\Phi = BA = (\mu_0 n I) A$

Need to find $L = \frac{nl\Phi}{I}$ in order to obtain $\varepsilon = -L \frac{\Delta I}{\Delta t}$

$\varepsilon = -L \frac{\Delta I}{\Delta t} = -\frac{nl\Phi}{I} \frac{\Delta I}{\Delta t} = -\frac{nl(\mu_0 n I A)}{I} \frac{\Delta I}{\Delta t} = -(\mu_0 n^2 l A) \frac{\Delta I}{\Delta t}$

$= -1.6 \times 10^{-3} \text{ V}$

Energy $= \frac{1}{2} LI^2 = \frac{1}{2} \cdot (\mu_0 n^2 l A) \cdot I^2$
Total energy density in an electromagnetic field

$\text{Total energy density} = (\text{electric} + \text{magnetic}) \text{ field energy density}$

$$\text{Energy density} = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2 \mu_0} B^2$$

### 22.9 Transformers

We can use one coil to induce an emf (voltage) in another coil by mutual induction.
The best part about a transformer is that the induced emf in the secondary coil is proportional to the turns ratio:

\[ \frac{N_s}{N_p} \]

\( N_s = \# \text{ of turns in the secondary coil} \)
\( N_p = \# \text{ of turns in the primary coil} \)

Thus, the more turns I have in the secondary coil, the higher the induced emf!

The iron core ensures the flux through each coil is the same, and we get:

\[ \frac{V_s}{V_p} = \frac{N_s}{N_p} \]

This is the transformer equation.

\( V_s = \text{Voltage in the secondary coil} \)
\( V_p = \text{Voltage in the primary coil} \)
Would a transformer work with DC (direct current) too?

1. Yes.
2. No. They only work with AC.
A transformer can either increase or decrease the primary voltage:

If $N_s > N_P$, then the transformer is called a step-up transformer.

If $N_s < N_P$, then the transformer is called a step-down transformer.

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**Example: Bug Zappers**

These devices plug into a standard house outlet at 120 V, and they have a step-up transformer inside which converts this primary voltage into 5000 V.

If the primary coil has 21 turns, how many turns does the secondary have?

From the transformer equation:

$$N_s = N_P \frac{V_s}{V_P} = (21) \frac{(5000)}{(120)} = 875 \text{ turns}$$

But what happens when I go from 120 V to 5000 V? I can’t get something for nothing!
Energy is just Power x Time, so the power in each coil must be equal:

\[ I_P V_P = I_S V_S \quad \Rightarrow \quad I_S = I_P \frac{V_P}{V_S} \]

Thus, if I have a step-up transformer, then \( V_S > V_P \), and the current in the secondary \((I_S)\) goes down!

This assumes no loss due to heat in the transformer, which for good ones, is about 1%.

We must have **Conservation of Energy**!