## Projectile Motion y(final) $\neq 0$



## Projectile Motion in general

You separate this into two 1D constant acceleration problems:

The horizontal motion: $a_{x}=0$

$$
\begin{aligned}
& v_{x}=v_{0 x} \\
& x-x_{0}=v_{0 x} t
\end{aligned}
$$

The vertical motion: $a_{y}=-g$
$v_{y}=v_{0 y}-g t$
$y-y_{0}=v_{0 y} t-\frac{g}{2} t^{2}$

$$
v_{y}^{2}-v_{0 y}^{2}=-2 g\left(y-y_{0}\right)
$$

$$
y-y_{0}=\frac{1}{2}\left(v_{0 y}+v_{y}\right) t
$$

$$
y-y_{0}=v_{y} t+\frac{1}{2} g t^{2}
$$

Write down what you know: pick the equations that let you solve the problem.

Problem 26: A stone is projected at a cliff of height $h$ with an initial speed of $42.0 \mathrm{~m} / \mathrm{s}$ directed at an angle $\theta_{0}=60$ degrees. The stone strikes at A, 5.50 s after launching.
Find (a) the height $h$ of the cliff, (b) the speed
 of the stone just before impact at A, and (c) the maximum height $H$.

What do you know? $\mathrm{v}_{0}, \theta_{0}, \mathrm{t}$ :
(a) we solve for $y=h$ :

$$
y-y_{0}=v_{0 y} t-\frac{g}{2} t^{2}
$$

which yields $h=51.8 \mathrm{~m}$ for $y_{0}=0, v_{0}=42.0 \mathrm{~m} / \mathrm{s}, q_{0}=60.0^{\circ}$ and $t=5.50 \mathrm{~s}$.
(b) The horizontal motion is steady, so $v_{x}=v_{0 x}=v_{0} \cos \theta_{0}$, but the vertical component of velocity varies according the equations before. Thus, the speed at impact is

$$
v=\sqrt{\left(v_{0} \cos \theta_{0}\right)^{2}+\left(v_{0} \sin \theta_{0}-g t\right)^{2}}=27.4 \mathrm{~m} / \mathrm{s}
$$

(c) Use $H=\frac{v_{0}^{2} \sin ^{2} \theta_{0}}{2 g}$

Problem 38: You throw a ball toward a wall at a speed of $25.0 \mathrm{~m} / \mathrm{s}$ and at an angle of 40 degrees. The wall is a distance $\mathrm{d}=22.0 \mathrm{~m}$ from the release point of the ball. (a) How far above the release point does the ball hit the wall? (b) What are the horizontal and vertical components of the its
 velocity as it hits the wall? (c) When it hits, has it passed the highest point on its trajectory?
What do you know, $\mathrm{v}_{0}$, the angle, x (final):
First find the time : $t=\frac{d}{v_{x}}=\frac{22 \mathrm{~m}}{25(\mathrm{~m} / \mathrm{s}) \cos 40^{\circ}}=1.15 \mathrm{~s}$
(a) $y-y_{0}=v_{0 y} t-\frac{g}{2} t^{2} \quad H=12 \mathrm{~m}$
(b) $v_{y}=v_{0} \cos 40 \quad v_{y}=v_{0 y}-g t=v_{0} \sin 40-g t \quad v_{y}=4.8 \mathrm{~m} / \mathrm{s}$
(c) The vertical velocity is positive so it hits the wall before max.

## Sample Problem 5-4

A block $S$ of mass $\mathbf{M}$, attached to another block H of mass $\mathbf{m}$ via a rope, is sliding on a frictionless surface.

a) What is the acceleration of block H ?
b) What is the tension in the cord?

1) Draw a free-body diagram showing all forces acting on body and the points at which these forces act.
2) Draw a convenient coordinate system and resolve forces into components.
3) Define direction of acceleration
4) Subdivide into differing systems if needed
5) Solve Newton's 2 nd law (vector) for each system and use equations to find unknowns.

Free Body Diagram:
Two Systems connected by T


System \#1 top S block $\begin{aligned} & \sum F_{x}=T-0=M a \\ & \\ & \sum F_{y}=F_{N}-M g=0\end{aligned}$

System \#2 top H block $\quad \sum F_{x}=0$

$$
\sum F_{y}=T-m g=-m a
$$

Combine and solve for a and T :

$$
\begin{aligned}
a & =\frac{m}{M+m} g \\
T & =m g-m a
\end{aligned}
$$

## Problem 5-39:

Elevator and its load have a combined mass of 1600 kg . Find the tension in the supporting cable when the elevator, originally moving downward at $12 \mathrm{~m} / \mathrm{s}$, is brought to rest with constant acceleration in a distance of 42 m .

Combination of kinematics and dynamics

Kinematics
Free-body diagram


$$
\begin{aligned}
& v_{0}=-12 \mathrm{~m} / \mathrm{s}, \quad v=0, \quad y-y_{0}=-42 \mathrm{~m} \\
& v^{2}-v_{0}^{2}=2 a\left(y-y_{0}\right) \\
& a=\frac{-v_{0}^{2}}{2\left(y-y_{0}\right)}=1.71 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Dynamics: Newton's $2^{\text {nd }}$ law

$$
\begin{aligned}
& T-m g=m a \\
& T=m(g+a)=18416 N
\end{aligned}
$$

Problem \#32: A 100 kg crate is pushed at constant speed up the frictionless $30.0^{\circ}$ ramp by a horizontal force $\vec{F}$. What the magnitudes of
a) $\vec{F}$ and
b) the force on the crate from the ramp?


We resolve this horizontal force into appropriate components.
(a) Newton's second law applied to the $x$-axis produces

$$
F \cos \theta-m g \sin \theta=m a
$$

For $a=0$, this yields $F=566 \mathrm{~N}$.

(b) Applying Newton's second law to the $y$ axis (where there is no acceleration), we have

$$
F_{N}-F \sin \theta-m g \cos \theta=0
$$

which yields the normal force $F_{N}=1.13 \times 10^{3} \mathrm{~N}$.


## Man in a Basket

A man hoists himself up in a basket. What tension T must he apply before he starts to accelerate?


From the diagram, $W=3 T$ so $T=\frac{1}{3} W$

What net force is applied to the ceiling?
(b) Free-body diagram of the unit

Problem 5-60: A man sits in a Bolsun's chair that hings from a massless rope, which runs over a massless, frictionless pulley and back down to the man's hand. The combined mass of the man and chair is 95.0 kg . With what force (magnitude must the man pull on the rope if he is to raise (a) with a constant velocity and (b) with an upward acceleration of $1.30 \mathrm{~m} / \mathrm{s}^{2}$.

The motion of the man-and-chair is positive if upward.
(a) When the man is grasping the rope, pulling with a force equal to the tensio rope, the total upward force on the man-and-chair due its two contact points with the rope is $2 T$. Thus, Newton's second law leads to

$$
2 T-m g=m a
$$

so that when $a=0$, the tension is $T=466 \mathrm{~N}$.
(b) When $a=+1.30 \mathrm{~m} / \mathrm{s}^{2}$ the equation in part (a) predicts that the tension will be $T=527 \mathrm{~N}$.

Problem 5-52: A constant horizontal Force $\mathbf{F}_{\mathrm{a}}$ is applied to block $\mathbf{A}$, which pushes against block B with a force of 20.0 N directed horizontally to the right. Same force is applied to block B: now block A pushes on Block A with a force of 10.0 N directed horizontally to the left. The blocks have a combined mass of 12.0 kg . What are the magnitudes of (a) their acceleration and (b) the force $\mathbf{F}_{\mathbf{a}}$ ?

(a)

(b)

We can ignore the vertical forces. For case (a) we have:

$$
\begin{align*}
& \vec{F}_{A}=\left(M_{A}+M_{B}\right) \vec{a}  \tag{b}\\
& \vec{F}_{A \rightarrow B}=M_{B} a=20 N
\end{align*}
$$

$$
\begin{aligned}
& \vec{F}_{a}=\left(M_{A}+M_{B}\right) \vec{a} \\
& \vec{F}_{B \rightarrow A}=M_{A} \vec{a}=10 \mathrm{~N} \\
& \vec{a}=2.5 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Combining these equations gives:

$$
\vec{F}_{a}=30 \hat{i} N
$$

Problem 5-52: A constant horizontal Force $\mathbf{F}_{\mathrm{a}}$ is applied to block $\mathbf{A}$, which pushes against block B with a force of 20.0 N directed horizontally to the right. Same force is applied to block B: now block A pushes on Block A with a force of 10.0 N directed horizontally to the left. The blocks have a combined mass of 12.0 kg . What are the magnitudes of (a) their acceleration and (b) the force $\mathbf{F}_{\mathrm{a}}$ ?

Look at Neuton's Third law for both cases:

(a)

(b)

$$
a=2.5 \mathrm{~m} / \mathrm{s}^{2}
$$

For case (a):

$$
a=2.5 \mathrm{~m} / \mathrm{s}^{2} \quad \vec{F}_{B \rightarrow A}=-20 \mathrm{Ni} \hat{i}
$$

$$
\vec{F}_{a}=30 N \hat{i}
$$

$$
\vec{F}_{A}-\vec{F}_{B \rightarrow A}=M_{A} \vec{a} \quad \vec{F}_{A \rightarrow B}=+20 N \hat{i}
$$

For case (b):

$$
\begin{aligned}
& a=2.5 \mathrm{~m} / \mathrm{s}^{2} \\
& \vec{F}_{A}-\vec{F}_{B}=M_{B} \vec{a}
\end{aligned}
$$

$$
\vec{F}_{B \rightarrow A}=+10 N \hat{i}
$$

$$
\vec{F}_{A \rightarrow B}=-20 N \hat{i}
$$

Problem \#65: The figure shows three blocks attached by cords that loop over frictionless pulleys. Block B lies on a frictionless table; the masses are $\mathrm{m}_{\mathrm{A}}=6.00 \mathrm{~kg}, \mathrm{~m}_{\mathrm{B}}=8.00 \mathrm{~kg}$ and $\mathrm{m}_{\mathrm{C}}=10 . \mathrm{kg}$. What is the tension is the rope at the right? What is the acceleration of block B?


Two Tensions $T_{L}$ and $T_{R}$. Three different Force Equations with up positive on A and C and left to right positive for B . Draw these!

$$
\begin{gathered}
T_{L}-M_{A} g=M_{A} a \quad T_{R}-T_{L}=M_{B} a \quad M_{C} g-T_{R}=M_{C} a \\
m_{A} \vec{g} \stackrel{\mathrm{~m}_{\mathrm{A}}+\mathrm{m}_{\mathrm{B}}+\mathrm{m}_{\mathrm{C}}}{\longleftrightarrow} m_{C} \vec{g}
\end{gathered}
$$

Find a: $\quad a=\frac{M_{C}-M_{A}}{M_{A}+M_{B}+M_{C}} g=(0.167) 9.8=1.63 \mathrm{~m} / \mathrm{s}^{2}$

Find $\mathrm{T}_{\mathrm{R}}: \quad T_{R}=M_{C}(g-a)=10(8.13)=81.7 N$

Problem \#67: The figure shows, as a function of time t , the force component $\mathrm{F}_{\mathrm{x}}$ that acts on a 3.00 kg ice block that can move only along the x axis. At $\mathrm{t}=0 \mathrm{~s}$, the block is moving in the positive direction of the axis, with a speed of $3.0 \mathrm{~m} / \mathrm{s}$. What are its (a) speed and (b) direction of travel at $\mathrm{t}=11 \mathrm{~s}$.

What are you going to do??? How can you solve this?

$$
a=\frac{d v}{d t}=F / \mathrm{m}
$$

$$
v(t)=\int_{0}^{t} \frac{F}{m} d t
$$


(a) The acceleration (which equals $F / m$ in this problem) is the derivative of the velocity. Thus, the velocity is the integral of $F / m$, so we find the "area" in the graph (15 units) and divide by the mass (3) to obtain $v-v_{o}=15 / 3=5$.
Since $v_{o}=3.0 \mathrm{~m} / \mathrm{s}$, then
(b) Our positive answer in part (a) implies points in the $+x$ direction.

