Chapter 7: Kinetic Energy and Work

Different energies:

- Kinetic/translation
- Gravitational potential
- Heat energy
- Electromagnetic energy
- Strain or elastic energy

Each energy is associated with a **"scalar"** which defines a state of a system at a given time.

Kinetic Energy

Kinetic Energy is associated with the state of motion

$$KE = \frac{1}{2}mv^2$$

Units of Joules: 1 J = kg·m^2/s^2

- KE depends on speed (v) not $\vec{v}$ (here $v^2 = \vec{v} \bullet \vec{v} = |\vec{v}|^2$)
- KE doesn't depend on which way something is moving or even if it's changing direction
- KE is ALWAYS a positive scalar

How much is "Kinetic Energy"



1) Electron (e-) moving in Copper

$m_e = 9.11\times10^{-31}$ kg and v ~ 1×10^6 m/s KE = 7×10^{-19} J (~ 4 eV)

2) Bullet traveling at 950 m/s (3100 ft/s).

m = 4.2 g and v ~ 950 m/s (3100 ft/s) KE = 2000 J

3) Football Linebacker

m = 240 lbs and v ~ 18 mph (7 m/s)__KE = 2800 J

4) Aircraft Carrier Nimitz

m = 91,400 tons and v ~ 1 knot___________KE = 10 MJ

Work

If you apply a net force (~a↑ → v↑), KE ↑ and if you decelerate (a↓ → v ↓), KE ↓

Somehow force is related to KE energy…

If we transfer energy via a <u>force</u>, this is work.

"Doing work" is the act of transferring energy.

Work (W) is said to be done <u>on</u> an object by a force.

Energy transferred <u>to</u> an object is <u>positive</u> work.

Energy transferred <u>from</u> an object is <u>negative</u> work.

Work and Kinetic Energy

How to find an "alternate form" of Newton's 2nd Law that relates position and velocity.??

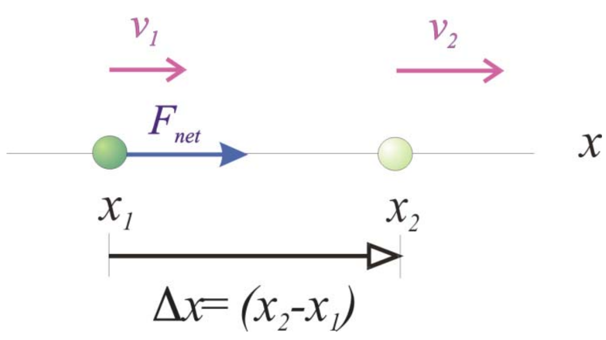
Start in 1-D (e.g. Bead along wire $\hat{x}$), we know …

$$F_{x,net} = ma_x = m\frac{dv}{dt}$$

$$\left(F_{x,net}\right)\left(\frac{dx}{dt}\right) = \left(m\frac{dv}{dt}\right)\left(\frac{dx}{dt}\right) = mv\frac{dv}{dt}$$

$$F_{x,net}\frac{dx}{dt} = \frac{d\left(\frac{1}{2}mv^2\right)}{dt}$$

$$F_{x,net}dx = d\left(\tfrac{1}{2}mv^2\right)$$



$$\int F_{x,net}dx = \int d\left(\tfrac{1}{2}mv^2\right) = KE_2 - KE_1$$

$$W_{net} \equiv \int F_{x,net}dx = KE_2 - KE_1 = \Delta KE$$

$$W_{net} = \Delta KE$$

Work-Kinetic Energy Theorem

Work-Kinetic Energy Theorem $\rightarrow$ **(change in the kinetic energy of an object) = (net work done on the particle)**

If $\vec{F}_{net}$ is not a function of $\vec{x}$, then

$$W_{net} \equiv \vec{F}_{net} \bullet \vec{d} = \Delta KE$$

No work is done on an object by a force unless there is a component of the force along the objects line of motion.

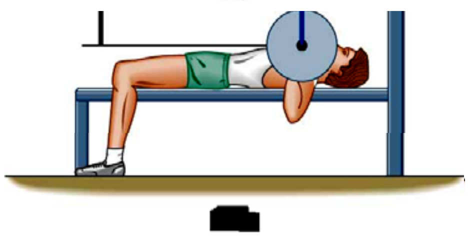
Question

Qualitatively, how much work am I doing on the bowling ball if I walk across the room with a constant velocity?

1. Quite a lot
2. $\boxed{\text{None}}$

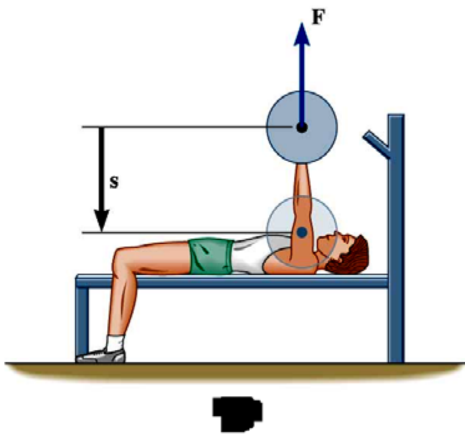
Positive and Negative Work

(a)



Weight lifting: apply a FORCE up and DISPLACE the barbell up…

both the force and displacement are in the +y direction so work is **<u>positive</u>**

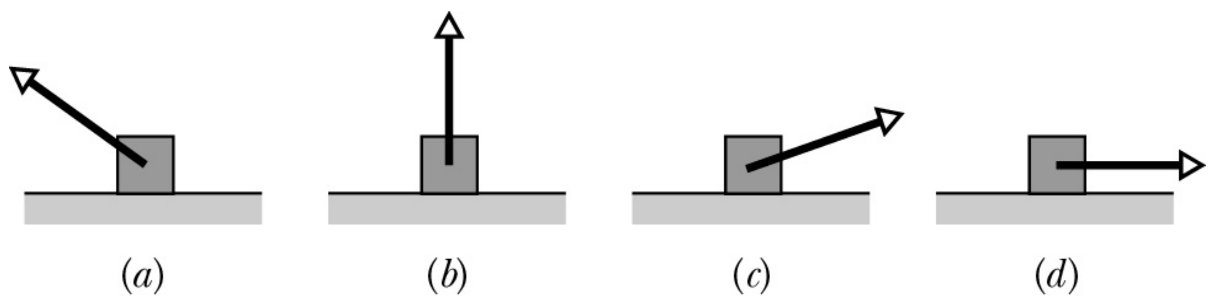


On the downward motion the FORCE is still up and

the force is in the +y but the displacement is in –y direction so work is **<u>negative</u>**

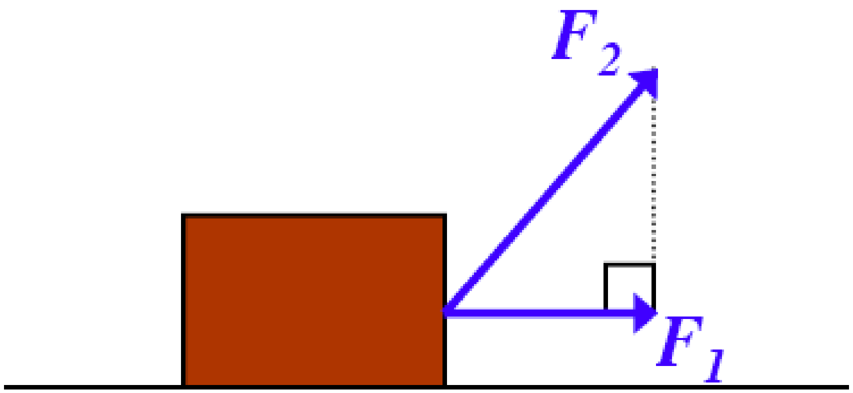
<u>External Force</u> acts on box moving rightward a distance d.

Rank: Work done on box by F



Question

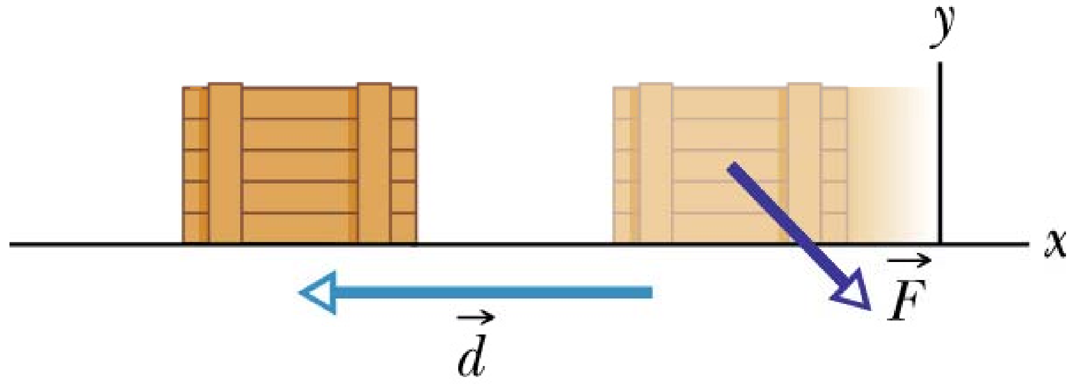
Two forces act on the box shown in the drawing, causing it to move across the floor. The two force vectors are drawn to scale. Which force does more work on the box?



1. F_1
2. F_2
3. *They're both zero ($F_1 = F_2 = 0$)*
4. *They're the same, but not zero ($F_1 = F_2 \neq 0$)*

Example

During a storm, a crate is sliding across a slick, oily parking lot through a displacement $\vec{d} = (-3.0m)\hat{i}$ while a steady wind pushes against the crate with a force $\vec{F} = (2.0N)\hat{i} + (-6.0N)\hat{j}$



(a) How much work does the force from the wind do on the crate during the displacement?

$$W_{net} \equiv \vec{F}_{net} \cdot \vec{d}$$

Negative Work

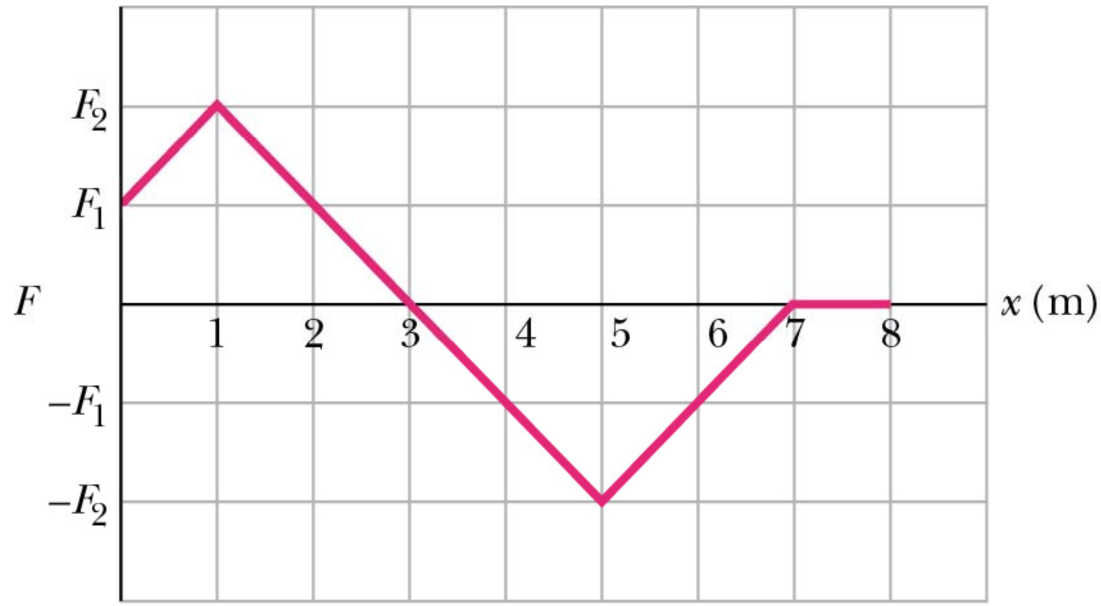
Energy transferred from object

KE decreases

(b) If the crate has a kinetic energy of 10 J at the beginning of displacement $\vec{d}$, what is the kinetic energy at the end of $\vec{d}$?

$$W_{net} = \Delta KE = (KE_f - KE_i) \qquad \text{or} \qquad KE_f = W_{net} + KE_i$$

Example



Question 7-4: The figure shows the values of a force **F**, directed along an **x** axis, that will act on a particle at the corresponding values of x. If the particle begins at rest a x=0, what is the particle's coordinate when it has

(a) the greatest speed

$$W_{net} \equiv \int \vec{F}_{net} \bullet d\vec{x} = \Delta KE = KE_f - KE_i$$

Area under F-x plot is W.

(b) the minimum speed

Question: The figure shows the values of a force **F**, directed along an **x** axis, that will act on a particle at the corresponding values of x. If the particle begins at rest a x=0, what is the particle's coordinate when it has

(a) the greatest speed

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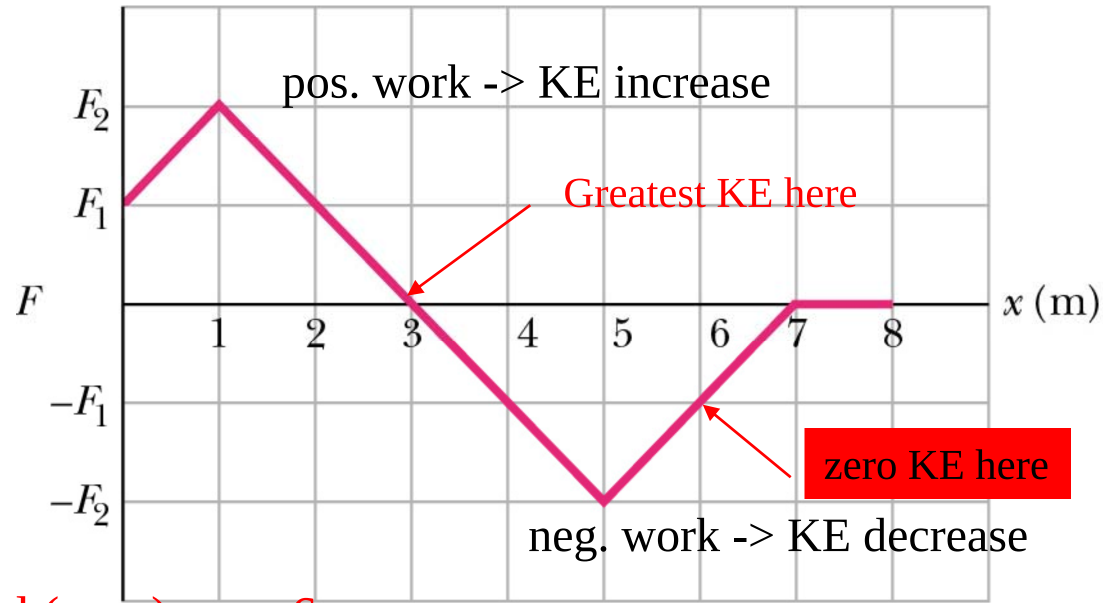


Greatest speed at x=3 m Minimum speed (zero) at x=6 m

Work

$$\sum_{all} W_i = W_{net} \equiv \vec{F}_{net} \cdot \vec{d} = \Delta KE = KE_f - KE_i$$

$$\sum_{all} W_i = W_{net}$$

Work is sum total of all transfer of energy (pos and neg)

$$W_{net} \equiv \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{net}(r) \cdot d\vec{r} = \vec{F}_{net} \cdot \vec{d} \qquad \text{where} \quad (\vec{r}_2 - \vec{r}_1) = \vec{d}$$

- No work is done on an object by a force unless there is a component of the force along the objects line of motion
- Sum of the area under net F-r plot yields net work

$$W_{net} = \Delta KE$$

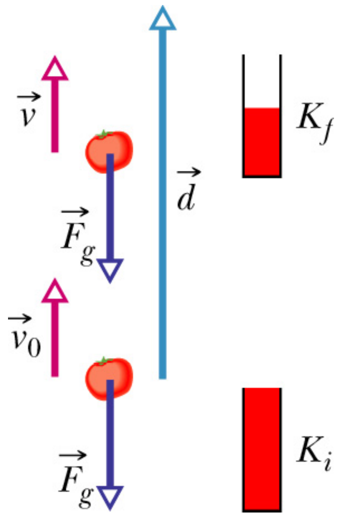
- If speed goes up then W is positive -- energy transferred to object
- If speed goes down then W is negative -- energy transferred from object
- If speed is constant then NO **NET** WORK IS DONE

$$\Delta KE = KE_f - KE_i$$

Change in kinetic energy is change in speed (i.e. v^2)

What about centripetal force?

Special Case: Work done by Constant Gravitational Force



$$W_g \equiv \int \vec{F}_g \bullet d\vec{x} = \vec{F}_g \bullet \vec{d}$$

$$where \quad F_g = (mg)\left(-\hat{j}\right) \quad and \quad g = 9.81 m/s^2$$

If an object is displaced <u>upward</u> (Δ y positive), then the work done by the **gravitational force** on the object is <u>negative</u>.

If an object is displaced <u>downward</u> (Δy negative), then the work done by the **gravitational force** on the object is <u>positive</u>.

What is the change in KE due to Gravitational Force

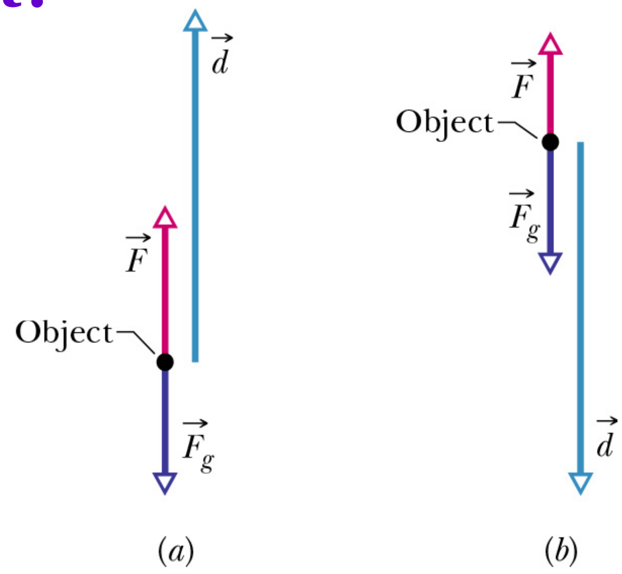
If the only force acting on an object is Gravitational Force then,

$$W_{net} = \left(W_g = \vec{F}_g \bullet \vec{d}\right) = \Delta KE = \left(KE_f - KE_i\right)$$

If an object is displaced <u>upward</u> (Δ y positive), the change in Kinetic Energy is <u>negative</u> (it slows down).

If an object is displaced <u>downward</u> (Δy negative), the change in Kinetic Energy is <u>positive</u> (it speeds up).

What work is needed to lift or lower an object?



In order to "lift" an object, we must apply an external force to counteract the gravitational force.

$$W_{net} \equiv W_g + W_{ext} = \Delta KE$$

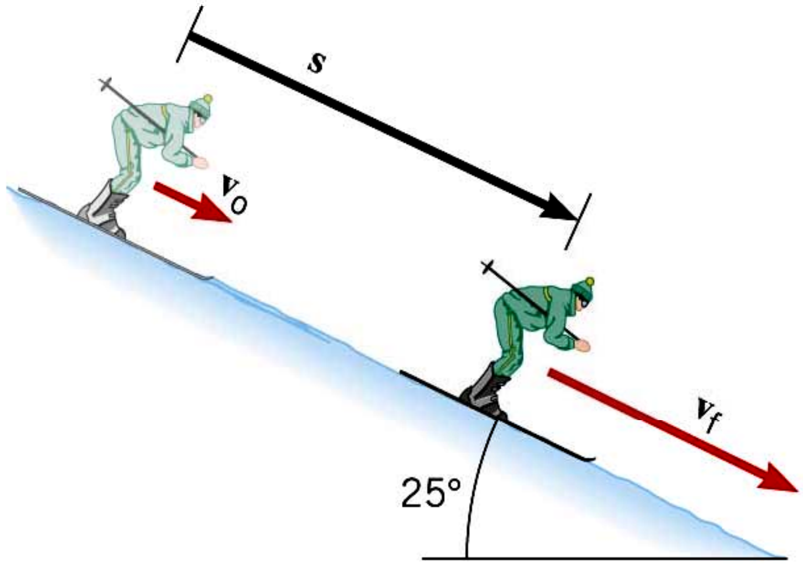
If $\Delta KE = 0$ (i.e. $v_f = v_i$), then $W_g = -W_{ext}$

If an object is displaced <u>upward</u> (Δ y positive), then the work done by the **External force** on the object is <u>positive</u>.

If an object is displaced <u>downward</u> (Δy negative), then the work done by the **External force** on the object is <u>negative</u>.

Example

Downhill Skier



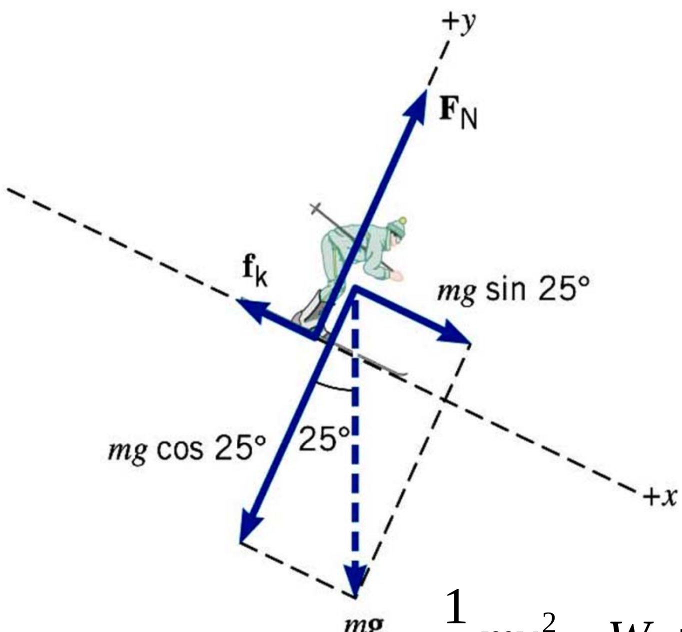
A 58-kg skier coasts down a 25° slope where the kinetic frictional force is f_k=70N. If the starting speed is v_o=3.6 m/s then what is the speed after a displacement of 57 m?

Along the direction of motion (+x) the forces are:

$$\sum F = mg\sin 25° - f_k$$

$$= 240N - 70N$$

$$= 170N$$



The work this force does is:

$$W = F \cdot s = (170N)(57m) = 9700J$$

This gives a change in kinetic energy that is:

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$\frac{1}{2}mv^2 = W + \frac{1}{2}mv_0^2 = 9700J + \frac{1}{2}(58kg)(3.6m/s)^2$$

$$= 10100J$$

$$v = \sqrt{\frac{2(10100J)}{58kg}}$$

$$= 19m/s$$

Special Case:

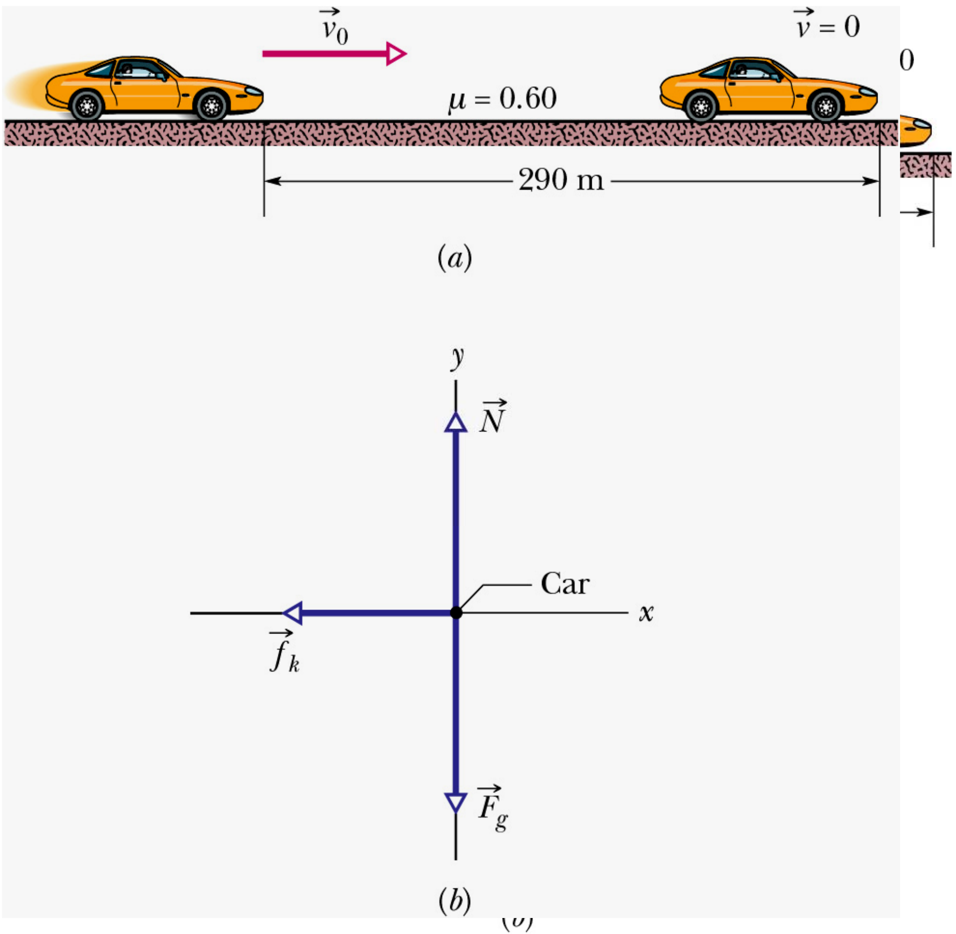
Work due to Friction:

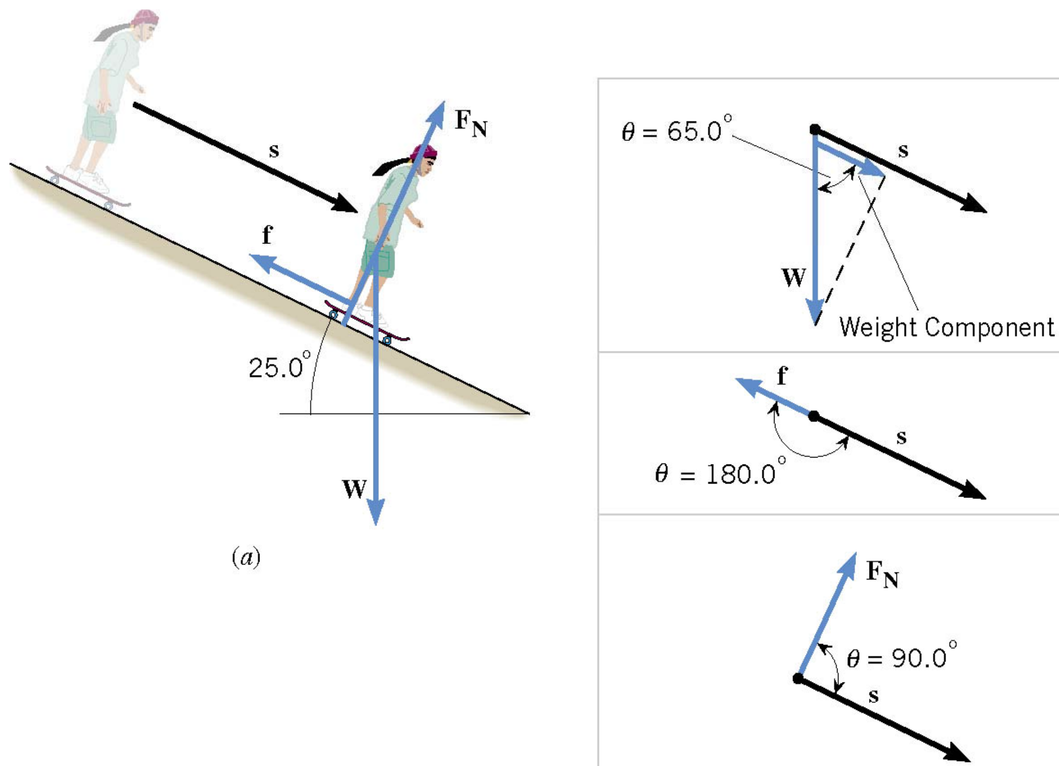
WHY?

WORK due to friction is ALWAYS NEGATIVE

- Energy is transferred OUT
- Kinetic energy decreases or $\Delta KE < 0$ (slow down)

Where did the energy go? THERMAL/Sound





(a) (b)