LOUISIANA STATE UNIVERSITY

Department of Physics & Astronomy

Algebra-based Physics II

Sep.10th, Chap. 19.5
Chap 20.1-4

Announcements:

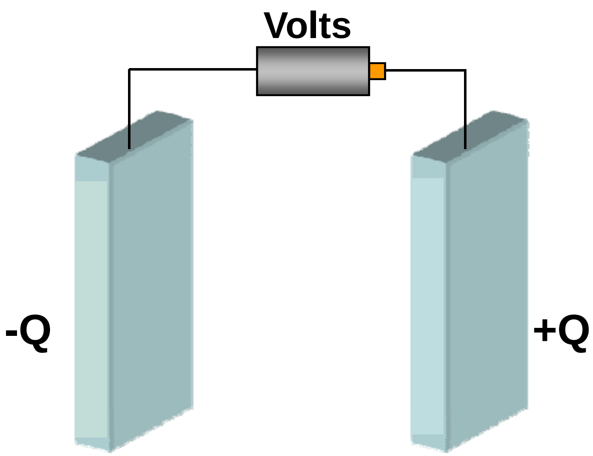
1. HW2 part B is due on Sunday.
2. HW3 is coming too!

Class Website:

http://www.phys.lsu.edu/~jzhang/teaching.html

Capacitance

$$\boxed{Q = CV}$$



C **is a new quantity called the Capacitance, describing the capability of storing charges in a capacitor.**

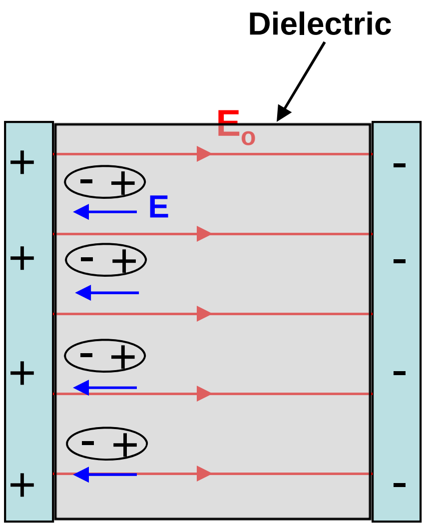
$$C = \frac{Q}{V} \rightarrow \frac{[\text{Charge}]}{[\text{Voltage}]} = \left[\frac{\text{C}}{\text{V}}\right] = [\text{Farad}] = [\text{F}]$$

We will find:

Capacitance of a capacitor depends on the structure of itself only!

Dielectrics

We can fill the space between the plates with some insulating material, say air, oil, paper, rubber, plastic, etc.



This material is called a <u>dielectric</u>.

So what effect does the dielectric have on the field between the plates?

Since the dielectric is an insulator, the charges in it aren't free to move, but they can separate slightly within each atom:

Each one of these atoms now produces a small internal electric field which points in the <u>opposite direction</u> to the field between the plates:

Thus, the net electric field between the plates is <u>reduced</u> by the dielectric.

The reduction of the field is represented by the following:

$$\kappa = \frac{E_o}{E}$$

E_o is the field without the dielectric
E is the field with the dielectric

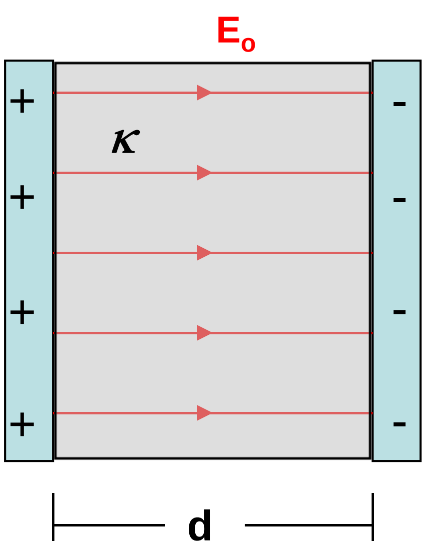
κ is called the <u>dielectric constant</u>, and it must be greater than 1.

$$\kappa = \frac{E_o}{E}$$

Since κ is the ratio of two electric fields, it's unitless.

Material	κ
Vacuum	1
Air	1.00054
Water	80.4

The larger κ is, the more it reduces the field between the plates!



Let's say the plates have surface area A and are separated by a distance d.

$$E = \frac{1}{\kappa} E_o = \frac{V}{d} \Rightarrow E_o = \frac{\kappa V}{d} = \frac{\sigma}{\varepsilon_o} = \frac{q}{\varepsilon_o A}$$

$$\Rightarrow \boxed{q = \left(\frac{\varepsilon_o A \kappa}{d}\right) V}$$

But, $q = CV$, **so**

$$\boxed{C = \frac{\varepsilon_o A \kappa}{d}}$$

Capacitors store charge - what about energy?

$$\boxed{EPE_{Stored} = \tfrac{1}{2}qV = \tfrac{1}{2}CV^2}$$

$$V = Ed \text{ and } C = \frac{\varepsilon_o \kappa A}{d}, \text{ so } \boxed{EPE_{Stored} = \tfrac{1}{2}\left(\frac{\varepsilon_o \kappa A}{d}\right)\left(E^2 d^2\right)}$$

Rearrange this:

$$EPE = \tfrac{1}{2}\kappa\varepsilon_o E^2 \underbrace{(Ad)}_{\text{Volume between the plates}} = \tfrac{1}{2}\kappa\varepsilon_o E^2 (\text{Vol})$$

$$\boxed{\frac{EPE}{\text{Vol}} = \text{Energy Density} = \tfrac{1}{2}\kappa\varepsilon_o E^2}$$

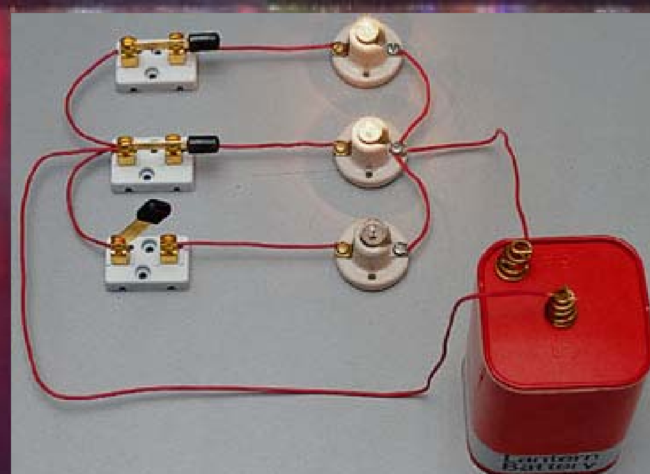
Units?

$$\left[\frac{\text{Energy}}{\text{Volume}}\right] = \left[\frac{\text{J}}{\text{m}^3}\right]$$

***This expression holds true for any electric fields, not just for capacitors!**

Chap. 20 Electric Circuits

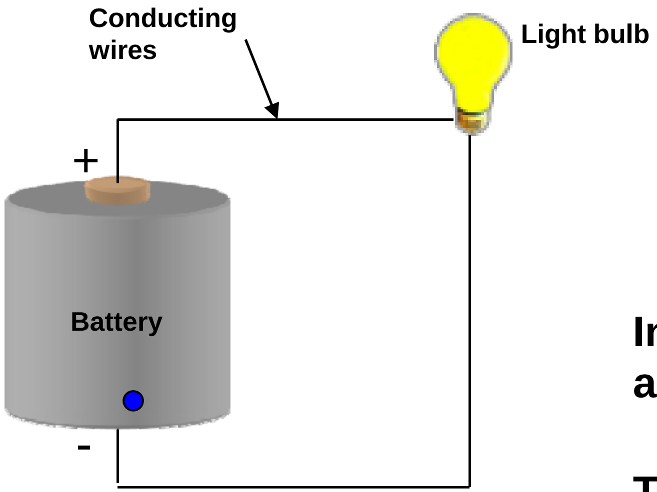
1. Current (I)
2. Ohm's Law
3. Resistance (R)
4. Resistivity (ρ)
5. Power (P)
6. Basic Circuits



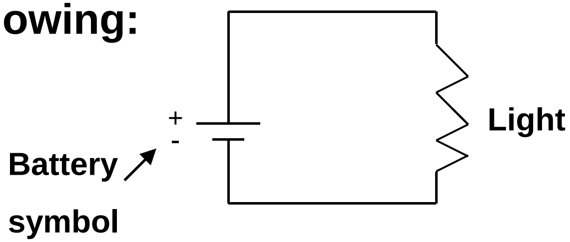
20.1 – Electromotive Force

Every electronic device depends on circuits.

Electrical energy is transferred from a power source, such as a battery, to a device, say a light bulb.



A diagram of this circuit would look like the following:



Inside a battery, a chemical reaction separates positive and negative charges, creating a potential difference.

This potential difference is equivalent to the battery's voltage, or emf (ε) (electromotive force).

This is not really a "force" but a potential.

Because of the emf of the battery, an electric field is produced within and parallel to the wires.

This creates a force on the charges in the wire and moves them around the circuit.

This flow of charge in a conductor is called electrical current (I).

A measure of the current is how much charge passes a certain point in a given time:

Electrical Current

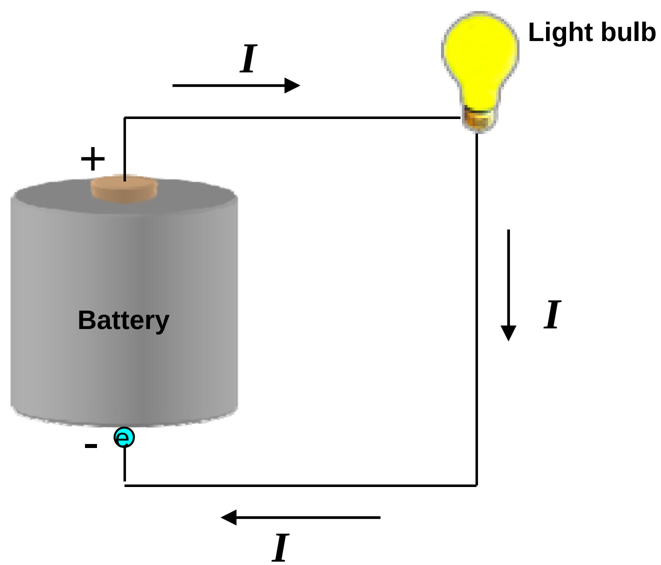
$$\boxed{I = \frac{\Delta q}{\Delta t}}$$

Units?

$$\left[\frac{\text{Charge}}{\text{time}}\right] = \left[\frac{\text{C}}{\text{s}}\right] = [\text{Ampere}] = [\text{A}]$$

If the current only moves in one direction, like with batteries, it's called Direct Current (DC).

If the current moves in both directions, like in your house, it's called Alternating Current (AC).



Electric current is due to the flow of moving electrons, but we will use the positive conventional current in the circuit diagrams.

So I shows the direction of "positive" charge flow from high potential to low potential.

20.2 – Ohm's Law

The flow of electric current is very analogous to the flow of water through a pipe:

The battery pushing the current acts like the water pump pushing the water.

The voltage of the battery is analogous to the pump pressure – the higher the pump pressure, the faster I can push the water through. Thus, the larger my battery voltage, the greater my current.

$$V \propto I$$

Let's make this an equality:

$$\boxed{V = IR}$$

This is Ohm's Law.

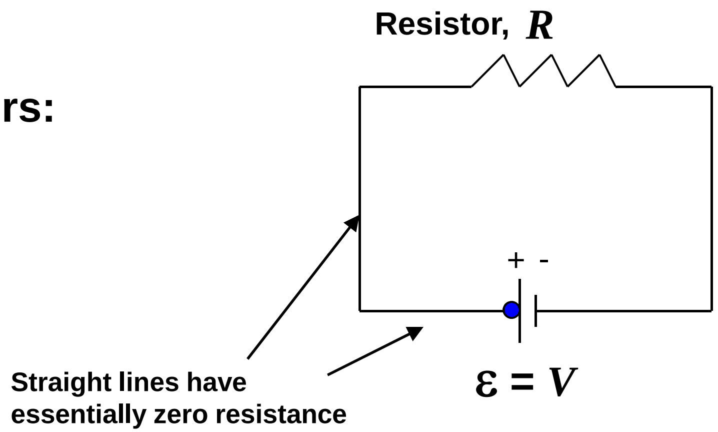
The proportionality constant, R, is called the electrical resistance.

Units?

$$R = \frac{V}{I}\left[\frac{\text{Volts}}{\text{Amp}}\right] = \left[\frac{\text{V}}{\text{A}}\right] = [\text{Ohm}] = [\Omega]$$

Define Resistor: A component of an electrical circuit that offers resistance to the flow of electric current.

Symbol for resistors:



<u>20.3 - Resistivity</u>

The electrical resistance of a conductor depends on its shape:

-Longer wires have more resistance

-Fatter wires have less resistance

Length

Thus, $$R \propto \frac{L}{A}.$$ **Let's make this an equality:** $$\boxed{R = \rho \frac{L}{A}}$$

Cross-sectional area

The proportionality constant, ρ, is the electrical resistivity.

<u>Units?</u>

$$\rho = R\frac{A}{L}\left[\frac{\Omega \cdot \mathrm{m}^2}{\mathrm{m}}\right] \rightarrow [\Omega \cdot \mathrm{m}]$$

Resistivity is an intrinsic property of materials, like density:

Every piece of copper has the same resistivity, but the resistance of any one piece depends on its size and shape.

$\rho, \boldsymbol{R}$

$\rho, \boldsymbol{R}$

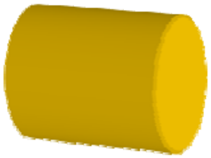


TABLE 20.1 Resistivities[a] of Various Materials

Material	Resistivity ρ ($\Omega \cdot$ m)	Material	Resistivity ρ ($\Omega \cdot$ m)
Conductors		*Semiconductors*	
Aluminum	2.82×10^{-8}	Carbon	3.5×10^{-5}
Copper	1.72×10^{-8}	Germanium	0.5^{b}
Gold	2.44×10^{-8}	Silicon	$20–2300^{b}$
Iron	9.7×10^{-8}	*Insulators*	
Mercury	95.8×10^{-8}	Mica	$10^{11}–10^{15}$
Nichrome (alloy)	100×10^{-8}	Rubber (hard)	$10^{13}–10^{16}$
Silver	1.59×10^{-8}	Teflon	10^{16}
Tungsten	5.6×10^{-8}	Wood (maple)	3×10^{10}

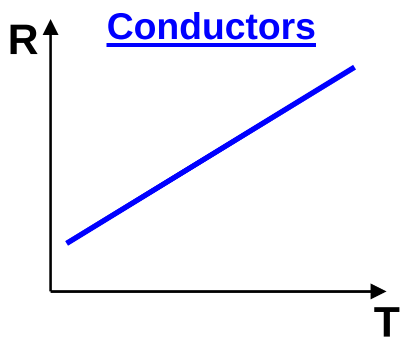
[a] The values pertain to temperatures near 20 °C.

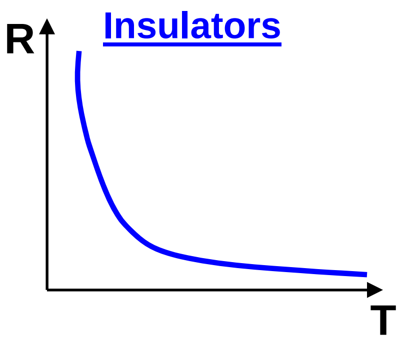
[b] Depending on purity.

Temperature Dependence of Resistivity

The resistance of most materials changes with temperature.

For good conductors (metals) the resistance decreases with decreasing temperature.





For insulators (poor conductors) the resistance increases with decreasing temperature.

For many materials, we find that:

$$\boxed{R = R_o\left[1 + \alpha(T - T_o)\right]}$$

R **= Resistance at temperature** T

R_o **= Resistance at temperature** T_o

α **is the temperature coefficient of resistivity** $\longrightarrow$

$\alpha > 0$ For metals

$\alpha < 0$ For insulators

20.4 Electrical Power

Our standard definition of power is: $\text{Power} = \dfrac{\text{Work}}{\text{Time}}.$

So what would electrical power be?

From the definition of potential: $V = \dfrac{W}{q} \Rightarrow \boxed{W = qV}$

Thus, $P = \dfrac{qV}{t} \Rightarrow \boxed{P = IV}$

We can write this different ways using Ohm's Law, *V=IR*:

$$\boxed{P = I^2 R} \qquad \boxed{P = \frac{V^2}{R}}$$

***So we have 3 ways of calculating electrical power depending on what other quantities are known.**

Electrical Energy

Work and Energy have the same units (Joules).

Thus, $\boxed{\text{Energy} = \text{Power} \times \text{Time}}$

Electrical companies, like Entergy, measure your monthly energy use this way, in units of *kilowatt·hours* (kWh).

For example, if you used an average power of 1500 W for 31 days (744 hours), your energy consumption would be:

$$E = (1.5\,\text{kW})(744\,\text{h}) = 1116\,\text{kWh}$$

At a cost of roughly \$0.13/kWh, this would be a monthly bill of \$145.

$$\boxed{1\,\text{kWh} = 3.60 \times 10^{6}\ \text{J}}$$

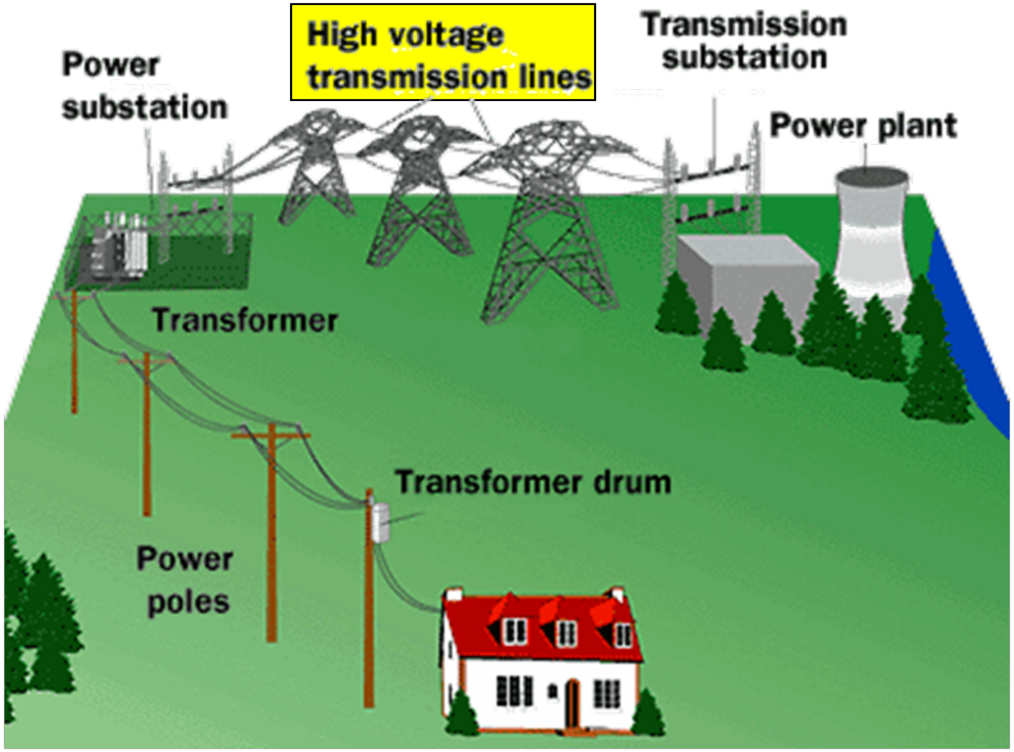
Power Transmission

Why is the electrical power transmitted at high voltages, instead of high currents???

Thermal losses (losing electrical power by converting it to heat) is proportional to I^2 for a given voltage and proportional to V.

$$\boxed{P \propto I^2}$$

$$\boxed{P \propto V}$$

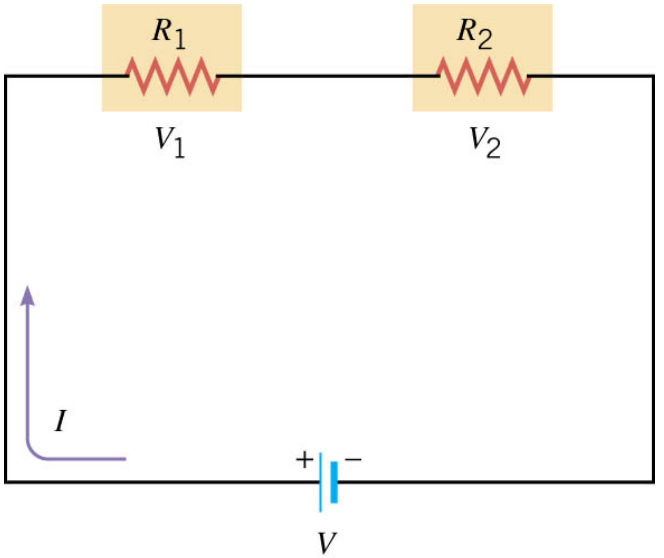


20.6 Series Circuits

Now let's add more than one component to the circuit!

There are several ways to hook these components together.

The first way is to wire them together in series:



The same current runs through two components connected in series.

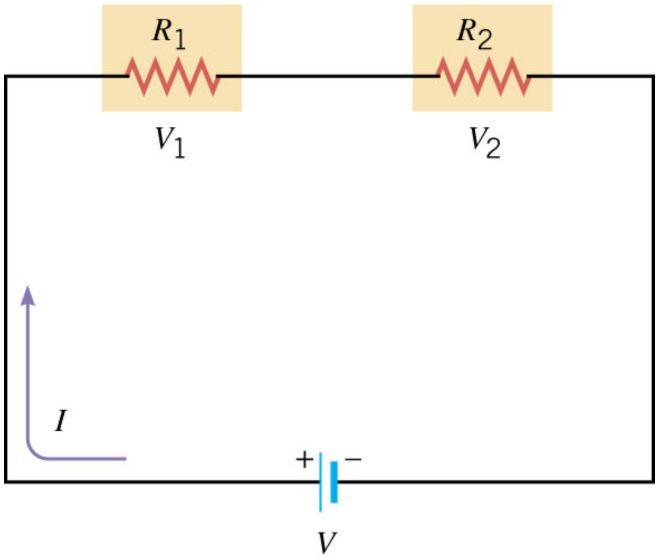
V_1 **and** V_2 **are called voltage drops.**

***We speak of currents running through resistors, and voltages drops across resistors.**

Thus, the current through resistor R_1 **is** I**, and the voltage drop across** R_1 **is** V_1**.**

How would we find the net resistance (equivalent resistance, R_{eq}) for resistors connected in series?

For resistors connected in series, the sum of the voltage drops across all the resistors must equal the battery voltage.



Thus, $$V = V_1 + V_2$$

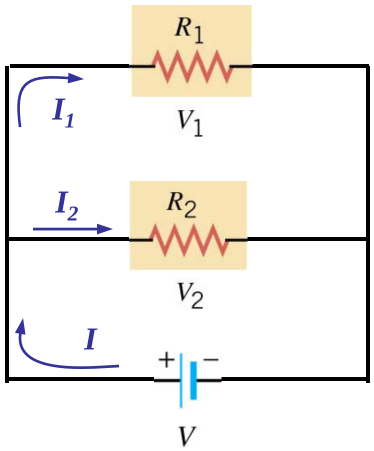
But from Ohm's Law: $$\cancel{I}R_{eq} = \cancel{I}R_1 + \cancel{I}R_2 \Rightarrow R_{eq} = R_1 + R_2$$

Thus, for resistors wired up in <u>series</u>, the equivalent resistance is:

$$\boxed{R_{eq} = R_1 + R_2 + R_3 + \cdots}$$

i.e. you just add them!!!

20.7 Parallel Circuits



For resistors connected in parallel, the voltage drop across each resistor is the same.

The current through each might be different. It splits: $I = I_1 + I_2$.

Thus, $$V_1 = V_2 = V.$$

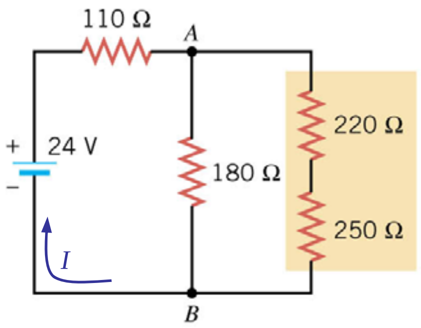
From Ohm's Law: $$R_{eq} = V/I = \frac{V}{I_1 + I_2} = \frac{\cancel{V}}{\frac{\cancel{V_1}}{R_1} + \frac{\cancel{V_2}}{R_2}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Thus, $$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots}$$ **for resistors in parallel.**

20.8 Series and Parallel Circuits

Now let's hook resistors up both in series and in parallel in the same circuit!

What is the current I in the following circuit?



We need to find the equivalent resistance!

Thus, $R_{eq} = 240\,\Omega$

$$I = \frac{V}{R_{eq}} = \frac{24}{240} = \boxed{0.1\,\mathrm{A}}$$

20.9 Internal Resistance

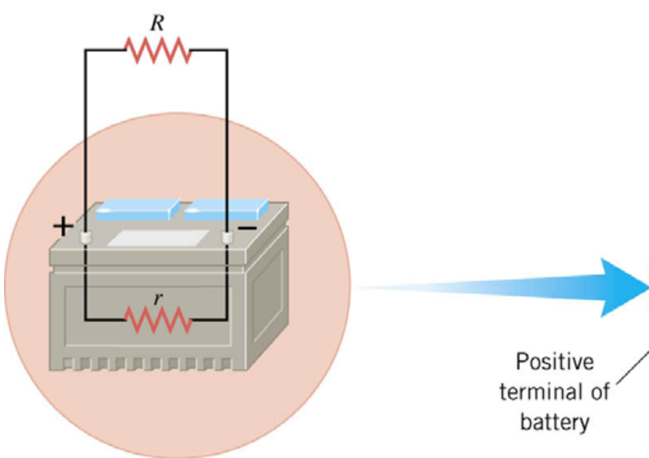
So far we've just considered batteries and generators as contributing their emf to a circuit.

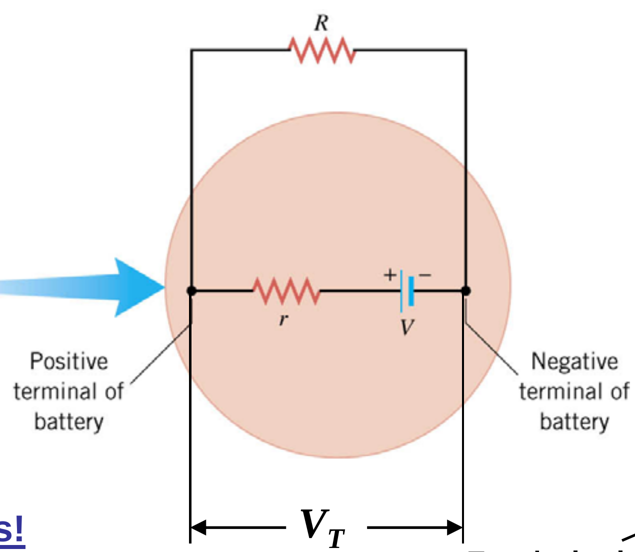
In reality, they too have some resistance.

This is called internal resistance, *r*.

In batteries it's due to the chemicals, and in generators it's due to wire resistance.

So, if a battery is connected to a load resistor, *R*, then the internal resistance, *r*, is in series with the load:





Notice: *R* and *r* are in series!

Thus, the voltage across the battery (known as the terminal voltage, V_T) is less than the full voltage V, since some is lost across *r*.

Thus, $$\boxed{V_T = V - V_r}$$

Terminal voltage **Battery voltage** **Voltage drop across the internal resistance, *r***

Kirchhoff's Rules

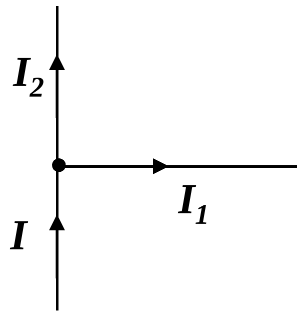
In many circuits, applying the series or parallel methods is not sufficient to analyze them.

There are two other rules we can use called Kirchhoff's Rules:

1. Junction Rule – Current into a junction has to equal current out.

It is based on conservation of charge.

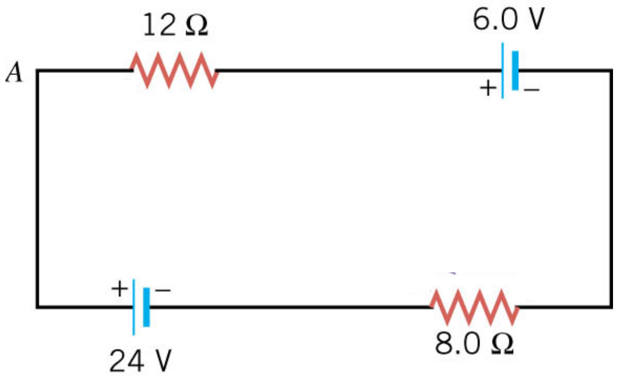
I flows into junction, and I_1 and I_2 flow out, thus:



$$I = I_1 + I_2$$

2. Loop Rule – Around any closed circuit loop, the sum of the potential (voltage) drops has to equal the sum of the potential rises.

It is based on conservation of energy.



Here's an example with the loop rule. We have a closed circuit loop with multiple batteries. What is the current in the circuit?

1. Choose the direction of the current(s) in each loop.
2. Label the resistors from + to – in the direction of the current flow.

Solution

Start at point A and go around the loop clockwise, and make a list of the potential drops and rises as we go all the way around.

Drops	Rises
$12I$	24
6	
$8I$	

Now apply the loop rule: $\sum \text{Drops} = \sum \text{Rises}$

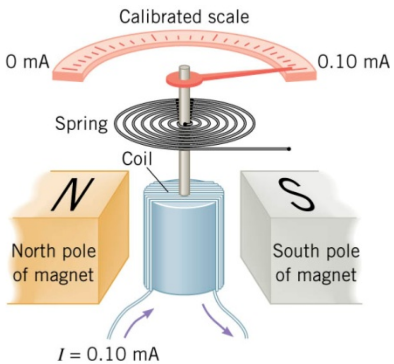
$$\Rightarrow 12I + 6 + 8I = 24 \;\Rightarrow 20I = 18$$

$$\Rightarrow I = \boxed{0.9\text{ A}}$$

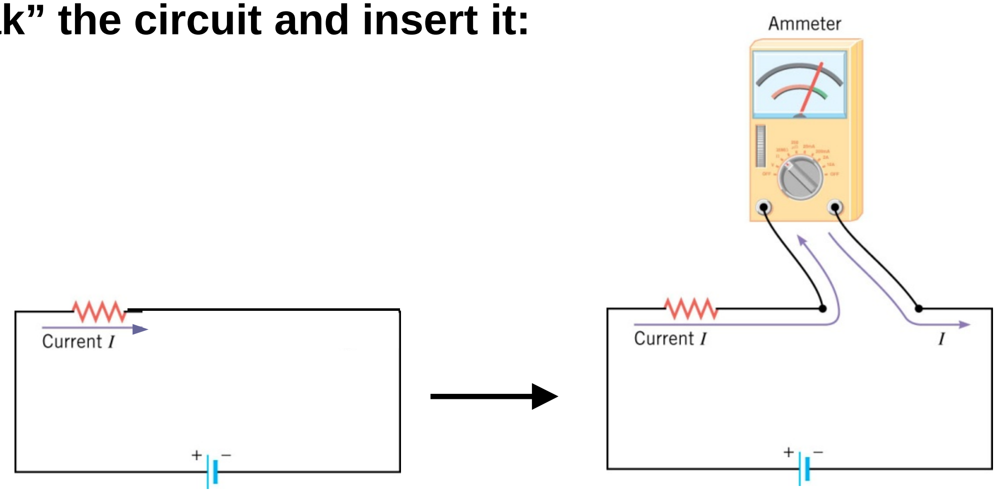
20.11 Measuring Voltages and Currents

Currents and voltages can be measured with devices called ammeters and voltmeters.

Both of these devices rely on the DC Galvanometer:

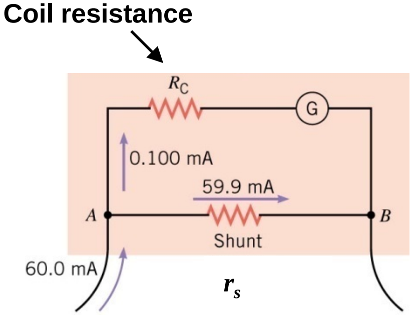


To measure the current with an ammeter, we must "break" the circuit and insert it:



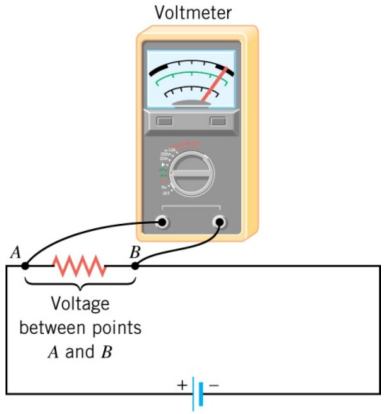
Inside the ammeter, a small resistor r_s is wired in parallel with the galvanometer:

Coil resistance



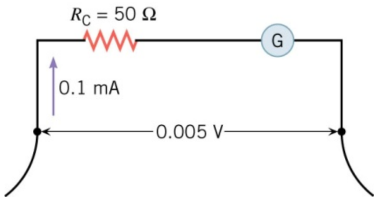
Almost all of the current passes through the shunt resistor, so the ammeter has very little effect on the circuit, i.e. nearly zero resistance.

Voltmeter – Measures the potential difference between two points in the circuit, i.e. across a resistor.



It does not have to be inserted in the circuit, i.e. the circuit does not have to be broken to use it.

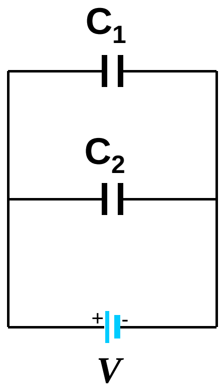
The voltmeter has a high resistance, so very little current actually flows through the voltmeter.



Thus, like the ammeter, it barely affects the circuit at all.

20.12 Capacitors in Series and Parallel

Let's first look at two capacitors connected in parallel:

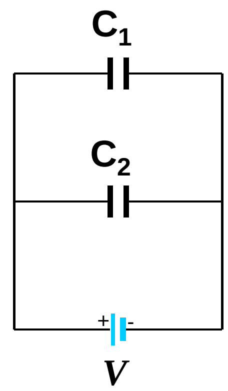


What is the equivalent capacitance?

Since they are in parallel, they each have the same voltage drop across them: $V_1 = V_2 = V$.

But the charge on each will, in general, be different:

$$Q_1 = C_1 V \text{ and } Q_2 = C_2 V$$



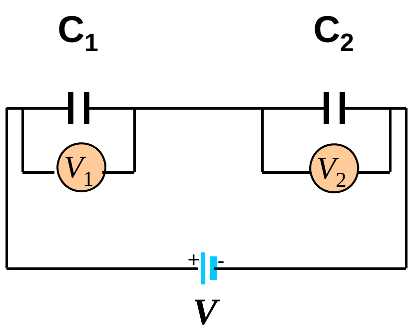
$$Q_{Tot} = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2)V = C_{eq}V$$

Thus,

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \cdots}$$

To find the equivalent capacitance for capacitors wired in <u>parallel</u>, you just add them!

Now consider two capacitors wired in <u>series:</u>



The voltage across each will now, in general, be different, but the <u>charge on each of the plates must be the same</u>.

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\Rightarrow Q = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} V = C_{eq}V$$

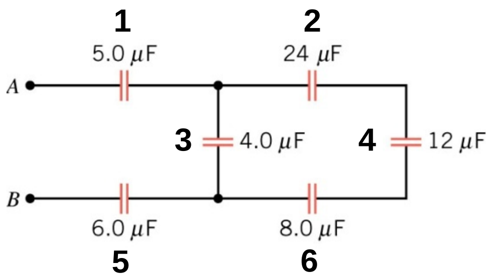
Thus,

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots}$$

For capacitors wired in <u>series</u>, the reciprocal of the equivalent capacitance is just the sum of the reciprocal capacitances.

***Notice that these rules are just the opposite for resistors in combination.**

Clicker Question 20-7 (cont.)



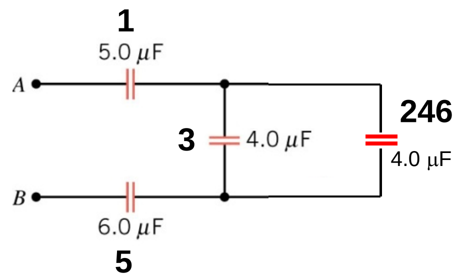
Let’s first number the capacitors:

Capacitors 2, 4, and 6 are in series, thus:

$$\frac{1}{C_{246}} = \frac{1}{C_2} + \frac{1}{C_4} + \frac{1}{C_6} = \frac{1}{24} + \frac{1}{12} + \frac{1}{8} = \frac{1}{4}$$

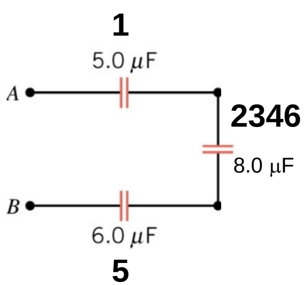
$$\Rightarrow C_{246} = 4.0\ \mu\text{F}$$

Now the circuit looks like this:



Now capacitor 3 is in parallel with capacitor 246: $C_{2346} = C_3 + C_{246} = 4 + 4 = 8$

Now the circuit looks like this:



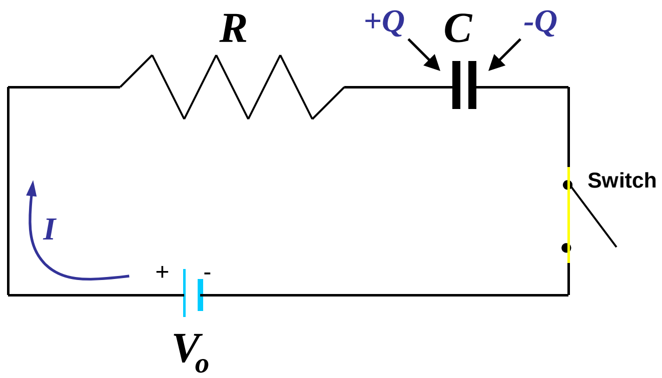
Now capacitors 1,2346, and 5 are in series:

$$\frac{1}{C_{123456}} = \frac{1}{C_1} + \frac{1}{C_{2346}} + \frac{1}{C_5} = \frac{1}{5} + \frac{1}{8} + \frac{1}{6} \approx \frac{1}{2} \Rightarrow C_{eq} = C_{123456} = \boxed{2.0\ \mu\text{F}}$$

20.13 RC Circuits

Many circuits contain both resistors and capacitors together.

Let's look at a simple circuit (called an RC circuit) with a resistor in series with a capacitor:

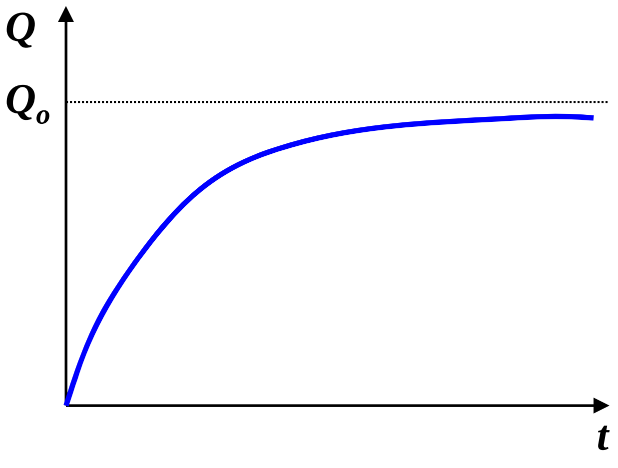


At this point, the switch is open, and the capacitor has no charge on its plates.

Now let's close the switch:

Once the switch is closed, a current starts to flow around the circuit, and charge builds up on the capacitor plates:

What does a plot of the charge on the capacitor versus time look like?



Notice that the charge builds up gradually and approaches an equilibrium value, Q_o.

If the capacitor is initially uncharged at $t = 0$, then the charge on the capacitor at some later time t is:

$$Q = Q_o\left[1 - e^{-t/RC}\right]$$

Charging Equation

e **is the base for natural log (ln), not $\log_{10}$.** $$e \approx 2.718$$

If t = 0, then $$Q = Q_o\left[1 - e^{0/RC}\right] = Q_o[1-1] = 0 \checkmark$$

We can get the voltage across the capacitor at any time by dividing by the charge by the capacitance, since $V = Q/C$:

$$\boxed{V = V_o\left[1 - e^{-t/RC}\right]}$$

Let's look at the product (RC)

What are the units of RC?

$$[\text{Resistance} \times \text{Capacitance}] = [\Omega \cdot \text{F}]$$

$$= \left[\left(\frac{\cancel{V} \cdot \text{s}}{\cancel{Q}}\right)\left(\frac{\cancel{Q}}{\cancel{V}}\right)\right] = [s]$$

Thus, RC has the units of time, and the product is known as the time constant of the circuit.

$$\boxed{\tau = RC}$$ **Time Constant**

Thus, we can write the charging equation as:

$$\boxed{Q = Q_o\left[1 - e^{-t/\tau}\right]}$$

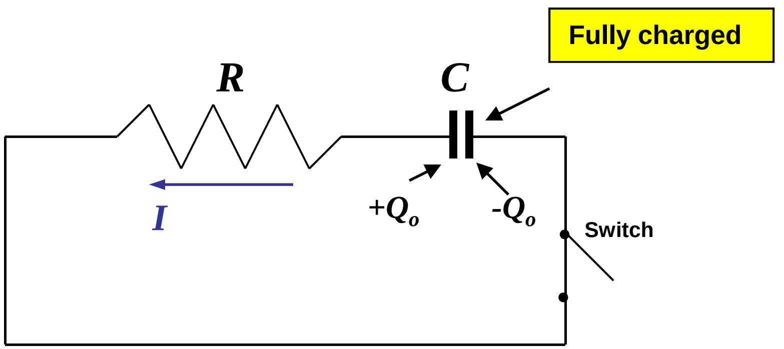
Let's start with an uncharged capacitor and then charge it for a length of time equal to one time constant, i.e. $t = \tau$:

Then, $$Q = Q_o\left[1-e^{-\tau/\tau}\right] = Q_o\left[1-e^{-1}\right] = Q_o(0.632) = \boxed{(63.2\%)Q_o}$$

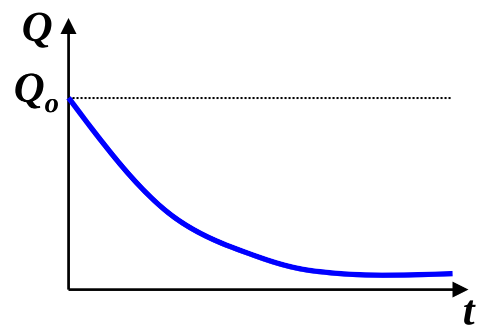
*Thus, charging for one time constant is the length of time it takes to accumulate 63.2% of the total charge on the capacitor.

Discharging

Now let's start with a capacitor that is charged all the way to Q_o and then discharge it through the resistor:



When we close the switch, a current flows through the resistor and we discharge the capacitor:



Now what does a plot of the charge on the capacitor versus time look like?

The functional form is: $$\boxed{Q = Q_o e^{-t/\tau}}$$

Discharging equation

Notice at t = 0, $$Q = Q_o e^{0/\tau} = Q_o \quad \surd$$

Now let $t = \tau$, one time constant: $$Q = Q_o e^{-\tau/\tau} = Q_o e^{-1} = Q_o(0.368) = (36.8\%)Q_o$$

Thus, one time constant is also the length of time it takes a fully charged capacitor to lose 63.2% of its charge.

One common use of the RC circuit is in pacemakers. What value of τ would be good for such a device???