

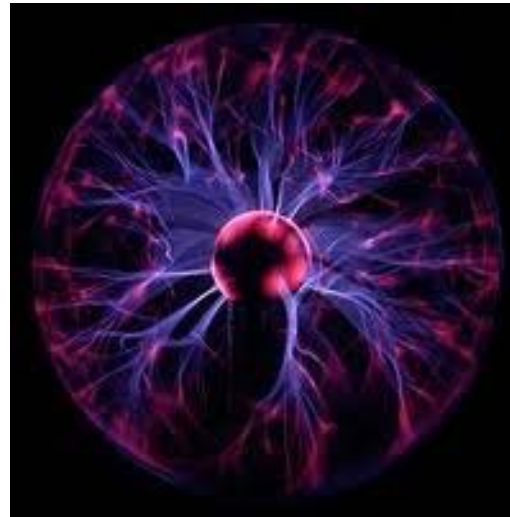
# Algebra-based Physics II

## Chapter 19

### Electric potential energy & The Electric potential

Why energy is stored in an electric field?

How to describe an field from energetic point of view?



Natural way of energy release

**Class Website:**

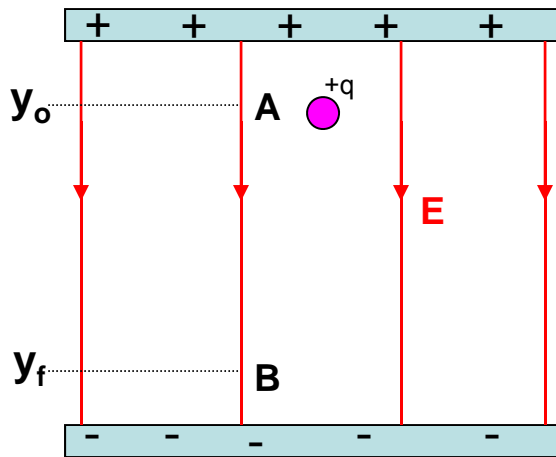
<http://www.phys.lsu.edu/~jzhang/teaching.html>



# Electric Potential Energy

The electric force also a **conservative force**

Let's place a positive point charge  $q$  in a uniform electric field and let it move from point A to B (no gravity):



How much work is done by the field in moving the charge from A to B?

\*Remember,  $W = F \cdot d$ , where  $F$  is the component of the constant force along the direction of the motion.

$$\text{Here, } F = qE \text{ , so } W = qE(y_f - y_o) = qE\Delta y$$

**Introduce:** Electrostatic Potential Energy (EPE)

# Algebra-based Physics II

Sep. 1<sup>st</sup>, Chap. 19.2-4

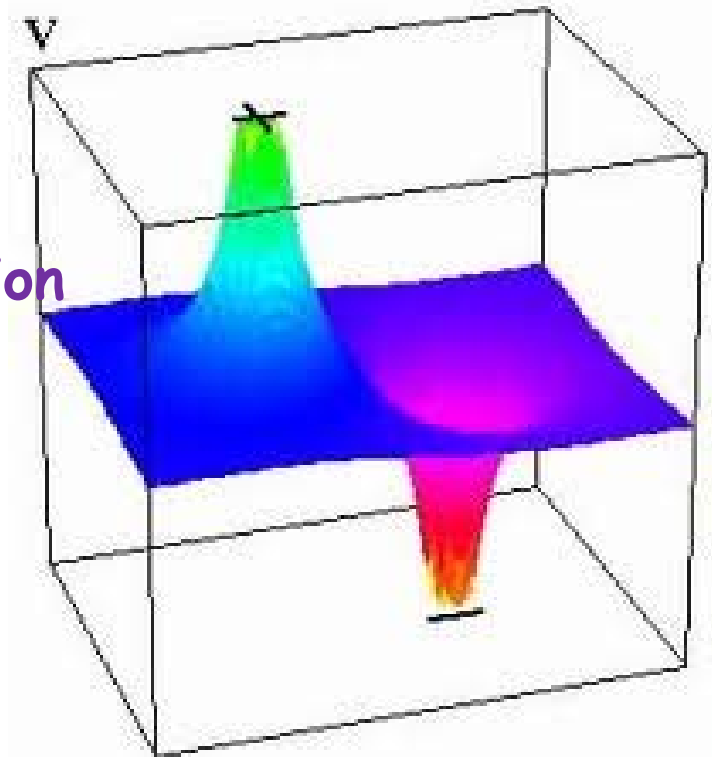
Electric potential

Electric potential distribution

Equipotential surfaces

Announcements:

1. HW2 (part A & B) posted.



electric potential of charges

Class Website:

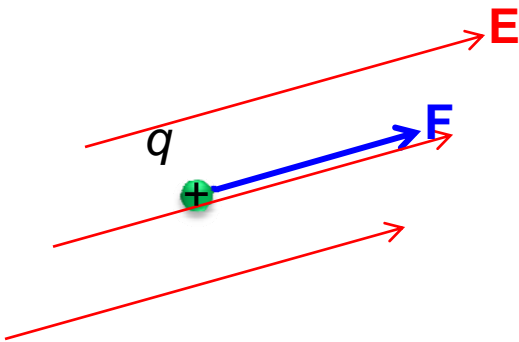
<http://www.phys.lsu.edu/~jzhang/teaching.html>

$$W_{AB} = -\Delta EPE = -(EPE_B - EPE_A) = EPE_A - EPE_B$$

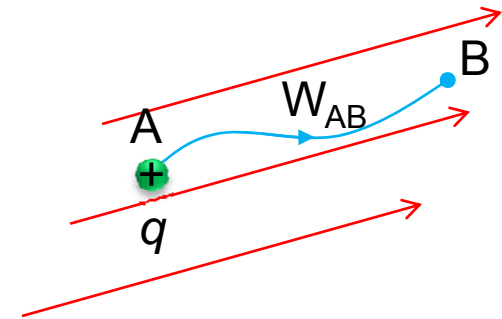
$\Delta EPE$  represents the increase of electric potential energy

The work done is equal to the **decrease** in electrostatic potential energy!

Charge feels electric force



Field force does work on charge



Force is conservative

$$\vec{E} \equiv \frac{\vec{F}}{q}$$

$$\frac{W_{AB}}{q} = \frac{-\Delta EPE}{q} = \frac{EPE_A}{q} - \frac{EPE_B}{q}$$

Define the electric field to describe the field distribution

$$V = \frac{EPE}{q} \quad \text{Depend on position}$$

$$V = \frac{EPE}{q}$$

The quantity on the right is the potential energy per unit charge. We call this the Electric Potential, V:

- The electric potential is a scalar !
- It belongs to the field only!
- The electric potential also describe the distribution of field

When charge is at different position, it has different EPEs, Thus there are different electric potential in the space

Units?  $\left[ \frac{\text{Energy}}{\text{Charge}} \right] = \left[ \frac{J}{C} \right] = [\text{Volt}] = [V]$

Review of Work:

1. Work is not a vector, but it can be either positive or negative:

**Positive** – Force is in the same direction as the motion

**Negative** – Force is in the opposite direction as the motion

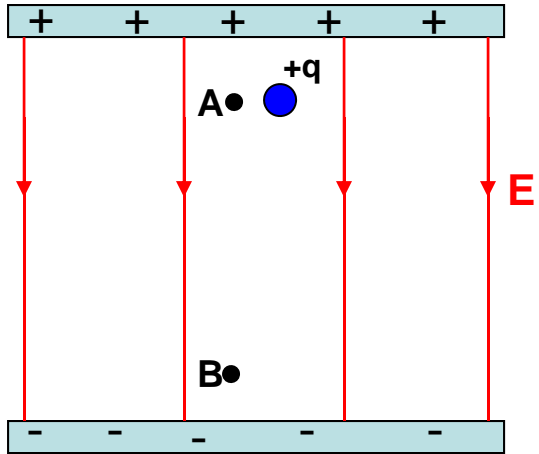
2. If **positive** work is done on an object, the object **speeds up**.

3. If **negative** work is done on an object, the object **slows down**.

# 19.2 Electric Potential Difference

We can talk about the value of the potential at different points in space:

For example, what is the difference in electrostatic potential between two points, A and B, in an electric field???



$$V_B - V_A = \frac{EPE_B}{q} - \frac{EPE_A}{q} = \frac{-W_{AB}}{q}$$

So,

$$\Delta V = V_B - V_A = \frac{-W_{AB}}{q}$$

Why is there a minus sign???

Let's say the charge at point A is positive:

If I release it, which way will it move? It moves down toward B!

Since the force is down and the motion is down, positive work is done on the charge. Thus,  $W_{AB}$  is positive.

This means that  $(V_B - V_A)$  is negative, or  $V_A > V_B$ .

We say that point A is at a **higher potential** than point B.

## Summary :

**Any charge, starting from rest, will accelerate from high to low potential energy region by electric force.**

$$W_{AB} = EPE_A - EPE_B$$

**Positive charges, starting from rest, will accelerate from regions of high potential and move toward regions of low potential.**

**Negative charges, starting from rest, will accelerate from regions of low potential and move toward regions of high potential.**

$$\Delta V = V_B - V_A = \frac{-W_{AB}}{q}$$



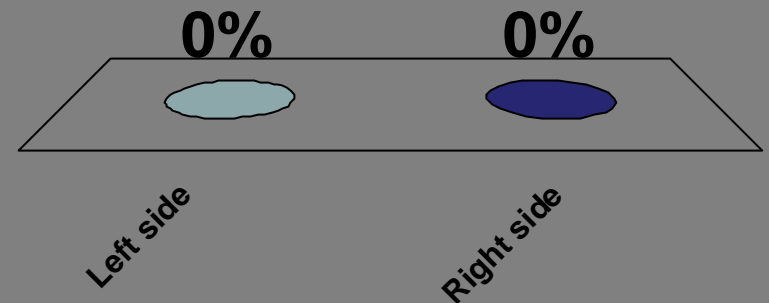
## Clicker Question 19-1

Under an external field ( $E$ ), Which side of space (left or right) for a positive charge has a lower potential energy?

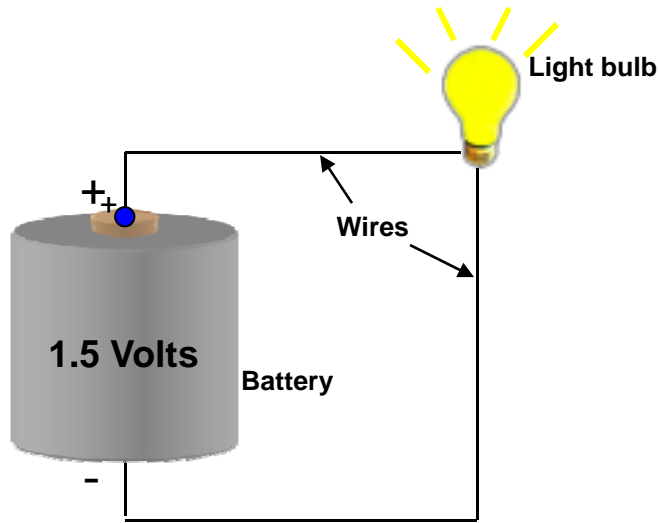


1. Left side
2. Right side

The external electric field lines start from left to right and a positive charge moves from high to low potential energy region, thus the right side has a lower potential energy for a positive charge.



One common object associated with voltages is a battery:

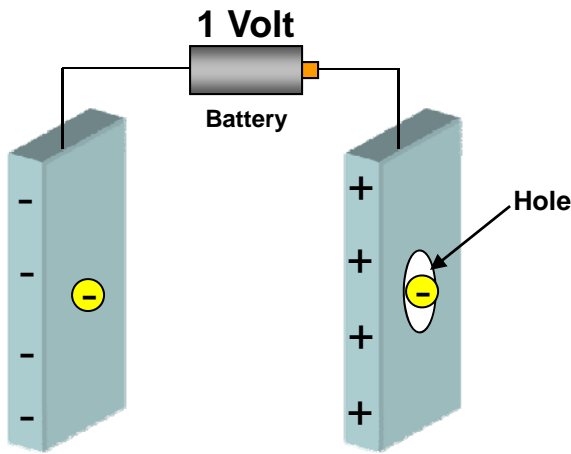


Notice that the positive charge moves from higher potential (+) to lower potential (-).

$$1.5 \text{ V} = 1.5 \frac{\text{J}}{\text{C}} \longrightarrow \text{The battery supplies 1.5 Joules of energy for every coulomb of charge.}$$

The word “volt” also appears in a unit of energy:

Let’s accelerate an electron from rest through a potential difference of 1 Volt:



The electron gets accelerated from low potential to high potential → It gains kinetic energy.

The energy gained by an electron when accelerated through a potential difference of 1 Volt = 1 electron volt = 1 eV.

If I accelerated an electron from rest through a potential difference of 50,000 V, then I know immediately that its kinetic energy is 50,000 eV.

Energy is usually expressed in Joules:

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

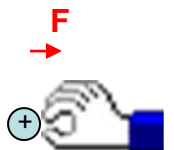
Just like in a gravitational field, in an electric field, potential energy (PE) can be converted into kinetic energy (KE):

Example: Let's bring a small positive test charge from very far away in toward a fixed, positive point charge:

As I push the charge in closer and closer, the repulsive **force** on it gets bigger and bigger:

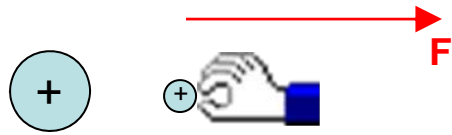


\*Thus, I have to do work on the charge to move it closer.



The work I do on the charge goes into increasing its potential energy!

Now release the charge.....



The charge converts its stored EPE into KE!!!

**Remember: The total mechanical energy of a system must be conserved.**

$$E_{Tot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh + \frac{1}{2}kx^2 + EPE$$

$\begin{matrix} \nearrow & \nearrow & \nearrow & \nearrow & \nearrow \\ KE_{Trans} & KE_{Rot} & PE_{Grav} & PE_{Elastic} & PE_{Electro} \end{matrix}$

And,....If the work done by nonconservative forces is zero, then:

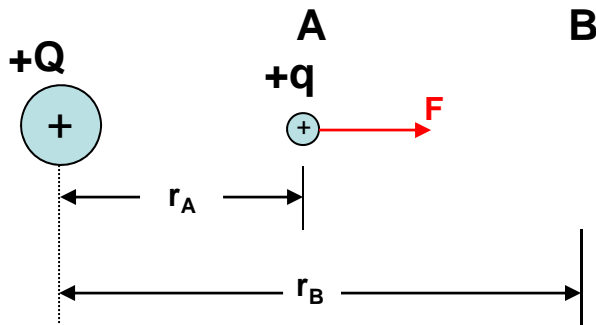
$$E_{Tot_f} = E_{Tot_o}$$

## 19.3 The Electric Potential of a Point Charge

An electric potential exists around charges.

What is the form of the potential for a point charge?

Let's place a positive test charge near a positive fixed point charge:



The electric field created by the point charge does work on the test charge and moves it to the right

**What is the work done by the field in moving the charge from A to B?**

Well,  $W = F \cdot d$ , and the force is given by Coulomb's Law:  $F = k \frac{qQ}{r^2}$

But, the force is not constant as the charge moves from  $r_A$  to  $r_B$ , since the force depends on  $r$ .

Thus, we have to use calculus and integrate the force over the distance:

$$\text{Result: } W_{AB} = k \frac{qQ}{r_A} - k \frac{qQ}{r_B}$$

From earlier, we know that:  $V_B - V_A = \frac{-W_{AB}}{q} = k \frac{Q}{r_B} - k \frac{Q}{r_A}$

If we let  $r_B$  be really far away, i.e.  $r_B \rightarrow \infty$ , then  $V_B = k \frac{Q}{r_B} \rightarrow 0$ .

\*This sets our zero potential at infinity.

Thus, we are left with:  $V_A = k \frac{Q}{r_A}$ .  $r_A$  is just some arbitrary distance from the point charge, so we drop the subscript:

$$V = k \frac{Q}{r}$$

**This is the electric potential due to a point charge.**

We now have 3 equations which kind of look similar:

$$F = k \frac{qQ}{r^2}$$

$$E = k \frac{Q}{r^2}$$

$$V = k \frac{Q}{r}$$

**Force between two charges.**

**Electric field of a point charge.**

**Electric potential of a point charge.**

# Algebra-based Physics II

Sep. 1<sup>st</sup>, Chap. 19.2-4

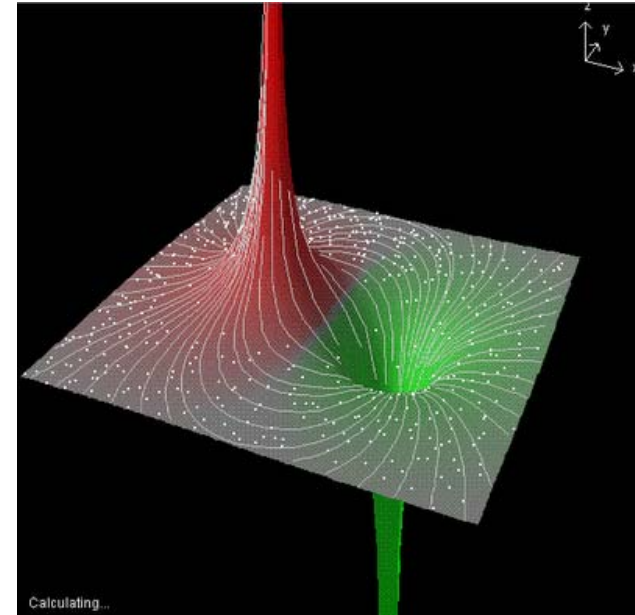
Electric potential

Electric potential distribution

Equipotential surfaces

Announcements:

1. HW2 (part A & B) posted.



electric potential of charges

Class Website:

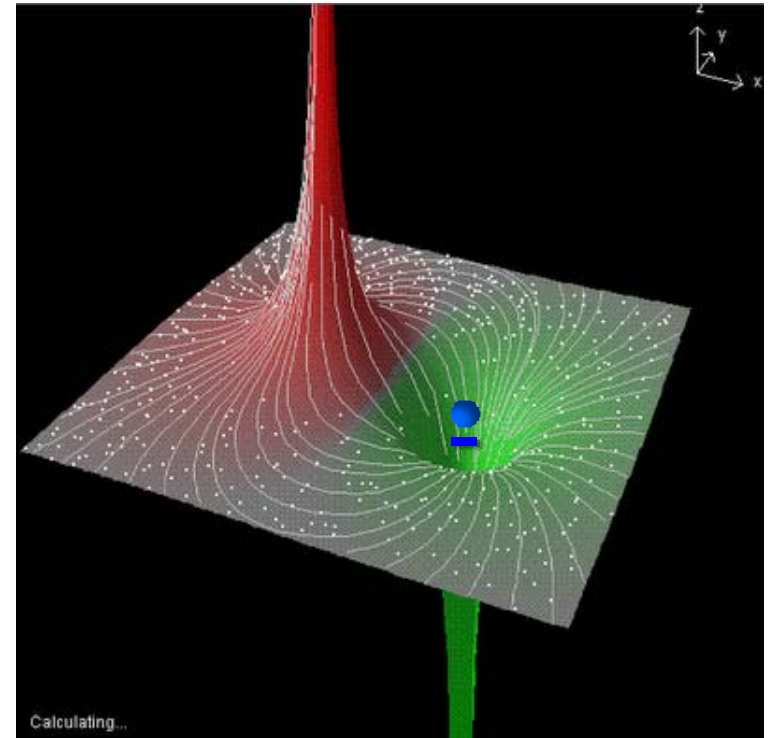
<http://www.phys.lsu.edu/~jzhang/teaching.html>

**Electric potential due to a point charge if we assume  $V = 0$  as  $r \rightarrow \infty$ :**

$$V = k \frac{Q}{r} = \begin{cases} \geq 0 & \text{(positive charge)} \\ \leq 0 & \text{(negative charge)} \end{cases}$$

**Electric potential energy with  $q$ :**

$$W_{ab} = -q(V_B - V_A)$$



**We now have 3 equations which kind of look similar:**

$$F = k \frac{qQ}{r^2}$$

$$E = k \frac{Q}{r^2}$$

$$V = k \frac{Q}{r}$$

**Force between two charges.**

**Electric field of a point charge.**

**Electric potential of a point charge.**



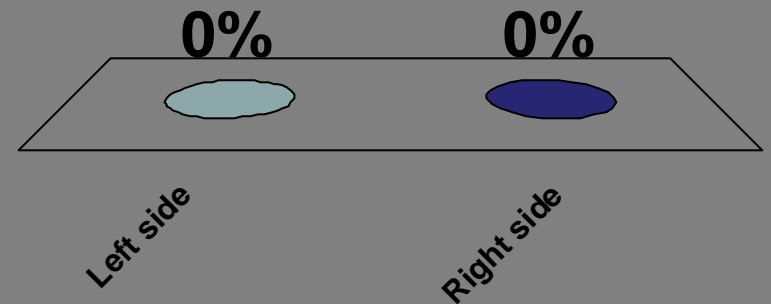
## Clicker Question 19-2

Which side of space (left or right) is at a higher potential?



1. Left side
2. Right side

The electric field points from left to right, and electric field lines start from regions of high potential, thus the left side is at a higher potential.



## Example 1:

Two electrons are initially held with a distance of  $5 \times 10^{-6}$  m. Then one of them is released and moves away to infinity, what is the ultimate kinetic energy of the electron?

The conservative electric force does the work and increases the kinetic energy.

USE energy conservation:

$$E_{mec}^i = E_{mec}^f$$

$$\frac{1}{2}m_e v_i^2 + EPE_i = \frac{1}{2}m_e v_f^2 + EPE_f \quad (v_i = 0, EPE_f = 0)$$

$$EPE_i = \frac{1}{2}m_e v_f^2 \Rightarrow v_f = \sqrt{\frac{2EPE_i}{m_e}}$$

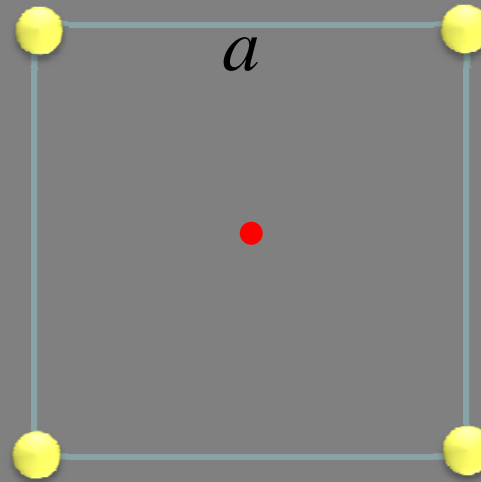
$$EPE_i = qV_r = e \cdot k \frac{e}{r_i} = k \frac{e^2}{r_i}$$

$$v_f = \sqrt{\frac{2ke^2}{m_e r_i}} = e \sqrt{\frac{2k}{m_e r_i}} \quad (r_i = 5 \times 10^{-6} \text{ m})$$

## Clicker Question 19-2

Four identical charges ( $q$ ) are located at the four corners of a square with side length  $a$ . What is the electric potential at the center of the square?

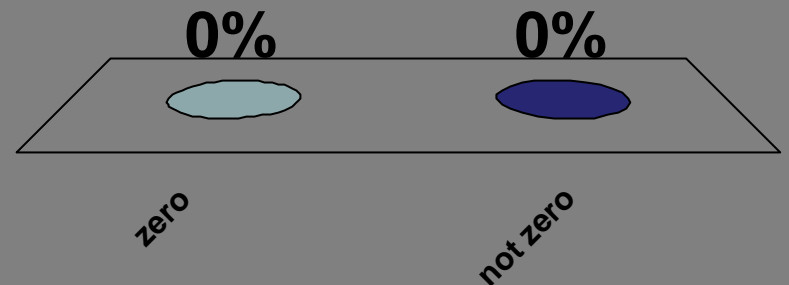
1. Zero
2. Not zero



$$V = \sum_j V_j = V_1 + V_2 + V_3 + V_4 = 4V_1$$

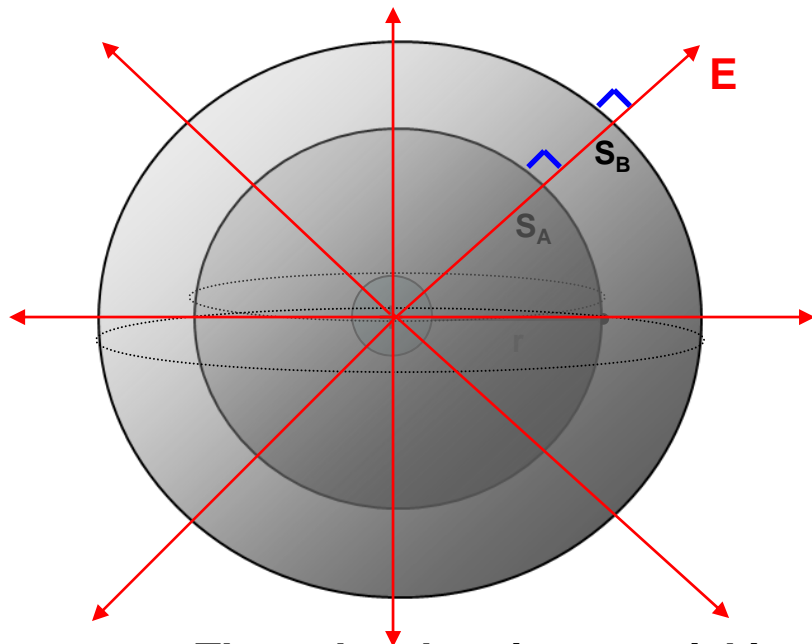
$$V_1 = k \frac{q}{r_1} = k \frac{q}{\frac{1}{\sqrt{2}}a} = k \frac{\sqrt{2}q}{a}$$

$$V = 4\sqrt{2} \frac{kq}{a}$$



## 19.4 Equipotential Surfaces

So the potential from a point charge is  $V = k \frac{Q}{r}$



This means the potential is the same in every direction around the point charge at a distance  $r$  away.

In 3D, this forms a spherical shell of radius  $r$  around the charge.

Thus, the electric potential is the same everywhere on this spherical surface (S<sub>A</sub>). It is called an equipotential surface.

**Equipotential surfaces are surfaces of constant potential.**

Let's look at another equipotential surface (S<sub>B</sub>) around the point charge:  
We know the **electric field** lines point everywhere radially outward:

**Notice:** The electric field lines are perpendicular to the equipotential surfaces.

Since  $S_A$  is closer to the positive charge than  $S_B$ ,  $S_A$  is at a higher potential than  $S_B$ .

Thus, electric field lines point in the direction of decreasing potential, i.e. they point from high potential to low potential.

Work?

The net electric force does no work as a charge moves on an equipotential surface.

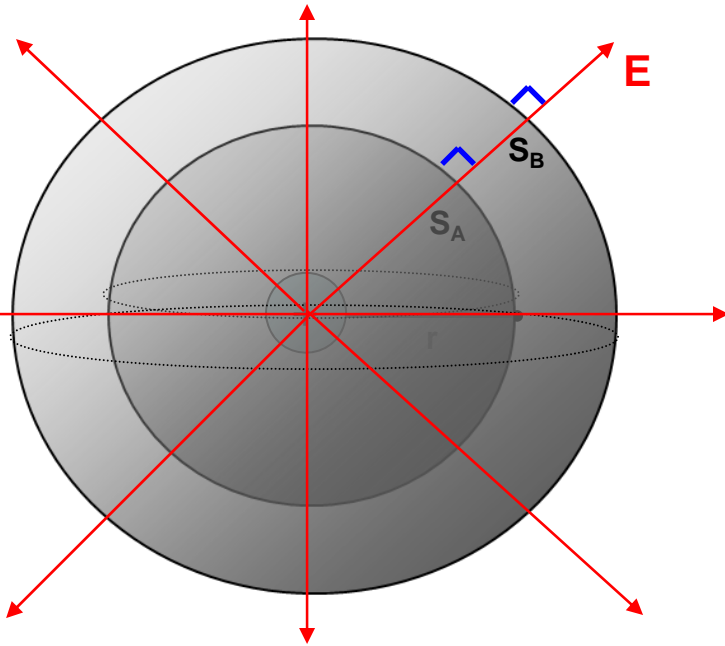
Why?

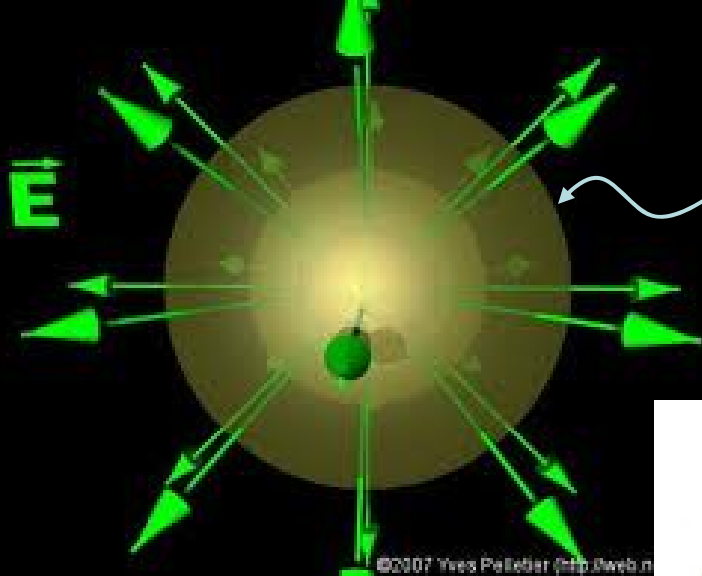
We defined  $V_B - V_A = \frac{-W_{AB}}{q}$ .

But, if we are on an equipotential surface, then  $V_A = V_B$ , and  $W_{AB} = 0$ .

Or....

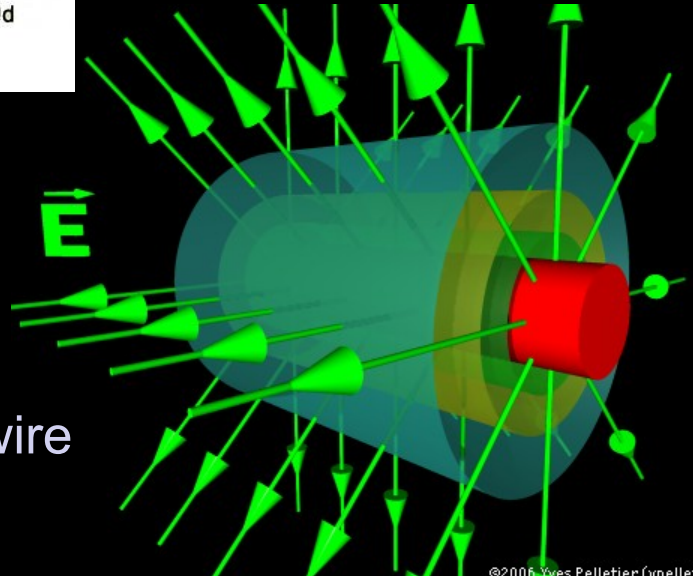
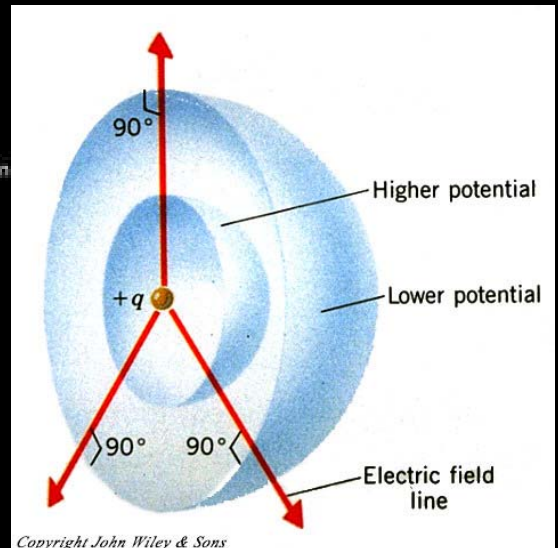
In order for the charge to feel a force along an equipotential surface, there must be a component of the field along the surface, but  $E$  is everywhere perpendicular to the equipotential surface.





equipotential surface

a point charge



a charged straight wire

# **Fields, Potentials, and Motion of Charges - Summary**

**Electric field lines start on positive charges and end on negative ones.**

**Positive charges accelerate from regions of high potential toward low potential.**

**Negative charges accelerate from regions of low potential toward high potential.**

**Equipotential surfaces are surfaces of constant potential.**

**Electric field lines are perpendicular to an equipotential surface.**

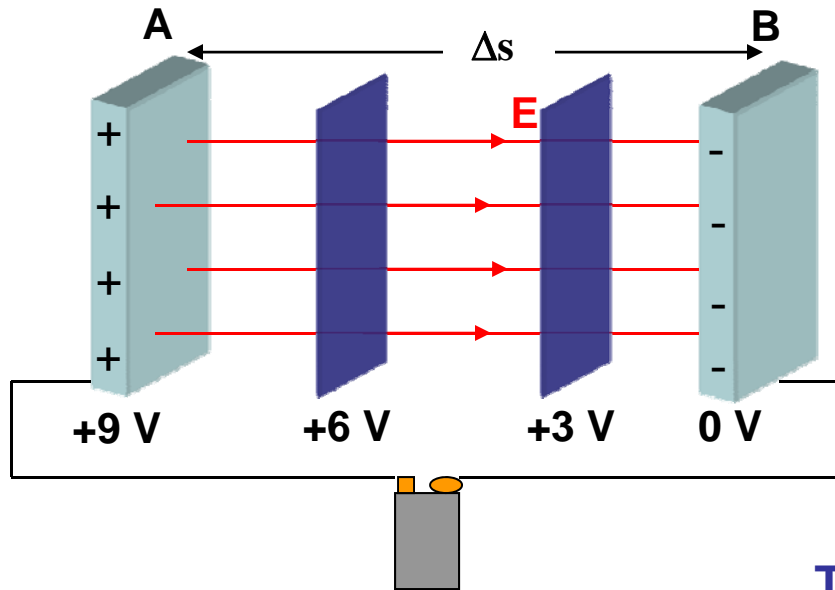
**Electric field lines are perpendicular to the surface of a conductor, thus a conductor is an equipotential surface!**

**Electric field lines point from regions of high potential toward low potential.**

**Therefore, positive charges move in the same direction as the electric field points, and negative charges move in the opposite direction of the electric field.**

**The electric force does no work as a charge moves on an equipotential surface.**

## Parallel plate capacitor



The positive plate is at a potential of +9 V and the negative plate is at 0 V.

What would the equipotential surfaces look like between the plates?

They would be a parallel set of planes!

Let the plates be separated by a distance  $\Delta s$ .

The electric field is then 
$$= \frac{[\text{Change in voltage}]}{[\text{Change in distance}]} \Rightarrow E = \frac{\Delta V}{\Delta s}$$

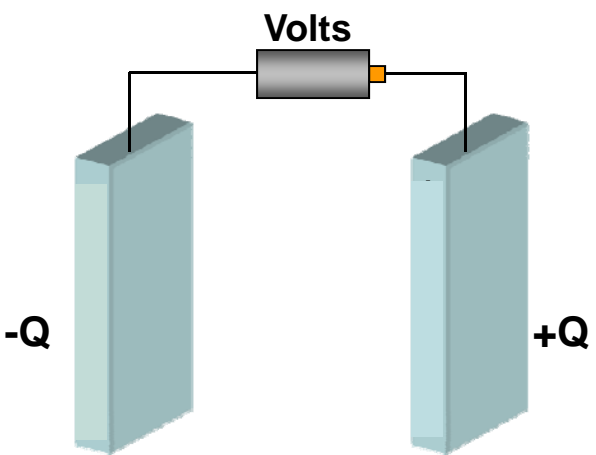
This is called the electric field gradient.

Thus, the electric field also has units of  $[V / m]$



# 19.5 Capacitors

Two oppositely charged conductors separated by some small distance.



We can charge the plates by connecting them to a battery:

The higher the voltage on our battery, the more charge we can put on each plate.

Thus,  $Q \propto V$

Make this an equality:

$$Q = CV$$

$C$  is a new quantity called the Capacitance.

Units?

$$C = \frac{Q}{V} \rightarrow \frac{[\text{Charge}]}{[\text{Voltage}]} = \left[ \frac{\text{C}}{\text{V}} \right] = [\text{Farad}] = [\text{F}]$$

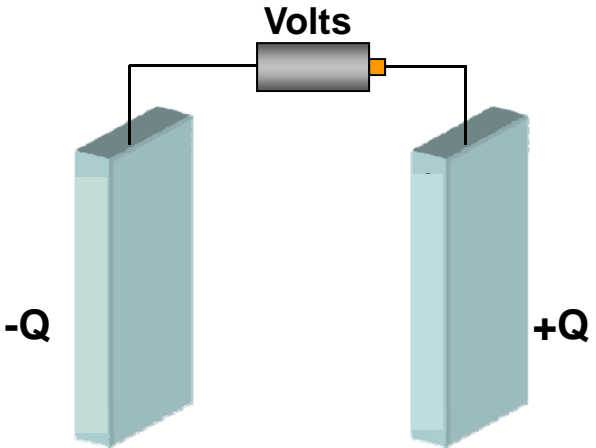
\*A farad is a very large capacitance. We often use microfarads ( $\mu\text{f}$ ) and picofarads ( $\text{pf}$ ).



**The larger the capacitance, the more charge it will hold!**

**Example:**

What voltage is needed to store  $7.2 \times 10^{-5} \text{ C}$  on the plates of a  $6.0 \mu\text{F}$  capacitor?



What do we know:

$$Q = 7.2 \times 10^{-5} \text{ C}$$

$$C = 6.0 \times 10^{-6} \text{ F}$$

**USE the definition of capacitance:**

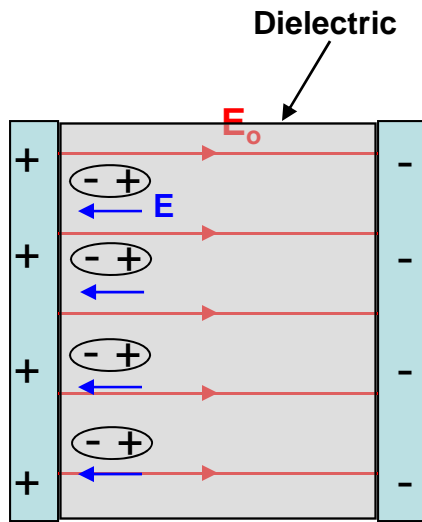
$$Q = CV \quad \text{so} \quad V = \frac{Q}{C} = \frac{7.2 \times 10^{-5}}{6 \times 10^{-6}} = 12 \text{ V}$$

**Reality:**



# Dielectrics

We can fill the space between the plates with some insulating material, say air, oil, paper, rubber, plastic, etc.



This material is called a dielectric.

So what effect does the dielectric have on the field between the plates?

Since the dielectric is an insulator, the charges in it aren't free to move, but they can separate slightly within each atom:

Each one of these atoms now produces a small internal electric field which points in the opposite direction to the field between the plates:

Thus, the net electric field between the plates is reduced by the dielectric.

The reduction of the field is represented by the following:

$$\kappa = \frac{E_0}{E}$$

$E_0$  is the field without the dielectric  
 $E$  is the field with the dielectric

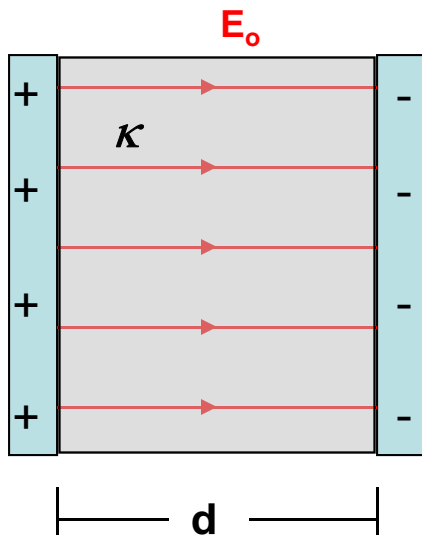
$\kappa$  is called the dielectric constant, and it must be greater than 1.

$$\kappa = \frac{E_o}{E}$$

Since  $\kappa$  is the ratio of two electric fields, it's unitless.

Material	$\kappa$
Vacuum	1
Air	1.00054
Water	80.4

The larger  $\kappa$  is, the more it reduces the field between the plates!



Let's say the plates have surface area  $A$  and are separated by a distance  $d$ .

$$E = \frac{1}{\kappa} E_o = \frac{V}{d} \Rightarrow E_o = \frac{\kappa V}{d} = \frac{\sigma}{\epsilon_o} = \frac{q}{\epsilon_o A}$$

$$\Rightarrow q = \left( \frac{\epsilon_o A \kappa}{d} \right) V$$

But,  $q = CV$ , so

$$C = \frac{\epsilon_o A \kappa}{d}$$

## Capacitors store charge - what about energy?

$$EPE_{\text{Stored}} = \frac{1}{2} qV = \frac{1}{2} CV^2$$

$$V = Ed \text{ and } C = \frac{\epsilon_o \kappa A}{d}, \text{ so } EPE_{\text{Stored}} = \frac{1}{2} \left( \frac{\epsilon_o \kappa A}{d} \right) (E^2 d^2)$$

Rearrange this:

$$EPE = \frac{1}{2} \kappa \epsilon_o E^2 (Ad) = \frac{1}{2} \kappa \epsilon_o E^2 (\text{Vol})$$

  
Volume between the plates

$$\frac{EPE}{\text{Vol}} = \text{Energy Density} = \frac{1}{2} \kappa \epsilon_o E^2$$

Units?

$$\left[ \frac{\text{Energy}}{\text{Volume}} \right] = \left[ \frac{\text{J}}{\text{m}^3} \right]$$

**\*This expression holds true for any electric fields, not just for capacitors!**