# Physics 2101
# Section 3
# May 7th: Chap. 20


Chapter 20—Irreversible processes

**Announcements:**

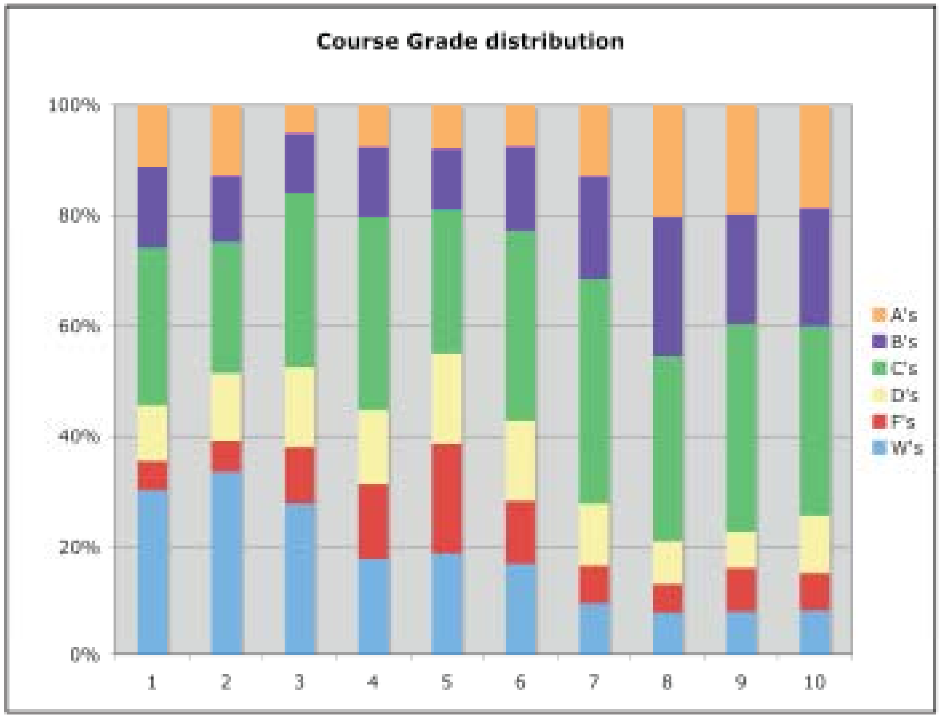
- Final Exam: May 11th (Tuesday), 7:30 AM at Howe-Russell 130
- Make up Final: May 15th (Saturday) 7:30 AM at Nicholson 119
- Final Exam for those who need extended time will be at Nicholson 109
- Review: Saturday 2PM at Nicholson 130

**Class Website:**

http://www.phys.lsu.edu/classes/spring2010/phys2101-3/

http://www.phys.lsu.edu/~jzhang/teaching.html

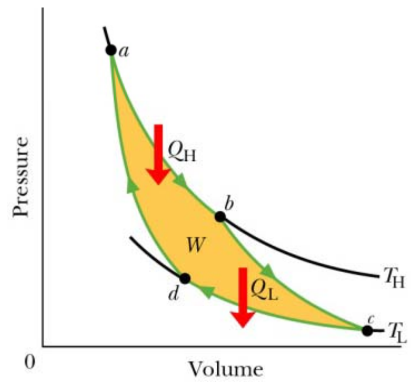
History of grades in 2101 for last 5 years:




**The rough guide lines are**
**A: < 18 %**
**B: 17 – 25 %**
**(with A+B < 40%)**
**C: 33-42 %**

I will post on Webassign you standing in the class ,i.e. 15 out of 59. This will give you a way to determine where you are

# Entropy: Reversible processes




1) For reversible process:

$$\Delta S_{cycle,rev} = 0 = \oint \frac{dQ}{T}$$

2) For isothermal gas process:

$$\Delta S_{isothermal} = \frac{Q}{T}$$

$$\Delta E_{int} = 0 \quad \Rightarrow \quad Q = W$$

$$W = nRT \ln\left(\frac{V_f}{V_i}\right)_{isothermal} \quad \Rightarrow \quad \Delta S_{isothermal} = nR \ln\left(\frac{V_f}{V_i}\right)$$

3) For liquids/solids - heat transfer:
(assuming no volume change)

$$Q_{heat\ capacity} = mc(T_f - T_i)$$
$$dQ_{heat\ capacity} = mc(dT)$$

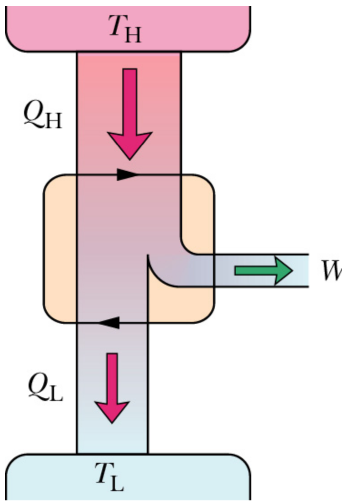
$$\Delta S_{\substack{liquid/solid \\ heat\ transfer}} = S_f - S_i = mc \ln\left(\frac{T_f}{T_i}\right)$$

4) For phase transition:

$$Q_{phase\ transition} = mL$$

$$\Delta S_{phase\ transition} = S_f - S_i = \frac{mL}{T}$$

# Heat Engines and Entropy




-Easy to produce thermal energy by doing work. How?

-Much harder to get work from thermal energy $\Rightarrow$ **engine**

<u>Conservation of energy</u>

$|Q_H| = |Q_L| + |W|$

$|Q_H|$ = heat added

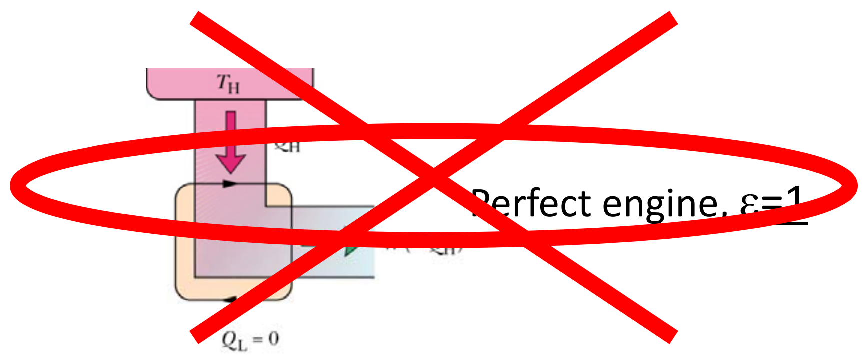
$|Q_L|$ = heat released

**2nd Law: There is no perfect heat engine (Kelvin-Plank statement)**

<u>What is the thermal efficiency, ε, of an engine?</u>

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|}$$

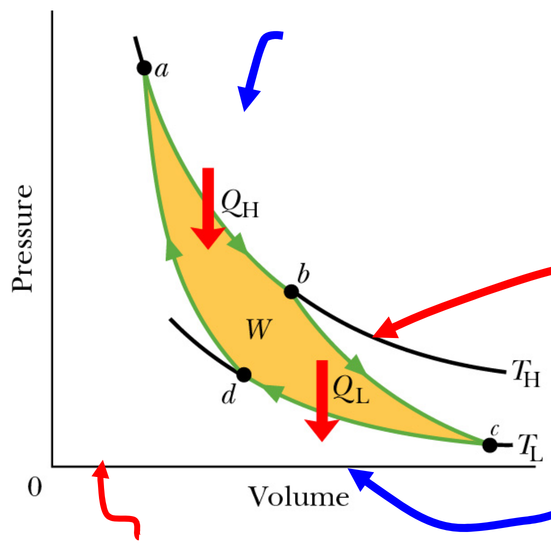
$$\varepsilon = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}$$



# How close to ideal can we get? Carnot Engine

All processes are reversible and no wasteful energy transfers occur

$$\varepsilon = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}$$




**1) a → b: Isothermal expansion**

$$\Delta E_{\text{int}} = 0 \quad \Rightarrow \quad Q_H = W = nRT_H \ln\left(\frac{V_b}{V_a}\right) > 0$$

$$\Delta S_H = \frac{Q_H}{T_H} = nR \ln\left(\frac{V_b}{V_a}\right) \quad > 0 \ \text{ positive}$$

**2) b → c: adiabatic expansion**

$$Q = 0 \quad \Rightarrow \quad \Delta S_{b\rightarrow c} = 0$$

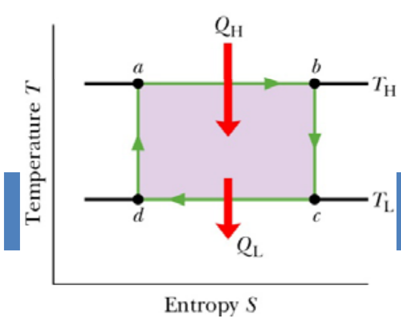
**3) c → d: Isothermal compression**

$$\Delta E_{\text{int}} = 0 \quad \Rightarrow \quad Q_L = W = nRT_L \ln\left(\frac{V_d}{V_c}\right) < 0$$

$$\Delta S_L = \frac{Q_L}{T_L} = nR \ln\left(\frac{V_d}{V_c}\right) \quad < 0 \ \text{ negative}$$

**4) d → a: adiabatic compression**

$$Q = 0 \quad \Rightarrow \quad \Delta S_{d\rightarrow a} = 0$$




$$0 = \Delta S_{cycle} = \Delta S_H + \Delta S_L$$

$$0 = \frac{|Q_H|}{T_H} - \frac{|Q_L|}{T_L}$$

$$\varepsilon_{carnot} = 1 - \frac{T_L}{T_H}$$

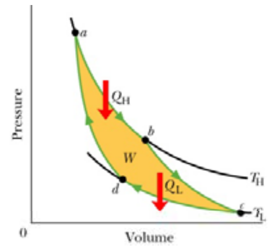
All Carnot engines operating between two constant temperatures $T_H$ & $T_L$ have the same efficiency.

An **irreversible** engine is less efficient

# Problem #1

$$\varepsilon = \frac{|W|}{|Q_H|} = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}$$

$$\varepsilon_{carnot} = 1 - \frac{T_L}{T_H}$$




Imagine a Carnot engine that operates between the temperatures of $T_H$ = 850 K and $T_L$ = 300 K. The engine performs 1200 J of work each cycle, which takes 0.25 s.

a) What is the efficiency of this engine?

The efficiency, ε, of a Carnot engine is ONLY determined by the ratio $T_L/T_H$:

$$\varepsilon = 1 - \frac{T_L}{T_H} = 1 - \frac{300\text{ K}}{850\text{ K}} \cong 65\%$$

b) What is the average power of this engine?

Power is found from Work done per cycle/time per cycle:

$$P_{ave} = W/\Delta t = \left(1200\text{ J}/0.25\text{ s}\right) = 4800\text{ W}$$

c) How much energy $|Q_H|$ is extracted as heat from the high temperature reservoir every cycle?

For any engine, the efficiency, ε, is defined as the work the engine does per cycle divided by the energy it absorbs as heat per cycle:

$$\varepsilon = \frac{|W|}{|Q_H|} \rightarrow |Q_H| = \frac{|W|}{\varepsilon} = \frac{1200\text{ J}}{65\%} \cong 1855\text{ J}$$

d) How much energy $|Q_L|$ is delivered as heat to the low temperature reservoir every cycle?

For a Carnot engine, from the 1st Law of thermo says that the net heat transfer per cycle is equal to the net work done:

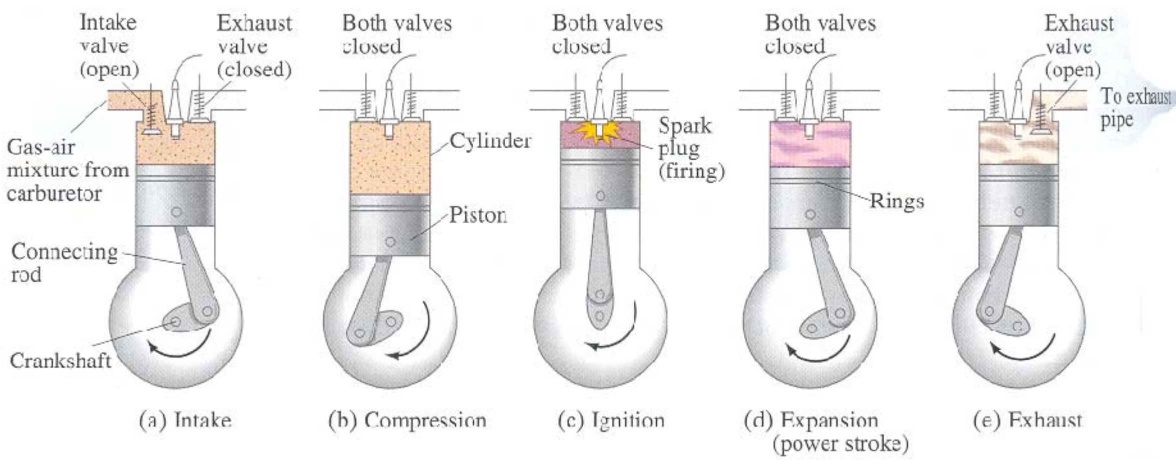
$$W = |Q_H| - |Q_L| \rightarrow |Q_L| = |Q_H| - W = 1855\text{ J} - 1200\text{ J} = 655\text{ J}$$

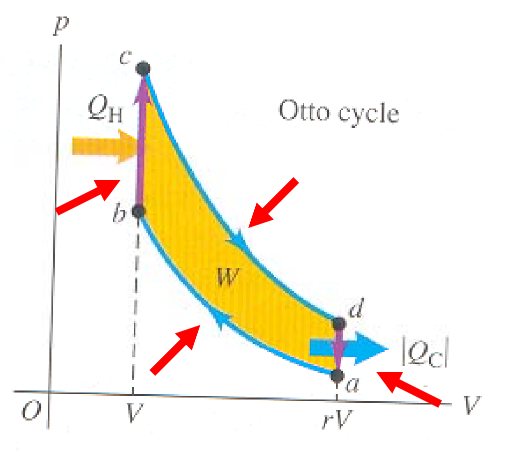
e) What is the entropy change ΔS of the working substance for the energy transfer to it from the high-temperature and low-temperature reservoir?

Along the hot and cool isotherms, the entropy changes are:

$$\Delta S_L = \frac{Q_L}{T_L} = \frac{-655\text{ J}}{300\text{ K}} = -2.18\text{ J/K} \qquad \Delta S_H = \frac{Q_H}{T_H} = \frac{1855\text{ J}}{850\text{ K}} = 2.18\text{ J/K} \qquad \rightarrow \Delta S_{tot} = 0$$

# Otto cycle: 4-stroke internal combustion engine




a→ b : adiabatic compression stroke

b → c : isochoric expansion due to fuel ignition

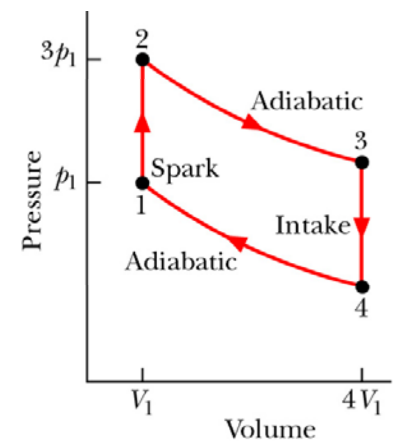
c → d : adiabatic expansion stroke

d → a : isochoric compression due to exhaust valve

**What is the efficiency of the Otto cycle?**

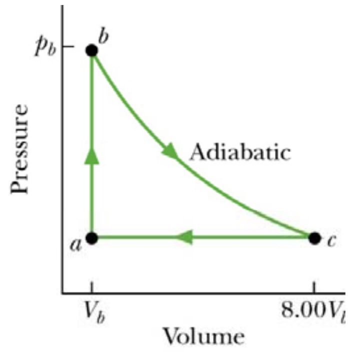
Assume the compression ratio of 4:1 ($V_4$=4$V_1$), the gas mixture can be regarded as an ideal gas monatomic gas:

**Theoretical : 40%**
**Reality : 20%**



# Problem

$$\varepsilon = \frac{|W|}{|Q_H|} = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}$$




**Problem 20-29** One mole of a monatomic ideal gas is taken through the reversible cycle shown. $V_c$=8.00$V_b$ , $p_b$=10.0 atm and $V_b$=$10^{-3}m^3$.) What's the energy added to the gas as heat? b) the energy leaving the gas as heat, c) What's the net work done by the gas? d) What's the efficiency of the cycle?

At the state b: $P_b V_b = RT_b$ so that $T_b = P_b V_b / R$

$$b \Rightarrow c\text{: } T_b V_b^{\gamma-1} = T_c V_c^{\gamma-1} \ (\gamma = \frac{C_p}{C_v} = \frac{5}{3}),\ \text{ so that } T_c = T_b (\frac{V_b}{V_c})^{\gamma-1} = T_b (\frac{1}{8})^{\gamma-1} = \frac{T_b}{8^{\gamma-1}}$$

$$c \Rightarrow a\ (p_a = p_c):\ \frac{V_a}{V_c} = \frac{T_a}{T_c};\ \text{ so } T_a = (\frac{V_a}{V_c})T_c = (\frac{V_b}{V_c})T_c = \frac{T_c}{8} = \frac{T_b}{8^{\gamma}}$$

Energy added to the gas as heat:

$$Q_{ab} = nC_V \Delta T_{ab} = \frac{3R}{2}(T_b - T_a)$$

Energy leaving from the gas as heat:

$$Q_{ca} = nC_p \Delta T_{ca} = \frac{5R}{2}(T_c - T_a)$$

Net work:

$$W_{net} = W_{bc} + W_{ca} = R(T_b + T_a - 2T_c)$$

$$W_{bc} = -(\Delta E_{in})_{bc} = R(T_b - T_c)$$

$$W_{ca} = p_a \Delta V_{ca} = R\Delta T_{ac} = R(T_a - T_c)$$

# Problem #1

**Entropy change with phase transition**

What is the entropy change of a 12.0 g ice cube that melts completely in a bucket of water whose temperature is just above the freezing point of water?

How do we find the change in entropy? $$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T}$$

In this case, what is the heat transferred to the system?? $$Q_{phase\ transition} = mL$$

Plugging this in, we can calculate the entropy change:
(this is a reversible isothermal process!!) $$\Delta S_{phase\ change} = \frac{mL}{T}$$

Thus:

$$\Delta S_{ice} = \frac{(12\ \mathrm{g})(333\ \mathrm{J/g})}{(273\ \mathrm{K})} = 14.6\ \mathrm{J/K} \geq 0$$

# Problem #2

**Solid: entropy change with temperature**

A 50.0 g block of copper whose temperature is 400 K is placed in an insulating box with a 100 g block of lead whose temperature is 200 K ($c_{Cu}$=386 J/kg/K & $c_{Pb}$=128 J/kg/K )

a) What is the equilibrium temperature of the two-block system? Review question

We know from Chapt. 19 that in an insulating box, the net heat flow is zero, thus:

$$\sum Q = 0 = m_{Cu}c_{Cu}\left(T_f - T_{i,Cu}\right) + m_{Pb}c_{Pb}\left(T_f - T_{i,Pb}\right)$$

$$T_f = \frac{m_{Cu}c_{Cu}T_{i,Cu} + m_{Pb}c_{Pb}T_{i,Pb}}{m_{Cu}c_{Cu} + m_{Pb}c_{Pb}} = 320\text{ K}$$

b) What is the change in internal energy of the <u>two-block system</u> between the initial state and the equilibrium state?

Since the system (Cu + Pb) is thermally insulated and no work is done, the change in the internal energy is zero!

c) What is the change in the entropy of the <u>two-block system</u>?

Knowing $\Delta S_{rev} = S_f - S_i = \int_i^f \frac{dQ}{T}$ what is the heat transfer?

$$dQ_{heat\ capacity} = mc(dT)$$

$$\Delta S = \int_i^f \frac{mc(dT)}{T} = mc\ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S_{tot} = \Delta S_{Cu} + \Delta S_{Pb} = m_{Cu}c_{Cu}\ln\left(\frac{320\text{ K}}{400\text{ K}}\right) + m_{Pb}c_{Pb}\ln\left(\frac{320\text{ K}}{200\text{ K}}\right)$$

$$= -4.3\text{ J/K} + 6.0\text{ J/K}$$

$$= 1.7\text{ J/K} \quad \text{(positive!!)}$$

# Problem #3

**Gas: entropy change during process**

An ideal gas undergoes an isothermal expansion at 77°C from 1.3 l to 3.4 l. Then entropy change during this process is 22 J/K. How many moles of gas are there?

What is the change in entropy during an isothermal process?

$$\Delta S_{isothermal} = \int_i^f \frac{dQ}{T} = \frac{Q}{T} \qquad \Delta E_{\text{int},isothermal} = 0$$

What do we know from 1st Law of Thermo?

$$\Delta E_{\text{int},isothermal} = 0 \;\Rightarrow\; Q = W = \int p dV \;\Rightarrow Q = nRT\ln\left(\frac{V_f}{V_i}\right)$$

Putting together, we find that:

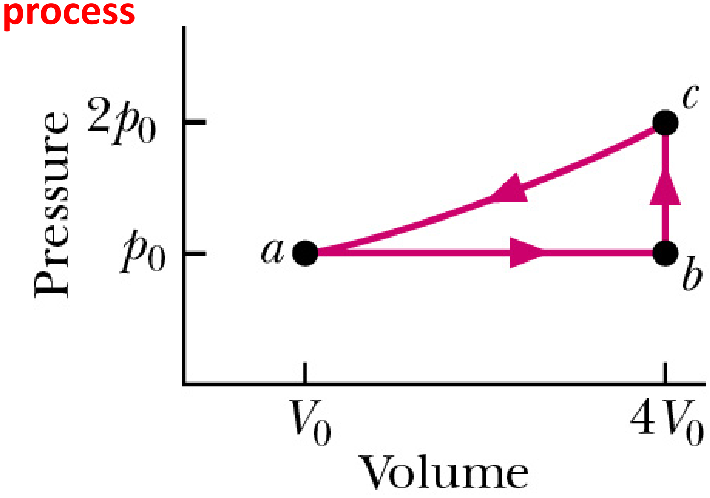
$$n = \frac{\Delta S_{isothermal}}{R\ln\left(\frac{V_f}{V_i}\right)} = \frac{22\ \text{J/K}}{(8.31\ \text{J/mole}\cdot\text{K})\ln\left(\frac{3.4\ \text{l}}{1.3\ \text{l}}\right)} = 2.75\ \text{moles}$$

# Problem #4

**Gas: entropy change during process**

One mole of an ideal monatomic gas is taken through the cycle shown.

a) How much work is done by the gas in going from state a to state c along abc?




Work done is: $W_{net} = W_{a\to b} + W_{b\to c} = \left[ \int_{V_0}^{4V_0} p_0 dV \right]_{isochoric} + \left[ \int_{4V_0}^{4V_0} pdV \right]_{isobaric}$

$$W_{net} = W_{a\to b} + W_{b\to c} = (4V_0 - V_0)p_0 + 0 = 3V_0 p_0$$

What are the changes in internal energy and entropy going:

b) from b to c ?

Internal energy is given by change in temperature:

$$\Delta E_{int,b\to c} = \tfrac{3}{2} nR\Delta T = \tfrac{3}{2} nR(T_f - T_i)$$
$$= \tfrac{3}{2} nR\left( \frac{2p_0 \cdot 4V_0}{nR} - \frac{p_0 \cdot 4V_0}{nR} \right) = 6p_0 V_0$$

The entropy is found from:

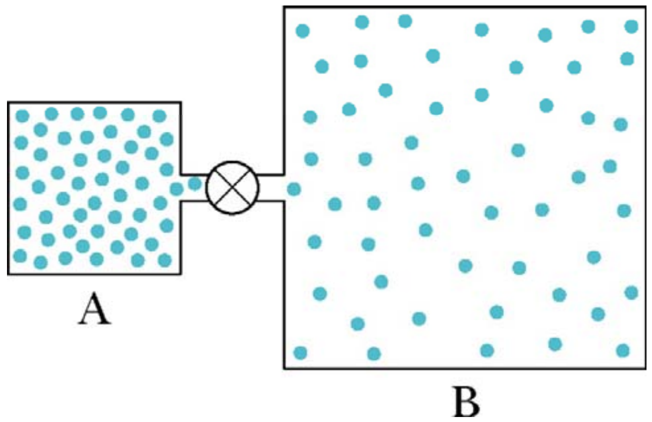
$$\Delta S_{b\to c} = \int_i^f \frac{dQ}{T} = \int_i^f \frac{[dE_{int} + W_{by}]}{T}$$
$$= \left[ \int_i^f \frac{[\tfrac{3}{2} nRdT + 0]}{T} \right]_{isochoric} = \tfrac{3}{2} nR \ln\left( \frac{T_f}{T_i} \right)_{\substack{isochoric \\ expansion}}$$
$$= \tfrac{3}{2} R \ln\left( \frac{T_f}{T_i} \right) = \tfrac{3}{2} R \ln\left( \frac{8p_0 V_0}{4p_0 V_0} \right)$$
$$= \tfrac{3}{2} R \ln(2)$$

c) through one complete cycle ?

For a complete cycle:

$$\Delta E_{int,cycle} = \Delta S_{rev,cycle} = 0$$

19-17: Container A holds an ideal gas at a pressure of $5.0 \times 10^5$ Pa at a T= 300 K. It is connected to a thin tube and a closed valve to container b, with four times the volume of A. Container B holds the same ideal gas at a pressure of $1.0 \times 10^5$ Pa and T=400K. The valve is opened to allow the pressures to equalize, but the temperature of each container is maintained. What then is the pressure in the two containers?




Use ideal gas Law

$$n = n_a + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right) = Const.$$

After valve is opened

$$p'_A = \frac{Rn'_A T_A}{V_A} = p'_B$$

$$p'_B = \frac{Rn'_B T_B}{4V_A}$$

or

$$n'_B = \left(\frac{4T_A}{T_B}\right) n'_A$$

$$n = n'_A + n'_B = n'_A\left(1 + \frac{4T_A}{T_B}\right) = n_a + n_B = \frac{V_A}{R}\left(\frac{p_A}{T_A} + \frac{4p_B}{T_B}\right)$$

$$n'_A = \frac{V}{R}\frac{\left(p_A/T_A + 4p_B/T_B\right)}{1 + 4T_A/T_B}$$

# Final Answer

$$p' = \frac{n'_A RT_A}{V_A} = \frac{p_A + p_B \dfrac{4T_A}{T_B}}{1 + \dfrac{4T_A}{T_B}}$$