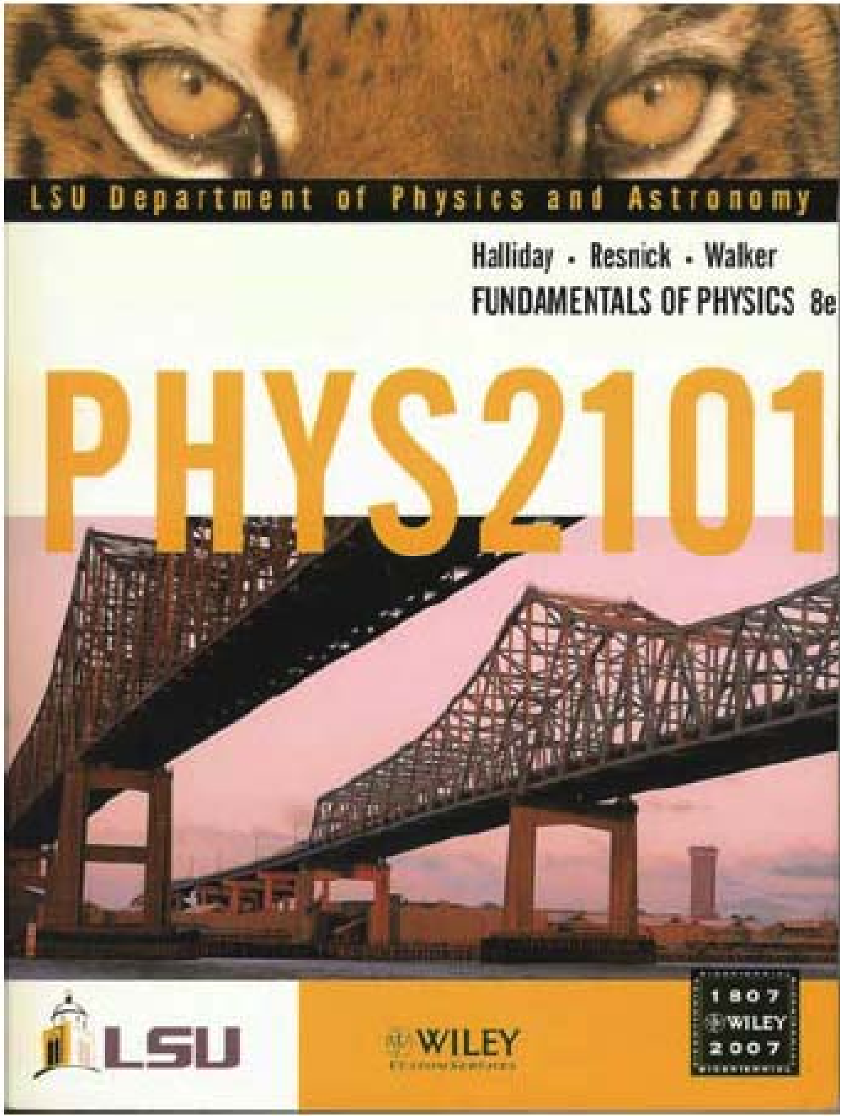


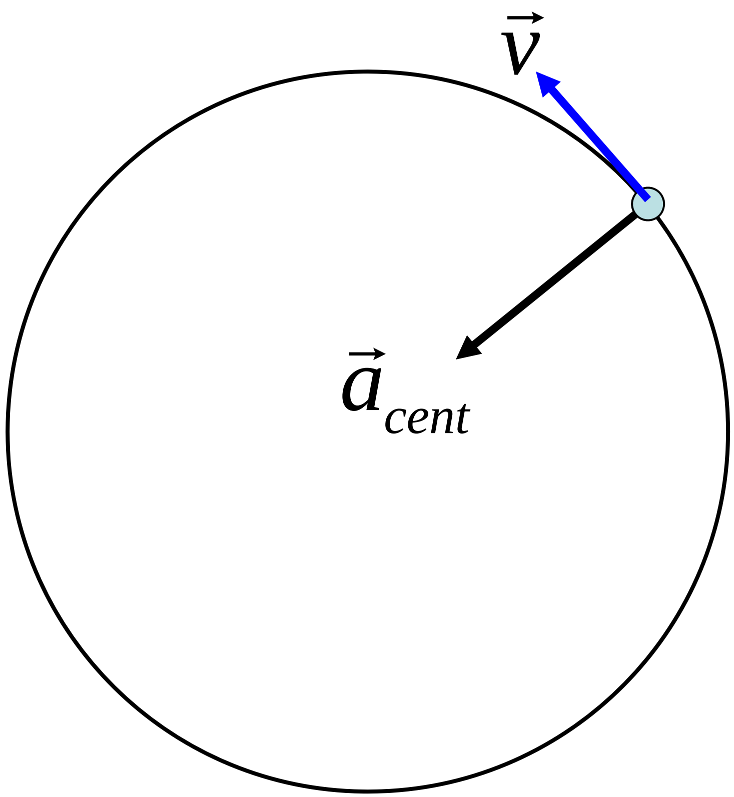
Announcements:

- 1^{st} –test at Lockett #6 (6:00PM, Feb 3^{rd})
- Formula sheet will be provided
- No other materials is needed
- Practice exam and answers are on the web

Uniform circular motion: Review

As you go around a circle, the velocity constantly changes direction… in order to change velocity, there must be an acceleration

A <u>centripetal acceleration</u> changes the direction of the object's velocity without changing the object's speed



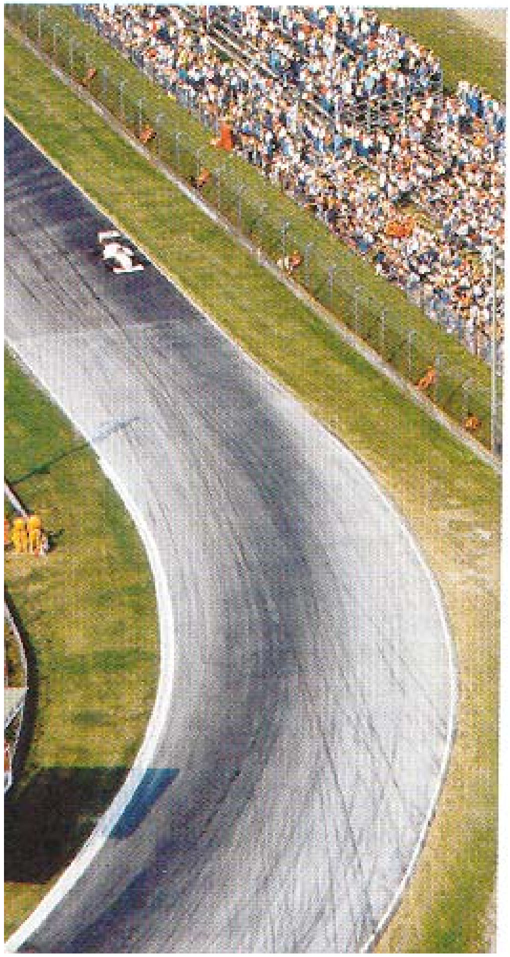
What provides "force" for centripetal acceleration?

A <u>centripetal force</u> accelerates a body by changing the direction of the body's velocity without changing the body's speed

$$F_{cent} = ma_{cent} = \frac{mv^2}{r}$$

- because v and r are constant, magnitude of F_{cent} (& a_{cent}) is constant.
- direction of F_{cent} (& a_{cent}) towards center

(constantly changing direction!!)



- tension (points along the direction of string/rope)
-friction (points parallel to surface - opposes motion)

- gravity (points downward with magnitude g)
- normal (points perpendicular to surface)

Uniform circular motion

From before: $a_{cent} = \dfrac{v^2}{r}$ centripetal acceleration

A centripetal force accelerates a body by changing the direction of the body's velocity without changing the body's speed

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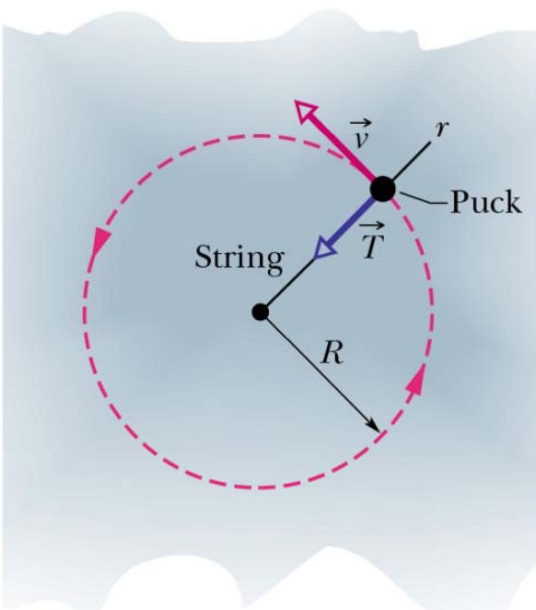
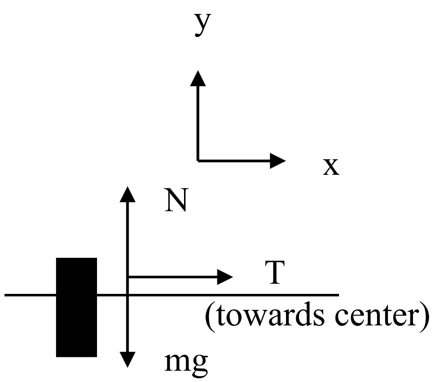


hockey puck on ice attached to string

what provides the centripetal force?

Answer: tension in string

$$T = ma_{cent} = m\left(\frac{v^2}{r}\right)$$

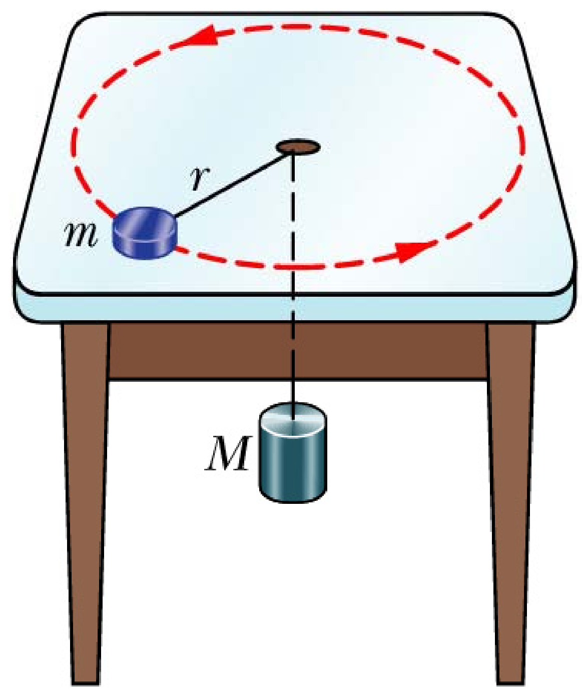


Rotations on a table

A mass M is attached to a string that passes through a hole in the center of a frictionless table. Attached to the other end of the string is a mass m moving in circular motion centered on the hole and a distance r from it.

(a) What velocity must the mass m have in order for mass M to remain at a constant height?

(b)What is the period of the mass m?



Turning a Corner

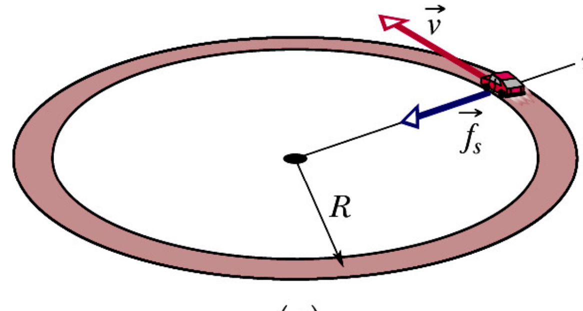
You turn a corner with your car --> How do you do that ?
(your speed and radius of motion are constant)

what friction is needed
for the car not to slip?

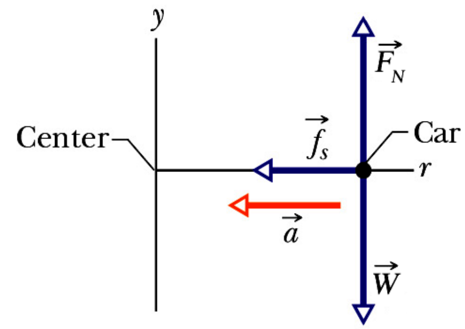
Friction provides the **centripeta**

$$\hat{x}: \quad \sum F_x = -f_S = -ma_x = m\Big(-$$

$$\hat{y}: \quad \sum F_y = F_N - mg = ma_y = 0$$



(a)



$$m\left(\frac{v^2}{R}\right) = f_S$$

If we use $f_S^{\max}$ then we can extrac ... go:

$$m\left(\frac{v^2}{R}\right) = f_S^{\max} = \mu_S F$$

$$v^{\max} = \sqrt{\mu_S g R}$$

what happens if

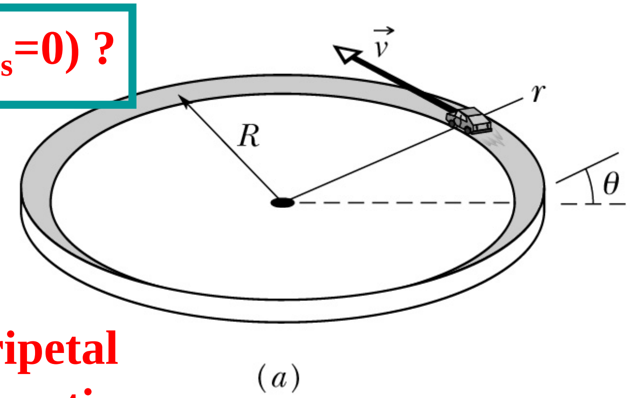
$$\mu_s > \frac{v^2}{gR}$$

$$\mu_s < \frac{v^2}{gR}$$

Can you guarantee NO SLIP? If μ_s=0 … BANK THE CURVE

What angle θ is needed to guarantee no-slip driving (here μ_s=0) ?

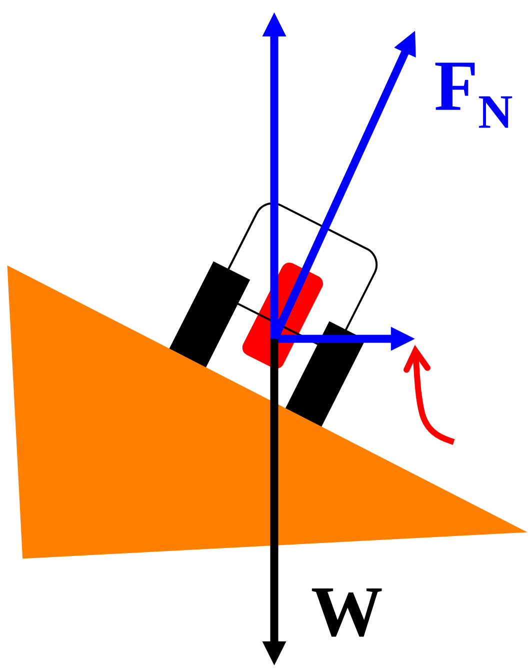
$$\hat{x}: \quad \sum F_x = F_N \sin\theta = ma_x = m\left(\frac{v^2}{R}\right)$$



centripetal acceleration

$$\hat{y}: \quad \sum F_y = F_N \cos\theta - mg = ma_y = 0$$

$$\longrightarrow F_N \sin\theta = m\left(\frac{v^2}{R}\right)$$

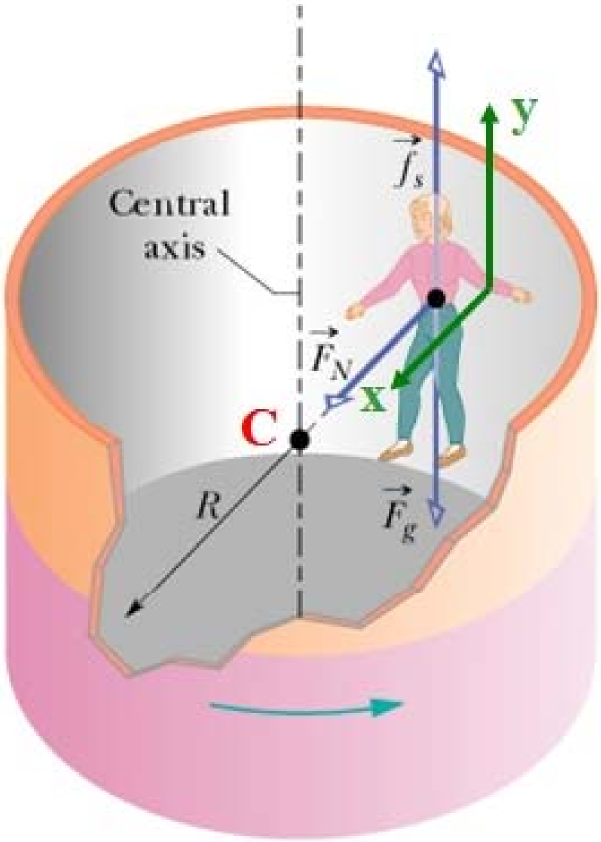


$$\longrightarrow F_N \cos\theta = mg$$

centripetal acceleration

$$\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)$$

So for a 35 mph (15.6 m/s) “suggested” speed with a 50 m radius-of-curvature exit ramp gives an angle of θ = 26.5°



Sample problem 6-8: The Rotor is a large hollow cylinder of radius R that is rotated rapidly around its central axis with a speed v. A rider of mass m stands on the Rotor floor with his/her back against the Rotor wall. Cylinder and rider begin to turn. When the speed v reaches some predetermined value, the Rotor floor abruptly falls away. The rider does not fall but instead remains pinned against the Rotor wall. The coefficient of static friction μ_s between the Rotor wall and the rider is given.

We draw a free-body diagram for the rider using the axes shown in the figure. The normal reaction F_N is the centripetal force.

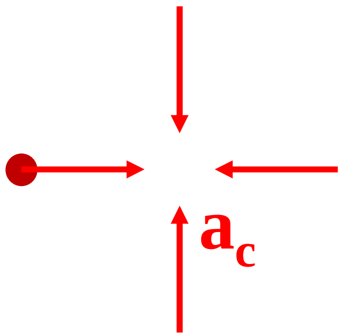
$$F_{x,\text{net}} = F_N = ma = \frac{mv^2}{R} \quad \text{(eq. 1)}$$

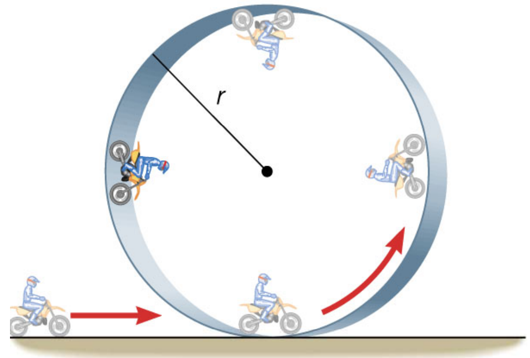
$$F_{y,\text{net}} = f_s - mg = 0, \quad f_s = \mu_s F_N \rightarrow mg = \mu_s F_N \quad \text{(eqs. 2)}$$

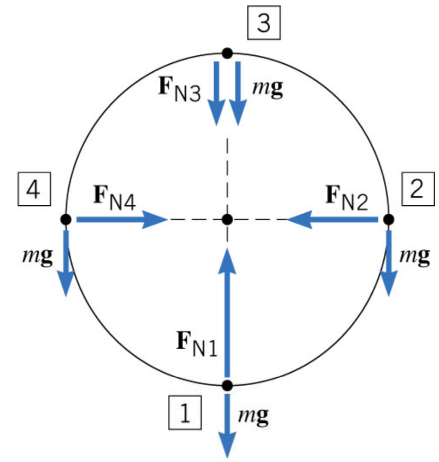
If we combine eq. 1 and eqs. 2 we get: $mg = \mu_s \dfrac{mv^2}{R} \rightarrow v^2 = \dfrac{Rg}{\mu_s} \rightarrow v_{\min} = \sqrt{\dfrac{Rg}{\mu_s}}.$

Vertical Circular Motion

What happens in vertical circular motion? What is going on with the centripetal acceleration?







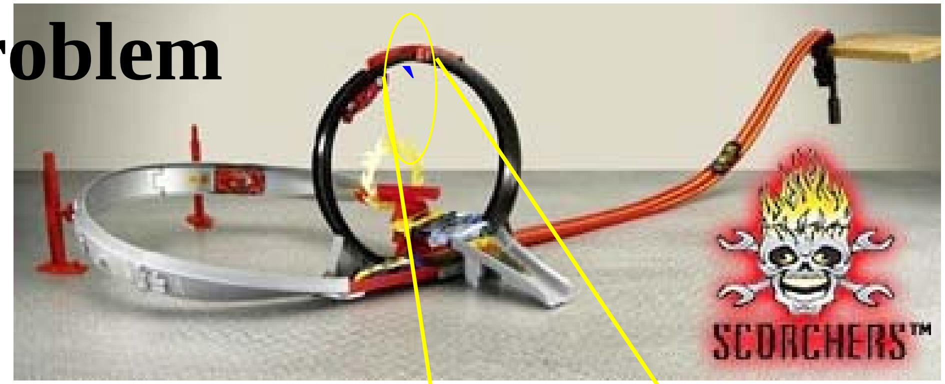
There must always be a centripetal acceleration pointed toward the axis of rotation.

In a cyclist doing a loop-de-loop, the SUM OF FORCES provides the a_c.

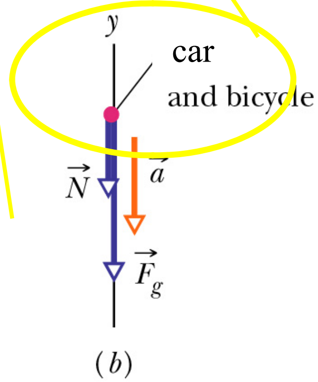
The two forces that provide a_c include the WEIGHT and the NORMAL FORCE. Since the F_N is always ⊥ to the surface, and W points down, the magnitude of F_N varies around the loop.

The same thing applies if you are swinging a pail of water holding on to the handle… Your TENSION T will vary around the orbit.

One last "circular" motion problem



What **minimum velocity** *(constant)* must the car have at the top of the loop to remain on the track?



Use Newton's 2nd Law knowing the forces at the top of the loop

$$\hat{y}: \quad -N - F_g = m\left(-\frac{v^2}{r}\right)$$

$$F_N = N$$

If car "just remains on track", N → 0 or

$$\hat{y}: -F_g = -(mg) = m\left(-\frac{v^2}{r}\right) \quad \text{or} \quad v = \sqrt{gr}$$

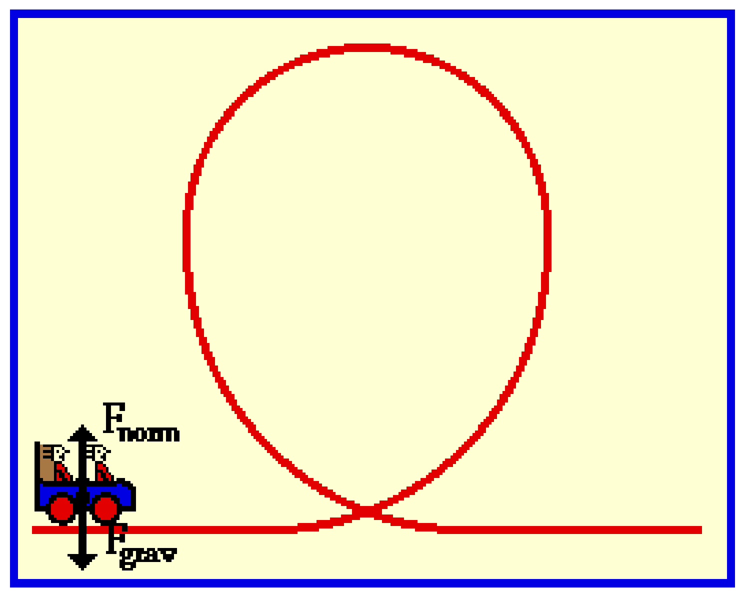
so minimum velocity is $v_{\min} = \sqrt{gr}$

If constant v, how does N change around loop?

At TOP $\Rightarrow \hat{y}: \quad N = m\left(\frac{v^2}{r}\right) - mg$ N small

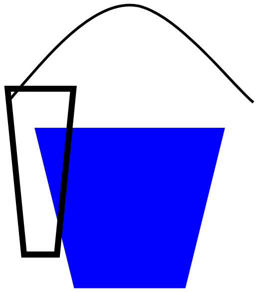
At SIDES $\Rightarrow \hat{x}: \quad \pm N = m\left(\pm\frac{v^2}{r}\right)$

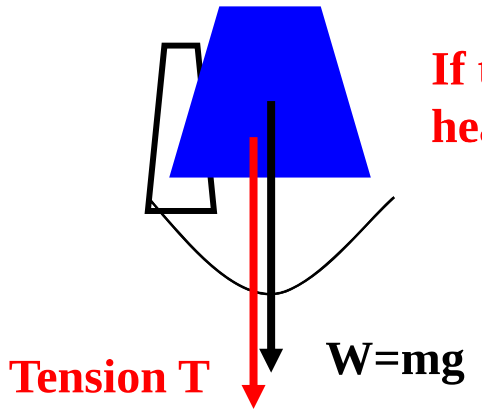
At BOTTOM $\Rightarrow \hat{y}: \quad N = m\left(\frac{v^2}{r}\right) + mg$ N large



Swing a pail of Water over your Head

Why doesn't it fall out? What does a free body diagram look like for this?





If the pail isnt moving, water falls out on your head

When you swing it around, you apply an additional tension T, and when the pail is above your head, the free body diagram here is complete.

WHY DOESN'T IT FALL ON YOUR HEAD??

These forces provide the centripetal acceleration for the circular motion… the force equation is:

$$m\frac{v^2}{r} = T + W$$

The slowest speed you can swing it is the point where T = 0 (this is <u>tension</u>, not period!). Then you must have a velocity that is:

$$m\frac{v^2}{r} = mg$$

an "arm" of 60 cm gives a period of the "orbit"

$$v^2 = rg \qquad v = \sqrt{(0.6m)(9.8\tfrac{m}{s^2})} = 2.4\tfrac{m}{s} \qquad T = \frac{2\pi r}{v} = 1.5s$$