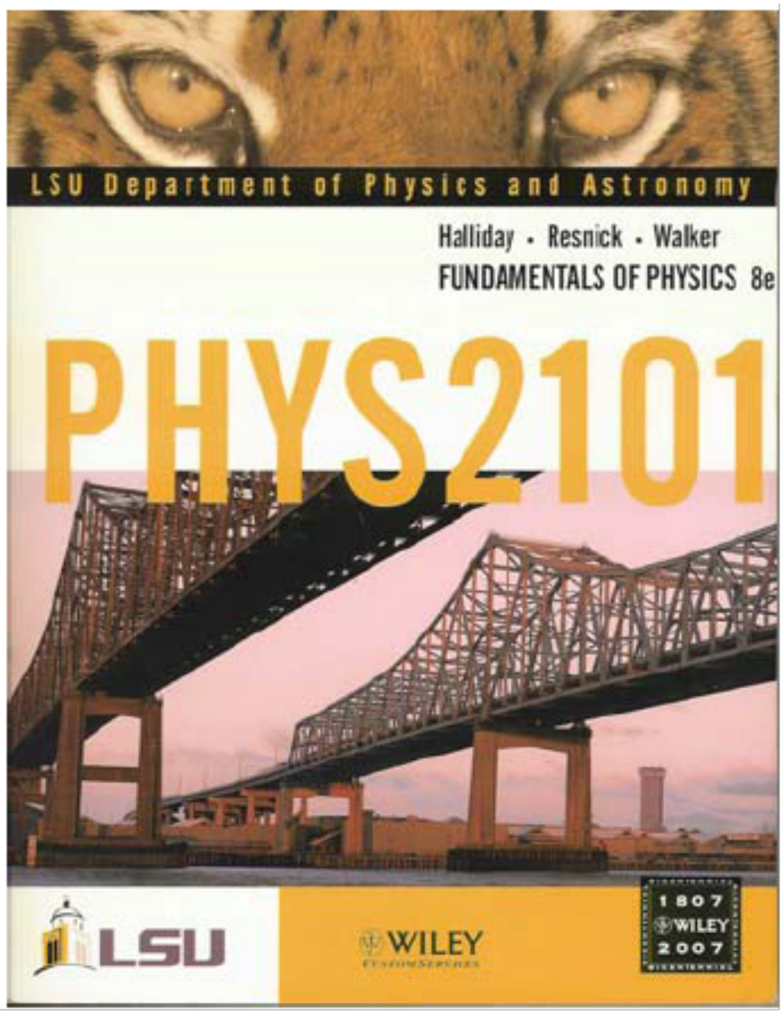


Physics 2101 for Technical Students Section 3

Feb. 17th: Ch.8 - 9

Announcements: Quiz#3

When: Feb. 26

Where: in class

What: HW supplemental #5

Test#2 (ch. 7-9) will be at 6 PM, March 3 (6 Lockett)

Class Website:

http://www.phys.lsu.edu/classes/spring2010/phys2101-5/

Quick Review:
Work Done by Friction – A non-Conservative Force

Mechanical energy:

Conservative System

$$E_{mech} = KE + U$$

Friction takes Energy out of the system—treat it like thermal energy

$$\Delta E_{mech} \neq 0 \text{ Lose Mechanical energy}$$

If there is no work done by an external force

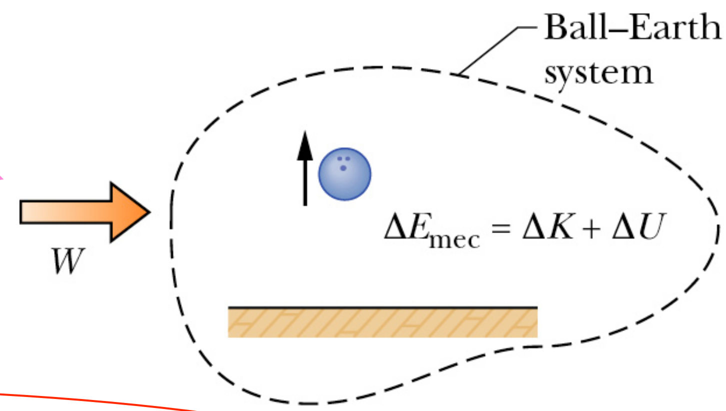
$\Delta E_{mech} + \Delta E_{Thermal} = 0$ -- *Conservation of Total Energy*

$$E_{mech}(final) = E_{mech}(initial) - E_{th}(final) + E_{th}(initial)$$

$$K_f + U_f = K_i + U_i - F_f \bullet displacement$$

Quick Review: Work done by External Force

Conserv Force	Other Forces
$F_{gravitation}$	$F_{applied}$
F_{spring}	$F_{friction}$
	$F_{air\ (drag)}$
	$F_{tension}$
	F_{normal}



Conservative Force in isolated system $\Delta E_{mech} = 0 \Rightarrow \Delta KE = -\Delta U$

If External force: $W_{ext,net} = \Delta E_{mech} = \Delta KE + \Delta U$

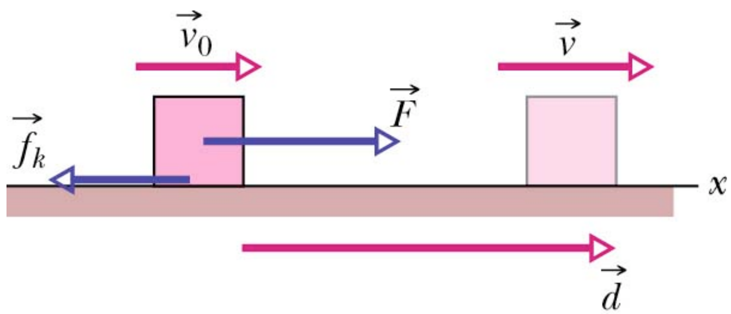
(NO FRICTION)

Positive if energy is transferred to system

Negative if energy is transferred from system

Quick Review:
Work done by External Force + Friction

Example: *Ext* Force with Friction



$$W_{ext,fric} = \Delta E_{mech} = \Delta KE + \Delta U$$

$$W_{ext} = \vec{F}_{ext} \bullet \vec{d}$$ *positive or negative*

$$W_{friction} = -\vec{f}_k \bullet \vec{d}$$ *always negative*

$$W_{ext,fric} = \left(\vec{F}_{ext} \bullet \vec{d} - \vec{f}_k \bullet \vec{d}\right) = \Delta E_{mech}$$

Define: $\Delta E_{therm} = \vec{f}_k \bullet \vec{d}$ *(increase in thermal energy by sliding)*

$$W_{ext,net} = \Delta E_{mech} + \Delta E_{therm} = \left(\Delta KE + \Delta U\right) + \Delta E_{therm}$$

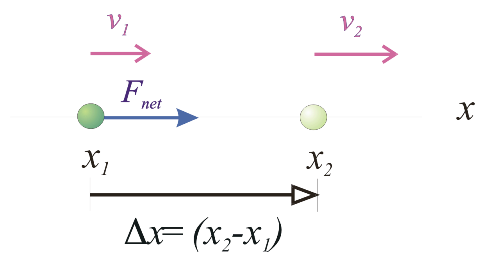
From Potential Energy Curves to Force Curves

Finding a conservative force from potential energy

$$\vec{F}(x) \qquad\qquad U(x)$$

In 1-D: $$\Delta U(x) = -W_{cons} = -\int_{x_1}^{x_2} F\,dx$$

where F is a slowly varying, internal force acting on a particle in system



then $$\Delta U(x) \approx -F(x)\Delta x$$

Now go backwards, say you know the change in potential energy at some point and you want to know the force at that point…

(in the differential limit)

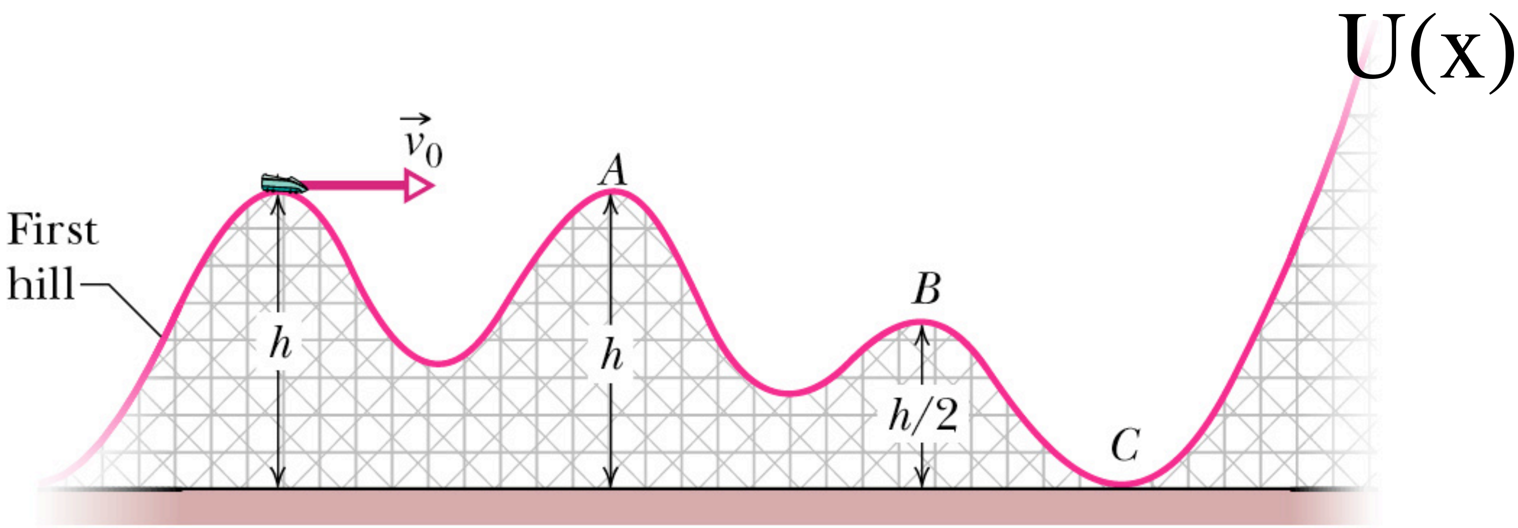
$$F(x) = -\frac{dU(x)}{dx}$$

$$F_{grav}(y) = -\frac{dU_{grav}(y)}{dy} = -\frac{(mgy)}{dy} = -mg$$

$$F_{spring}(x) = -\frac{dU_{spring}(x)}{dx} = -\frac{\left(\frac{1}{2}kx^2\right)}{dx} = -kx$$

Potential Energy Curve

The potential energy as a function of the a system with 1-D movement along x-axis while gravitational force does work on it:

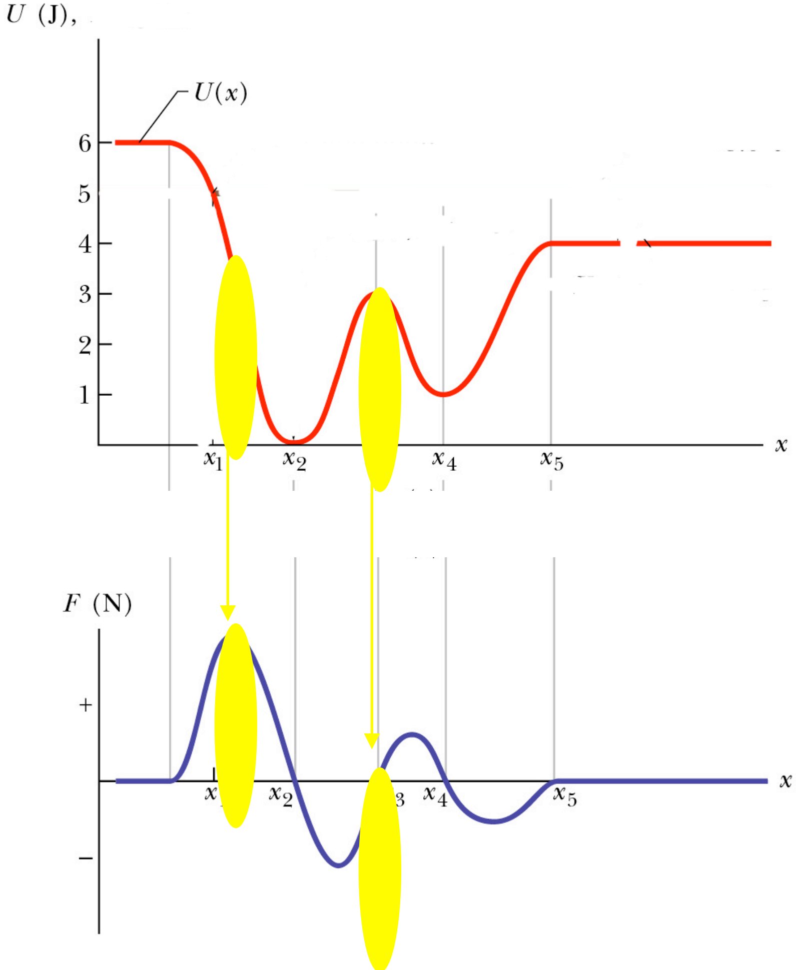


$$E_{mech} = KE(x) + U(x) \quad \Rightarrow \quad KE(x) = E_{mech} - U(x)$$

$$F(x) = -\frac{dU(x)}{dx}$$

Potential Energy Curve

Plot of *U*(*x*), the potential energy as a function of the a system with 1-D movement along *x*-axis while a conservative force does work on it:



$$F(x) = -\frac{dU(x)}{dx}$$

F(x) is negative slope of tangent to *U(x)*

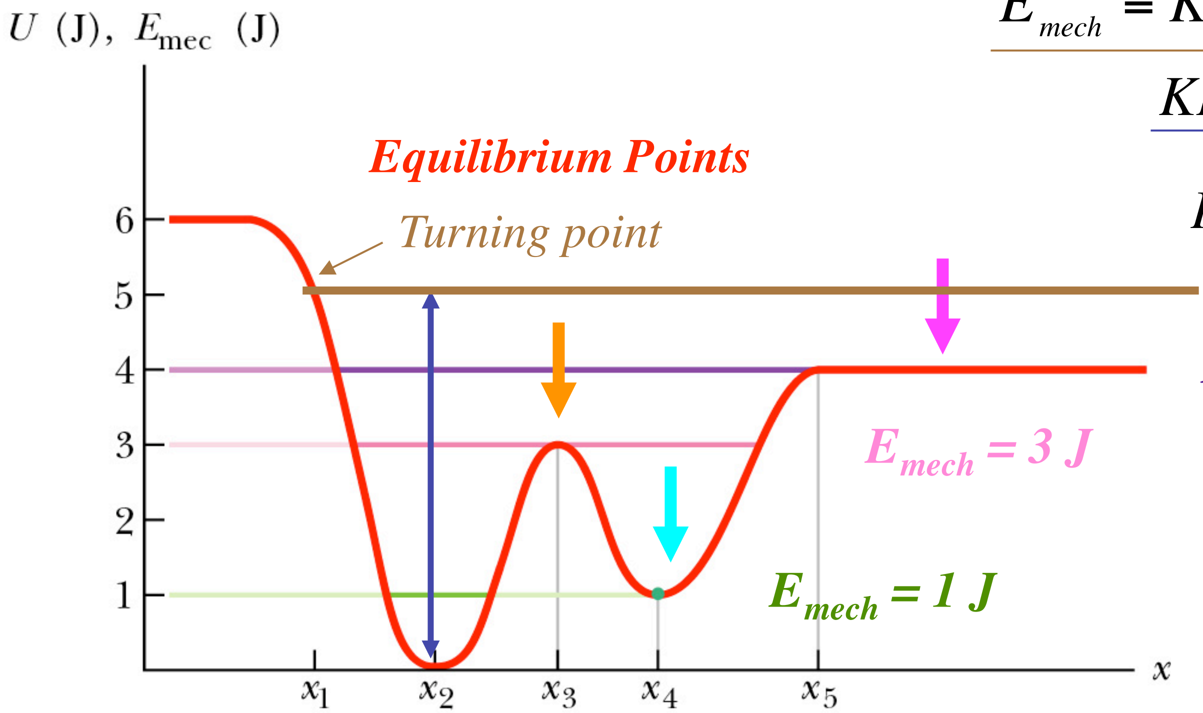
Potential Energy Curve

Plot of *U*(*x*), the potential energy as a function of the a system with 1-D movement along *x*-axis:

$$E_{mech} = KE(x) + U(x) = \text{constant}$$

$$KE(x) = E_{mech} - U(x)$$

$$F(x) = -\frac{dU(x)}{dx}$$



Equilibrium positions: where slope of *U*(*x*) curve is zero *[i.e. F(x) = 0 ; NO FORCE]*

-> ***Neutral*** *vs* ***Unstable*** *vs* ***Stable*** *Equilibrium*

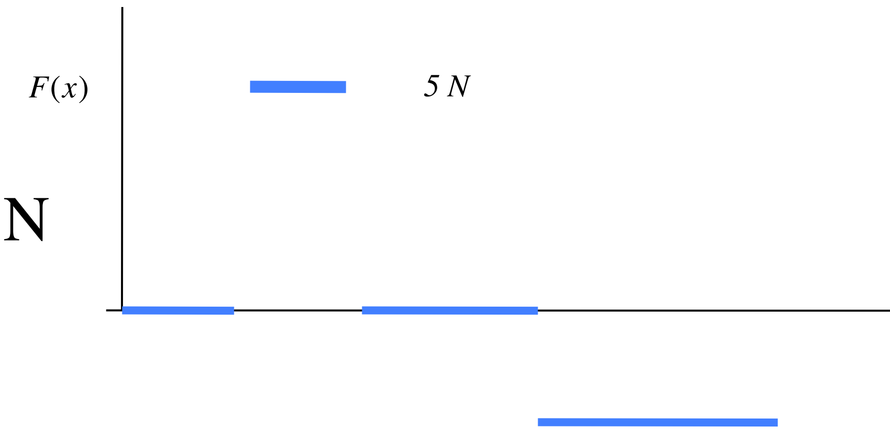
- Stable: *KE=0 ; F=0 & if move left or right → move back*
- Unstable: *KE=0 ; F=0 but if move left or right → force to move away*
- Neutral: *KE=0 ;* $E_{mech}=U$ *→ stationary particle*

Problem #121: A conservative force F(x) acts on a 2.0 kg particle. The potential energy U(x) is shown in the figure. When the particle is at x=2.0 m it has a velocity of -1.5m/s.

(a) find the magnitude and direction of the force at this position.

(b) What is the range of the allowed motion?

(c) What is the velocity at x=7.0 m?



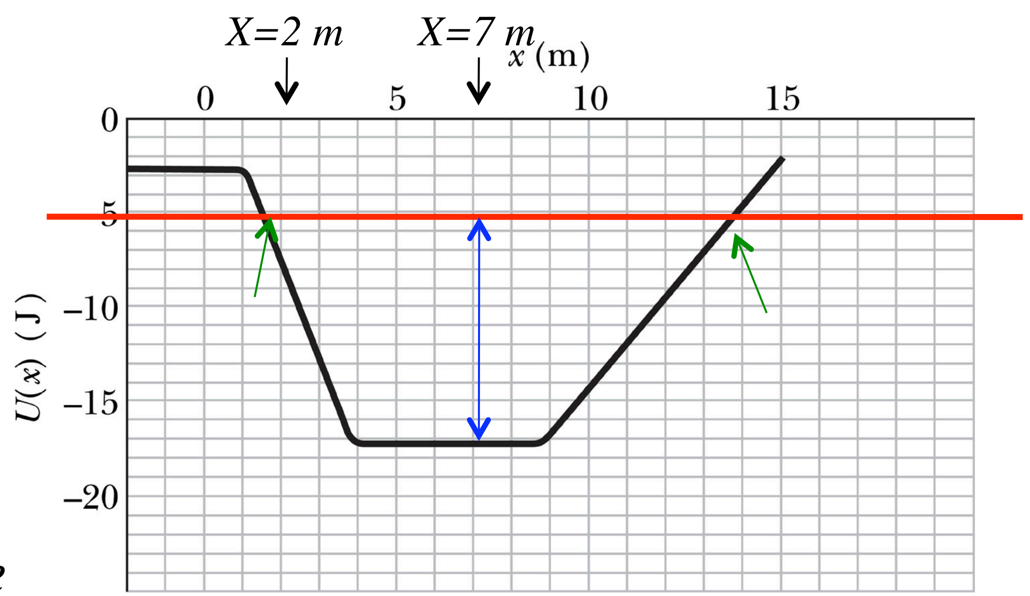
(a) F(x=2)=-dU/dx=-(-17.5+2.5)/(4-1)=5 N

(b) $K_i = \dfrac{mv^2}{2} = 2.25J$

$E_{mec} = K_i + U_i = 2.25J - 7.5J = -5J$

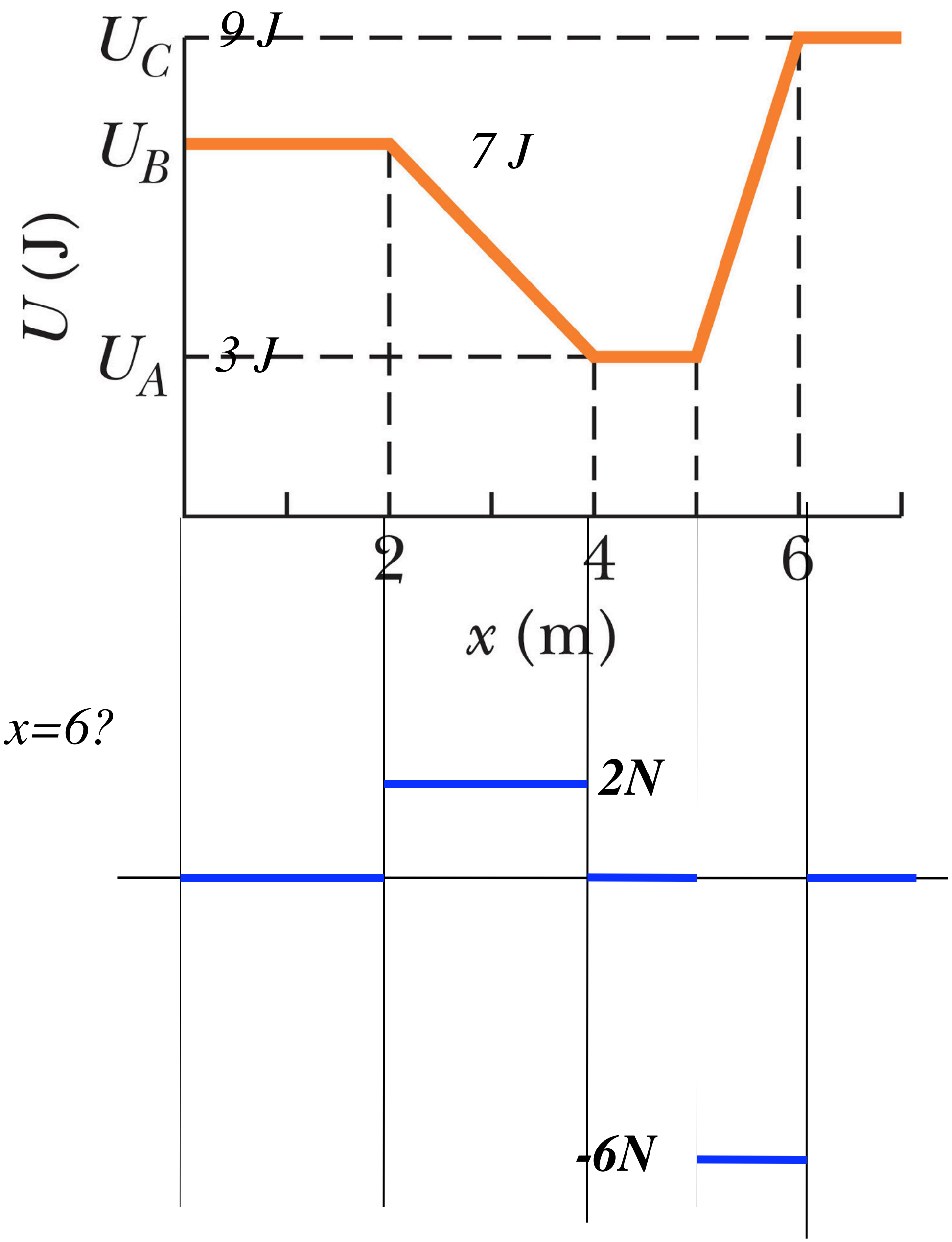
Range given by green arrows.

(c) $K_i = \dfrac{mv^2}{2} = -5 + 17J = 12J$



What is direction and magnitude of the force at 2, 7, 12 m?

Supplemental Homework #4:
Look at the figure for potential vs x. The force acts on a 0.50 kg particles and U_A=3 J, U_B=7 J, U_C=9 J. (a) Calculate the magnitude and direction of the force in all five regions. (b) Draw the plot.



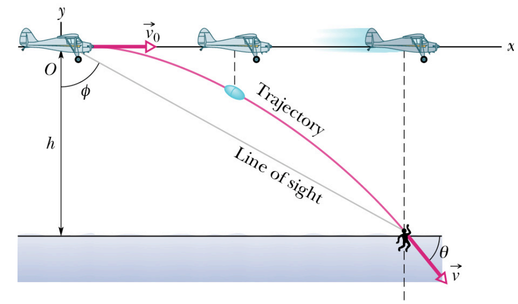
What happens at x=2, x=4, x=5, and x=6?

Chapter 9:

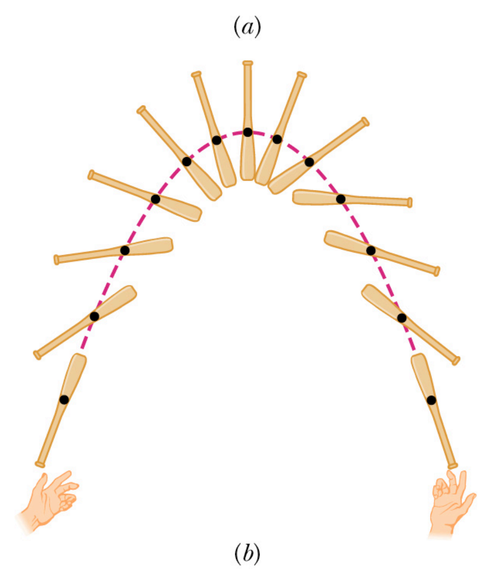
Center of Mass
+
Linear Momentum

The Center of Mass

Up to now, we've taken boxes, balls, pigs, penguins to be particles. We know how to apply Newton's laws to determine dynamics of a ***point*** particle.



Now we have a *real* mass (system of individual particles). How do we do this?



1) We find the <u>Center of Mass (COM) :</u>

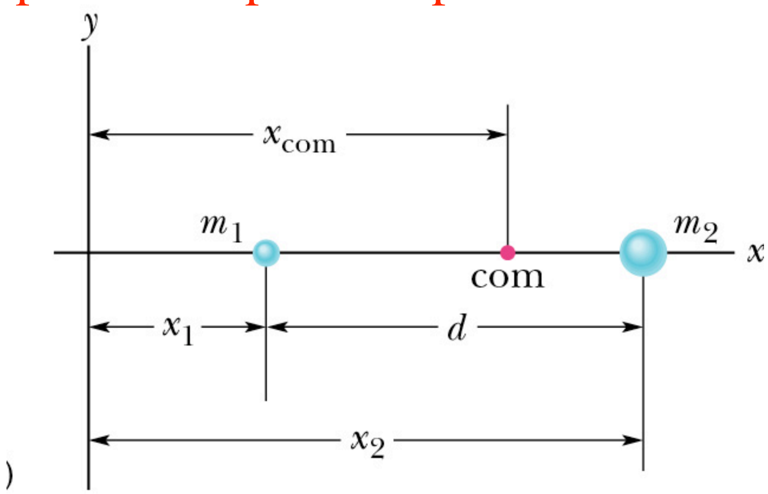
A point that moves as though all mass of a system were concentrated at that point *(doesn't have to be on the object)*

How to determine COM (*C*enter *O*f *M*ass) ?

Experimentally: *Find the point where it balances*
(as if all external forces are applied there)

Mathematically: *Find the "effective position"*

Simplest example: two particles



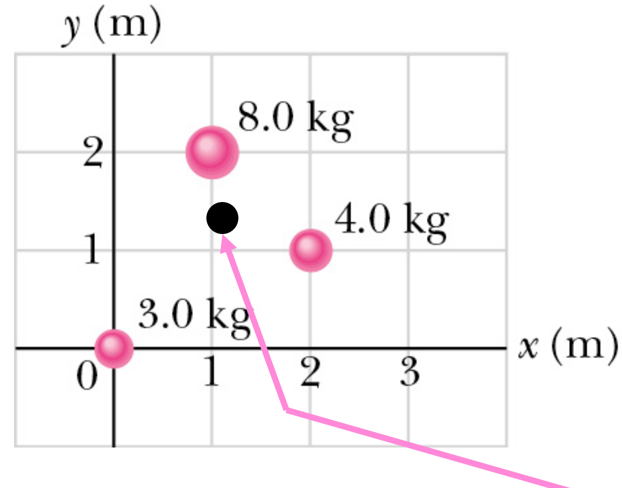
$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

$$= \frac{m_1(0) + m_2(0)}{m_1 + m_2} = 0$$

Note: Use symmetry!

Sample problem 9-3



What is the COM of the 3-particle system shown in the figure.

1) What is total mass

[redacted]

In General: How to determine COM?

N particles

with total mass M

$$M = \sum_{i=1}^{N} m_i$$

$$x_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$$

$$z_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i$$

In Vector notation:

Position of i^{th} particle: $\vec{r}_i = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$

Center of Mass : $\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j} + z_{com}\hat{k} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i$

Here N is large but still "countable"

Checkpoint

Checkpoint 1: ***The figure shows a uniform square plate from which four identical squares at the corners will be removed.*** *(answer all in terms of quadrants,*

a) Where is the COM of the plate originally?

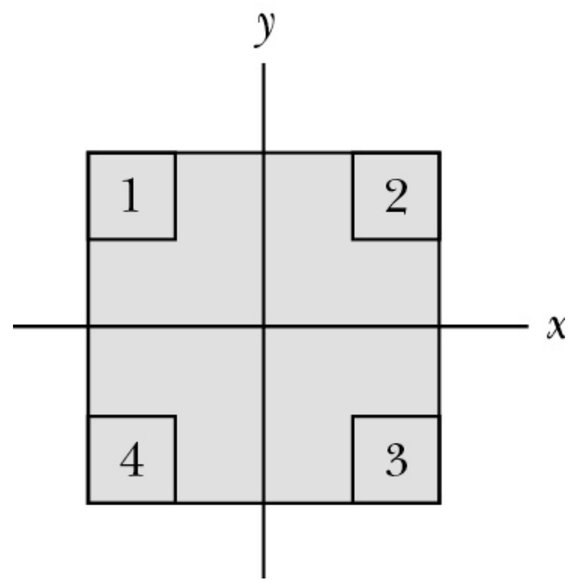
b) Where is the COM after removal of square 1?

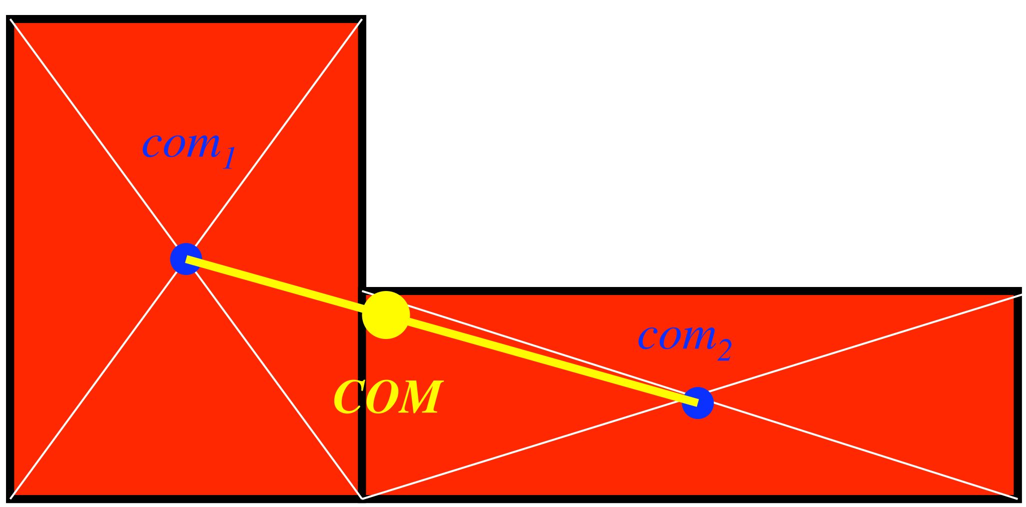
c) Where is the COM after removal of square 1 & 2?

d) Where is the COM after removal of square 1, 2 ,& 3?

e) Where is the COM after removal of all four squares?

e) Where is the COM after removal of all squares?





Checkpoint

CheckPoint #1: ***The figure shows a uniform square plate from which four identical squares at the corners will be removed.*** *(answer all in terms of quadrants, axes, or points)*

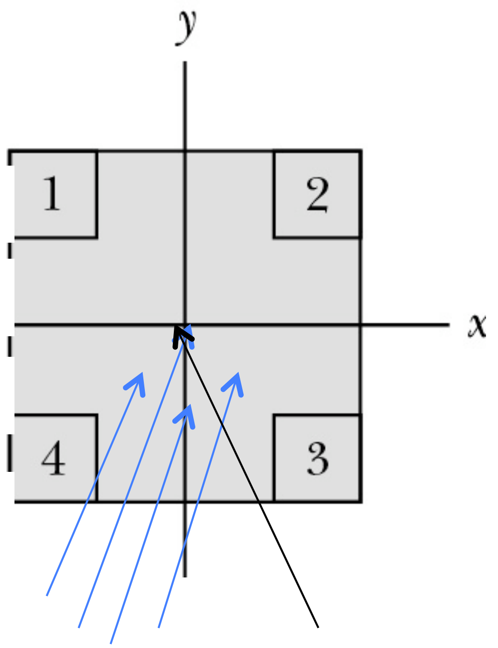
a) *Where is the COM of the plate originally?*

b) *Where is the COM after removal of square 1?*

c) *Where is the COM after removal of square 1 & 2?*

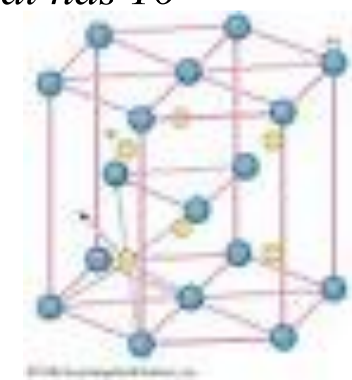
d) *Where is the COM after removal of square 1, 2 ,& 3?*

e) *Where is the COM after removal of all four squares?*



COM: Solid Bodies

What happens if N gets VERY large? For example a kilogram of material has 10^{26} atoms. Counting is impossible….



Treat as a continuum of material…

$$x_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i \longrightarrow x_{com} = \frac{1}{M}\int x\,dm \longrightarrow x_{com} = \frac{1}{V}\int x\,dV$$

$$y_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i \longrightarrow y_{com} = \frac{1}{M}\int y\,dm$$

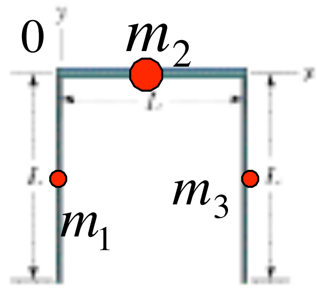
$$z_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i \longrightarrow z_{com} = \frac{1}{M}\int z\,dm$$

$$M = \sum_{i=1}^{N} m_i \longrightarrow \rho = \frac{dm}{dV} = \frac{M}{V}$$

Here "mass density" replaces mass

If the figure below, three uniform thin rods, each of length L = 18 cm, form an inverted U. The vertical rods each have mass of 14 g; the horizontal rod has mass of 44 g. Where is the center of mass of the assembly (x, y)?

(____ , ____) cm



$$\text{From symmetry} \Rightarrow x_{cm} = \frac{L}{2} = 9cm$$

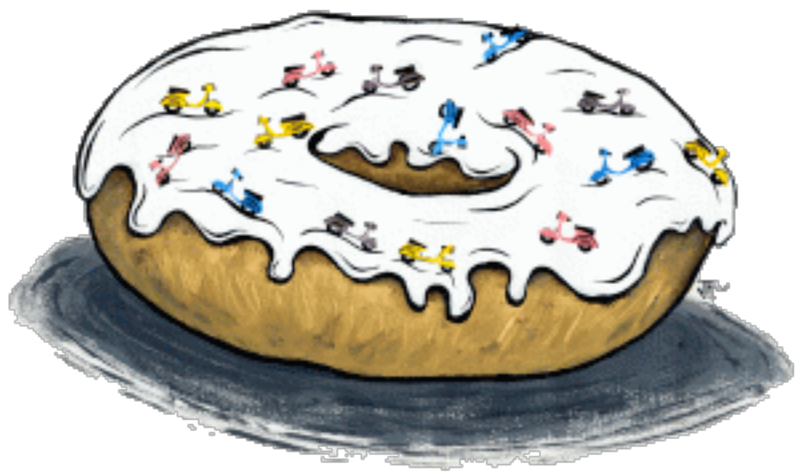
$$y_{cm} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3}{m_1 + m_2 + m_3}$$

$$= \frac{(-\frac{L}{2}) \times 14 + (0) \times 44 + \left(-\frac{L}{2}\right) \times 14}{14 + 44 + 14} = -3.5cm$$

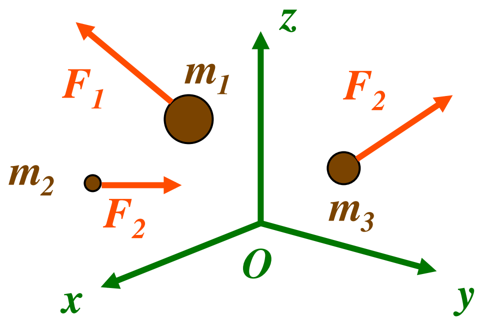
Rules of Center of Mass

(1) Center of mass of a symmetric object always lies on an axis of symmetry.

(2) Center of mass of an object does NOT need to be on the object.



Newton's 2nd Law for a System of Particles



Consider a system of n particles of masses $m_1, m_2, m_3, ..., m_n$ and position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, ..., \vec{r}_n$, respectively.
The position vector of the center of mass is given by

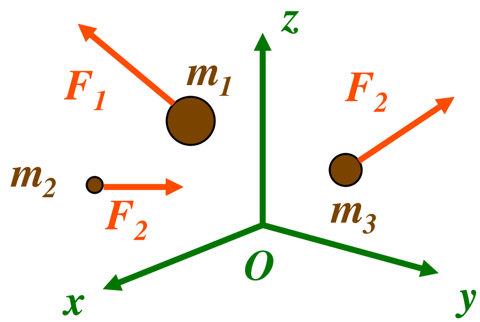
$M\vec{r}_{\text{com}} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ... + m_n\vec{r}_n$. We take the time derivative of both sides $\rightarrow$

$$M\frac{d}{dt}\vec{r}_{\text{com}} = m_1\frac{d}{dt}\vec{r}_1 + m_2\frac{d}{dt}\vec{r}_2 + m_3\frac{d}{dt}\vec{r}_3 + ... + m_n\frac{d}{dt}\vec{r}_n \rightarrow$$

$M\vec{v}_{\text{com}} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ... + m_n\vec{v}_n$. Here $\vec{v}_{\text{com}}$ is the velocity of the com and $\vec{v}_i$ is the velocity of the ith particle. We take the time derivative once more $\rightarrow$

$$M\frac{d}{dt}\vec{v}_{\text{com}} = m_1\frac{d}{dt}\vec{v}_1 + m_2\frac{d}{dt}\vec{v}_2 + m_3\frac{d}{dt}\vec{v}_3 + ... + m_n\frac{d}{dt}\vec{v}_n \rightarrow$$

$M\vec{a}_{\text{com}} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ... + m_n\vec{a}_n$. Here $\vec{a}_{\text{com}}$ is the acceleration of the com and $\vec{a}_i$ is the acceleration of the ith particle.



$$M\vec{a}_{\text{com}} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots + m_n\vec{a}_n.$$

We apply Newton's second law for the ith particle:

$m_i\vec{a}_i = \vec{F}_i$. Here $\vec{F}_i$ is the net force on the ith particle,

$$M\vec{a}_{\text{com}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots + \vec{F}_n.$$

The force $\vec{F}_i$ can be decomposed into two components: applied and internal:

$\vec{F}_i = \vec{F}_i^{\text{app}} + \vec{F}_i^{\text{int}}$. The above equation takes the form:

$$M\vec{a}_{\text{com}} = \left(\vec{F}_1^{\text{app}} + \vec{F}_1^{\text{int}}\right) + \left(\vec{F}_2^{\text{app}} + \vec{F}_2^{\text{int}}\right) + \left(\vec{F}_3^{\text{app}} + \vec{F}_3^{\text{int}}\right) + \ldots + \left(\vec{F}_n^{\text{app}} + \vec{F}_n^{\text{int}}\right) \rightarrow$$

$$M\vec{a}_{\text{com}} = \left(\vec{F}_1^{\text{app}} + \vec{F}_2^{\text{app}} + \vec{F}_3^{\text{app}} + \ldots + \vec{F}_n^{\text{app}}\right) + \left(\vec{F}_1^{\text{int}} + \vec{F}_2^{\text{int}} + \vec{F}_3^{\text{int}} + \ldots + \vec{F}_n^{\text{int}}\right)$$

The sum in the first set of parentheses on the RHS of the equation above is just $\vec{F}_{\text{net}}$.

The sum in the second set of parentheses on the RHS vanishes by virtue of Newton's third law.

The equation of motion for the center of mass becomes $M\vec{a}_{\text{com}} = \vec{F}_{\text{net}}$.

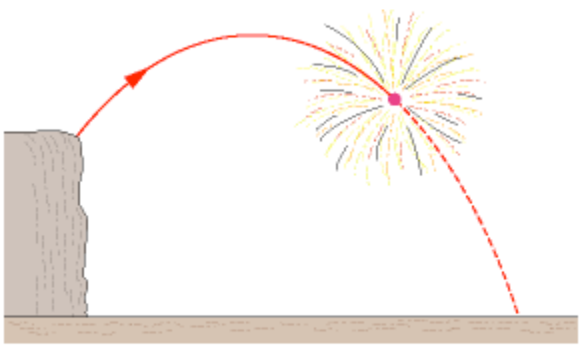
In terms of components we have:

$$F_{\text{net},x} = Ma_{\text{com},x} \qquad F_{\text{net},y} = Ma_{\text{com},y} \qquad F_{\text{net},z} = Ma_{\text{com},z}$$

Newton's law

- For a single particle, $\vec{F} = m\vec{a}$
- For each particle in a system of particles, $\vec{F}_i = m_i\vec{a}_i$
- For a system of particles, $\vec{F}_{ext} = M\vec{a}_{com}$

Net forces!



***Internal forces** that cancel each other:*

- *tension;*
- *explosions;*
- *electromagnetic attractions or repulsions,…*

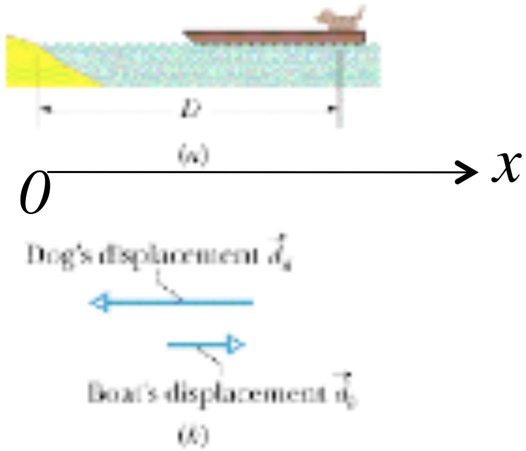
Checkpoint

Two skaters on frictionless ice hold opposite ends of a pole of negligible mass. An axis runs along the pole, and the origin of the axis is at the COM. On skater, Fred, weighs twice as much as the other skater, Ethel. Where do the skaters meet if (a) Fred pulls hand over hand along the pole so as to to draw himself to Ethel, (b) Ethel pulls hand over hand to draw herslef to Fred, and (c) both skaters pull hand over hand?

- Center of Mass

In figure (*a*), a 4.1 kg dog stands on a 21 kg flatboat at distance $D = 6.1$ m from the shore. It walks 2.0 m along the boat toward shore and then stops. Assuming no friction between the boat and the water, find how far the dog is then from the shore.

m



Step I: choose a system $\Rightarrow boat(m_b) + dog(m_d)$

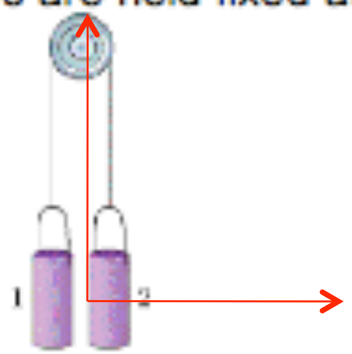
Step II: Find net force $F^x_{net} = 0 \Rightarrow$ COM remains unchanged

Step III: choose coordinating system

Step IV:

$$x_{cm} = \frac{\left(D - \frac{L_b}{2}\right)m_b + Dm_d}{m_b + m_d} = \frac{\left(x + 2 - \frac{L_b}{2}\right)m_b + xm_d}{m_b + m_d}$$

In the figure below, two identical containers of sugar are connected by a cord that passes over a frictionless pulley. The cord and pulley have negligible mass, each container and its sugar together have a mass of 780 g, the centers of the containers are separated by 50 mm, and the containers are held fixed at the same height.



a) *Where is the center of mass?*
b) *Transfer 20 g from one to other--CM now?*
c) *Release--How does CM move?*
d) *What is the acceleration?*

(a) $x_{cm} = 0, \quad y_{cm} = 0$

(b) Assume part of 1 transferred to 2 $x_{cm} = \dfrac{(-25) \times 760 + 25 \times 800}{760 + 800}$ $y_{cm} = 0$

(c) $y_{cm} < 0$

(d) $F_y = -m_1 g - m_2 g = (m_1 + m_2) a_{cm}$