Announcements:

• 1st test at Lockett #6 (6:00PM, Feb 3rd)
• Formula sheet will be provided
• No other materials is needed
• Practice exam and answers are on the web
Uniform circular motion: Review

As you go around a circle, the velocity constantly changes direction... in order to change velocity, there must be an acceleration

A centripetal acceleration changes the direction of the object's velocity without changing the object's speed
What provides “force” for centripetal acceleration?

A centripetal force accelerates a body by changing the direction of the body’s velocity without changing the body’s speed

\[ F_{\text{cent}} = ma_{\text{cent}} = \frac{mv^2}{r} \]

- because \( v \) and \( r \) are constant, magnitude of \( F_{\text{cent}} \) (\& \( a_{\text{cent}} \)) is constant.
- direction of \( F_{\text{cent}} \) (\& \( a_{\text{cent}} \)) towards center
  (constantly changing direction!!)

- tension (points along the direction of string/rope)
- friction (points parallel to surface - opposes motion)
- gravity (points downward with magnitude \( g \))
- normal (points perpendicular to surface)
Uniform circular motion

From before: \( a_{\text{cent}} = \frac{v^2}{r} \) centripetal acceleration

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- direction of \( F_{\text{cent}} \) (\& \( a_{\text{cent}} \)) towards center (constantly changing direction!!)

hockey puck on ice attached to string
what provides the centripetal force?
Answer: tension in string

\[ T = ma_{\text{cent}} = m \left( \frac{v^2}{r} \right) \]
Rotations on a table

A mass $M$ is attached to a string that passes through a hole in the center of a frictionless table. Attached to the other end of the string is a mass $m$ moving in circular motion centered on the hole and a distance $r$ from it.

(a) What velocity must the mass $m$ have in order for mass $M$ to remain at a constant height?

(b) What is the period of the mass $m$?
Turning a Corner

You turn a corner with your car --> How do you do that? (your speed and radius of motion are constant)

Friction provides the centripetal force:

\[ \sum F_x = -f_s = -ma_x = m\left( -\frac{v^2}{R} \right) \]

\[ \sum F_y = F_N - mg = ma_y = 0 \]

If we use \( f_s^{\text{max}} \) then we can extract the maximum speed we can go:

\[ m\left( \frac{v^2}{R} \right) = f_s^{\text{max}} = \mu_s F_N \]

\[ v^{\text{max}} = \sqrt{\mu_s gR} \]

what friction is needed for the car not to slip?

\[ \mu_s > \frac{v^2}{gR} \]

\[ \mu_s < \frac{v^2}{gR} \]
Can you guarantee NO SLIP? If $\mu_s = 0$ ... BANK THE CURVE

What angle $\theta$ is needed to guarantee no-slip driving (here $\mu_s = 0$)?

$\hat{x}: \sum F_x = F_N \sin \theta = m a_x = m \left( \frac{v^2}{R} \right)$

$\hat{y}: \sum F_y = F_N \cos \theta - mg = m a_y = 0$

$\rightarrow F_N \sin \theta = m \left( \frac{v^2}{R} \right)$

$\rightarrow F_N \cos \theta = mg$

$\theta = \tan^{-1} \left( \frac{v^2}{gR} \right)$

So for a 35 mph (15.6 m/s) “suggested” speed with a 50 m radius-of-curvature exit ramp gives an angle of $\theta = 26.5^\circ$
Sample problem 6-8: The Rotor is a large hollow cylinder of radius $R$ that is rotated rapidly around its central axis with a speed $v$. A rider of mass $m$ stands on the Rotor floor with his/her back against the Rotor wall. Cylinder and rider begin to turn. When the speed $v$ reaches some predetermined value, the Rotor floor abruptly falls away. The rider does not fall but instead remains pinned against the Rotor wall. The coefficient of static friction $\mu_s$ between the Rotor wall and the rider is given.

We draw a free-body diagram for the rider using the axes shown in the figure. The normal reaction $F_N$ is the centripetal force.

\[
F_{x,\text{net}} = F_N = ma = \frac{mv^2}{R} \quad \text{(eq. 1)}
\]

\[
F_{y,\text{net}} = f_s - mg = 0, \quad f_s = \mu_s F_N \rightarrow mg = \mu_s F_N \quad \text{(eqs. 2)}
\]

If we combine eq. 1 and eqs. 2 we get: $mg = \mu_s \frac{mv^2}{R} \rightarrow v^2 = \frac{Rg}{\mu_s} \rightarrow v_{\text{min}} = \sqrt{\frac{Rg}{\mu_s}}$. 
Vertical Circular Motion

What happens in vertical circular motion? What is going on with the centripetal acceleration?

There must always be a centripetal acceleration pointed toward the axis of rotation.

In a cyclist doing a loop-de-loop, the SUM OF FORCES provides the $a_c$.

The two forces that provide $a_c$ include the WEIGHT and the NORMAL FORCE. Since the $F_N$ is always $\perp$ to the surface, and $W$ points down, the magnitude of $F_N$ varies around the loop.

The same thing applies if you are swinging a pail of water holding on to the handle… Your TENSION $T$ will vary around the orbit.
One last “circular” motion problem

What **minimum velocity** *constant* must the car have at the top of the loop to remain on the track?

Use Newton’s 2nd Law knowing the forces at the top of the loop

\[ \hat{y}: \quad -N - F_g = m \left( -\frac{v^2}{r} \right) \]

\[ F_N = N \]

If car “just remains on track”, \( N \to 0 \) or

\[ \hat{y}: -F_g = -(mg) = m \left( -\frac{v^2}{r} \right) \quad \text{or} \quad v = \sqrt{gr} \]

so minimum velocity is

\[ v_{\text{min}} = \sqrt{gr} \]

If constant \( v \), how does \( N \) change around loop?

**At TOP**  \( \Rightarrow \hat{y}: \quad N = m \left( \frac{v^2}{r} \right) - mg \)

\( N \) small

**At SIDES**  \( \Rightarrow \hat{x}: \quad \pm N = m \left( \pm \frac{v^2}{r} \right) \)

**At BOTTOM**  \( \Rightarrow \hat{y}: \quad N = m \left( \frac{v^2}{r} \right) + mg \)

\( N \) large
Swing a pail of Water over your Head

Why doesn’t it fall out? What does a free body diagram look like for this?

If the pail isn’t moving, water falls out on your head

When you swing it around, you apply an additional tension T, and when the pail is above your head, the free body diagram here is complete.

WHY DOESN’T IT FALL ON YOUR HEAD??

These forces provide the centripetal acceleration for the circular motion… the force equation is:

\[ m \frac{v^2}{r} = T + W \]

The slowest speed you can swing it is the point where T = 0 (this is tension, not period!). Then you must have a velocity that is:

\[ m \frac{v^2}{r} = mg \]

an “arm” of 60 cm gives a period of the “orbit”

\[ v^2 = rg \quad v = \sqrt{(0.6m)(9.8 \frac{m}{s^2})} = 2.4 \frac{m}{s} \quad T = \frac{2\pi r}{v} = 1.5s \]